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BASIC CONCEPTS

Course Contents

1.1 Introduction to Engineering Thermodynamics
1.2 Microscopic & Macroscopic Point of View
1.3 Thermodynamic System & Control Volume
1.4 Thermodynamic Properties, Processes & Cycles
1.5 Thermodynamic Equilibrium
1.6 Quasi-static Process
1.7 Heat and Work
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1. Basic Concepts

1.1 Introduction to Engineering Thermodynamics

- **Thermodynamics** is the branch of science that deals with energy transfer and its effect on the state or condition of the system.
- Thermodynamics, basically entails four laws known as Zeroth, First, Second and Third law of thermodynamics.
  - **Zeroth law** deals with thermal equilibrium, relates to the concept of equality of temperature.
  - **First law** pertains to the conservation of energy and introduces the concept of internal energy.
  - **Second law** relates the direction of flow of heat, dictates limits on the conversion of heat into work and introduces the principle of increase of entropy.
  - **Third law** defines the absolute zero of entropy.
- These laws are based on experimental observations and have **No Mathematical Proof**.

**Application Areas of Engineering Thermodynamics**

- All activities in nature involve some interaction between energy and matter; thus, it is hard to imagine an area that does not relate to thermodynamics in some manner.
- Thermodynamics is commonly encountered in many engineering systems and other aspects of life, and one does not need to go very far to see some application areas of it. In fact, one does not need to go anywhere. The heart is constantly pumping blood to all parts of the human body, various energy conversions occur in trillions of body cells, and the body heat generated is constantly rejected to the environment. The human comfort is closely tied to the rate of this metabolic heat rejection. We try to control this heat transfer rate by adjusting our clothing to the environmental conditions.
- Some of the selected areas of application of engineering thermodynamics are:
  - Automobile engines
  - Turbines, Compressors & Pumps
  - Propulsion system for aircraft and rockets
  - Combustion systems
  - HVAC systems: Vapor compression & absorption refrigeration, Heat pumps
  - Cooling of electronic equipments
  - Power stations: Nuclear, Thermal, etc.
  - Alternative energy systems
  - Biomedical applications: Life-support systems, Artificial organs
Fig. 1.1 Application areas of engineering thermodynamics
## 1.2 Macroscopic and Microscopic Point of View

- It is well known that every substance is composed of a large number of molecules. The properties of the substance depend on the behavior of these molecules.
- The behavior of a system may be investigated from either a microscopic (Micro means small) or macroscopic (Macro means big or total) point of view.
- These approaches are discussed below in a comparative way:

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<th>Macroscopic Approach</th>
<th>Microscopic Approach</th>
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<tr>
<td>1</td>
<td>In this approach a certain quantity of matter is considered without taking into account the events occurring at molecular level.</td>
<td>The matter is considered to be comprised of a large number of tiny particles known as molecules, which moves randomly in chaotic fashion. The effect of molecular motion is considered.</td>
</tr>
<tr>
<td>2</td>
<td>Analysis is concerned with overall behavior of the system.</td>
<td>The Knowledge of the structure of matter is essential in analyzing the behavior of the system.</td>
</tr>
<tr>
<td>3</td>
<td>This approach is used in the study of classical thermodynamics.</td>
<td>This approach is used in the study of statistical thermodynamics.</td>
</tr>
<tr>
<td>4</td>
<td>A few properties are required to describe the system.</td>
<td>Large numbers of variables are required to describe the system.</td>
</tr>
<tr>
<td>5</td>
<td>The properties like pressure, temperature, etc. needed to describe the system, can be easily measured.</td>
<td>The properties like velocity, momentum, kinetic energy, etc. needed to describe the system, cannot be measured easily.</td>
</tr>
<tr>
<td>6</td>
<td>The properties of the system are their average values.</td>
<td>The properties are defined for each molecule individually.</td>
</tr>
<tr>
<td>7</td>
<td>This approach requires simple mathematical formulas for analyzing the system.</td>
<td>No. of molecules are very large so it requires advanced statistical and mathematical method to explain any change in the system.</td>
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</table>

- The macroscopic properties are the average properties of a large number of microscopic characteristics. Obviously, when both the methods are applied to a practical system, they give the same result.
1.3 Thermodynamic System and Control Volume

**Thermodynamic System**

“It is defined as a quantity of matter or a region in the space upon which attention is concentrated for the investigation or analysis of the thermodynamic problems i.e. heat transfer, work transfer, etc.”

**Surroundings or Environment**

“It is the matter or region outside the system”

**Boundary**

“The system and surroundings are separated by an envelope called boundary of the system”

**Types of boundary**

- Fixed or moving boundary
- Real or imaginary boundary

**Types of Thermodynamic System**

A. **Open System**

- In an open system mass and energy (in form of heat and work) both can transfer across the boundary.
- Most of the engineering devices are open system.
- **Examples:** Boiler, Turbine, Compressor, Pump, I.C. Engine, etc.

B. **Closed System**

- A closed system can exchange energy in the form of heat and work with its surroundings but there is **no mass transfer** across the system boundary.
- The mass within the system remains constant though its volume can change against a flexible boundary.
- Further, the physical nature and chemical composition of the mass may change.
- **Examples:** Cylinder bounded by a piston with certain quantity of fluid, Pressure cooker and Bomb calorimeter, etc.

C. **Isolated System**

- There is no interaction between system and surroundings.
- It is of fixed mass and energy, and hence there is no mass and energy transfer across the system boundary.
- **Examples:** The Universe and Perfectly insulated closed vessel (Thermo flask).
D. Adiabatic System

- Boundaries do not allow heat transfer to take place across them.
- An adiabatic system is thermally insulated from its environment.
- It can exchange energy in the form of work only. If it does not, it becomes isolated.
- **Example:** A perfectly insulated piston-cylinder arrangement.

![Diagram of thermodynamic systems]

**Fig. 1.3 Types of thermodynamic systems**

E. Homogeneous & Heterogeneous System

**Homogeneous System**

“A system which consists of a single phase is termed as homogeneous system.”

**Examples:**
- Mixture of air and water vapor
- Water + Nitric acid

**Heterogeneous System**

“A system which consists of two or more phases is called a heterogeneous system.”

**Examples:**
- Water + Steam
- Ice + Water
- Water + Oil
**Control Volume Concept**

- For thermodynamic analysis of an open system, such as an air compressor, turbine, etc. attention is focused on a certain volume in space surrounding the system, known as control volume.
- The control volume bounded by the surface is called “Control Surface”.
- Both mass and energy can cross the control surface. It may be physical or imaginary.

**Example of Control Volume:**
- Consider an air compressor (open system) as shown in Fig. 1.4. Since compressed air will leave the compressor and be replaced by fresh air, it is not convenient to choose a fixed mass as our system for the analysis.
- Instead we can concentrate our attention on the volume formed by compressor surfaces and consider the compressed air and fresh air streams as mass leaving and entering the control volume.

![Fig. 1.4 An example of control volume](image)

**Difference between System and Control Volume**

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<th>Control Volume</th>
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<tbody>
<tr>
<td>1</td>
<td>A system is a defined quantity of matter which is considered to analyze the problem.</td>
<td>A control volume is a certain volume which is considered to analyze the problem.</td>
</tr>
<tr>
<td>2</td>
<td>The system is separated from its surrounding by a boundary which may be real or imaginary and may change shape, volume and position relative to observer.</td>
<td>The C.V. is separated from its surrounding by a control surface which may be real or imaginary and normally fixed in shape &amp; position relative to observer.</td>
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1.4 Thermodynamic Properties, Processes and Cycles

**Thermodynamic Properties**

“A thermodynamic property refers to the characteristics which can be used to describe the physical condition or state of a system.”

**Examples** of thermodynamic properties are: Temperature, Pressure, Volume, Energy, Mass, Viscosity, Thermal conductivity, Modulus of elasticity, velocity, etc.

**Salient Aspects of a Thermodynamic Property**

- It is a *macroscopic characteristic* of the system.
- It has a unique value when the system is in a particular state, and this value does not depend on the previous states that the system passed through; that is, it is not a path function but it is a point function.
- Since a property is not dependent on the path, any change depends only on the initial and final states of the system. *Hence its differential is exact.*

**Types of Thermodynamic Properties**

1. **Intensive Property**
   - Intensive property is independent of the mass of the system. Its value remains same whether one considers the whole system or only a part of it.
   - **Examples:** Pressure, Temperature, Density, Viscosity, Thermal conductivity, Electrical potential, etc.

2. **Extensive Property**
   - Extensive property depends on the mass of the system.
   - **Examples:** Mass, Energy, Enthalpy, Volume, Entropy, etc.

3. **Specific Property**
   - Extensive properties per unit mass are called specific properties.
   - **Examples:** Specific volume \( v = \frac{V}{m} \) and specific total energy \( e = \frac{E}{m} \).

**Note:**

An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition, as shown in Fig. 1.5. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.
State

- “State refers to the condition of a system as described by its properties.” It gives a complete description of the system. At a given state, all the properties of a system have fixed values.
- If the value of even one property changes, the state will change to a different one, any such kind of operation is called Change of state.

Process and Path

- Any change that a system undergoes from one equilibrium state to another is called a process, and the series of states through which a system passes during a process is called the path of the process.
- To describe a process completely, one should specify the initial and final states of the process, as well as the path it follows, and the interactions with the surroundings.
- There are infinite ways for a system to change from one state to another state.

Cycle

- When a system in a given initial state goes through a number of different changes of state or processes and finally returns to its initial state, the system has undergone a cycle. Thus for a cycle the initial and final states are identical.
- Example: Steam (water) that circulates through a steam power plant undergoes a cycle.

Point Function

- When two properties locate a point on the graph (Co-ordinate axis) then those properties are called as Point Function.
- Examples: Pressure, Volume, Temperature, etc.
- It can be represented by an exact differential. i.e. \( \int_{1}^{2} dV = V_2 - V_1 \)
**Path Function**

- There are certain quantities which cannot be located on a graph (Co-ordinate axis) by a point but are given by the area or so, on that graph.

- In that case, the area on the graph, pertaining to the particular process, is a function of the path of the process, such quantities are called Path Functions.

- **Examples:** Heat, Work, etc.

- It can be represented by an inexact differential. Their change can not be written as difference between their end states.

- Thus,

\[ \int_1^2 \delta W \neq W_2 - W_1 \text{ and is shown as } W_{1-2} \]

\[ \int_1^2 \delta Q \neq Q_2 - Q_1 \text{ and is shown as } Q_{1-2} \]

- **Note:**
  The operator \( \delta \) is used to denote inexact differentials and \( d \) is used to denote exact differentials.

### 1.5 Thermodynamic Equilibrium

- A system is said to be in a state of thermodynamic equilibrium, if the conditions for the following three types of equilibrium are satisfied simultaneously:

  - **Mechanical Equilibrium:** There are no unbalanced forces within the system or between the surroundings. The pressure in the system is same at all points and does not change with respect to time.
  
  - **Thermal Equilibrium:** The temperature of the system does not change with time and has same value at all points of the system.
  
  - **Chemical Equilibrium:** No chemical reaction takes place in the system and the chemical composition which is same throughout the system does not vary with time.

- A system in thermodynamic equilibrium does not deliver anything.

### 1.6 Quasi-Static Process OR Quasi-Equilibrium Process

- "**Quasi**" means Almost slow or Infinitely slow.

- Consider a system of gas contained in a cylinder fitted with a piston upon which many very small pieces of weights are placed as shown in Fig.1.9(a).

- The upward force exerted by the gas just balances the weights on the piston and the system is initially in equilibrium state identified by pressure \( P_1 \), volume \( V_1 \) and temperature \( T_1 \).
When these weights are removed slowly, one at a time, the unbalanced potential is infinitesimally small.

The piston will slowly move upwards and at any particular instant of piston travel, the system would be almost close to state of equilibrium.

Every state passed by the system will be an equilibrium state.

The locus of a series of such equilibrium states is called a “Quasi-Static or Quasi-Equilibrium process.”

It should be pointed out that a quasi-equilibrium process is an idealized process and is not a true representation of an actual process. But many actual processes closely approximate it, and they can be modeled as quasi-equilibrium with negligible error.

Engineers are interested in quasi-equilibrium processes for two reasons. First, they are easy to analyze; second, work-producing devices deliver the most work when they operate on quasi-equilibrium processes. Therefore, quasi-equilibrium processes serve as standards to which actual processes can be compared.

Fig. 1.9(b) shows the p-v diagram of a compression process of a gas.

A quasi-static process is also called a reversible process. This process is a succession of equilibrium states and infinite slowness is its characteristic feature.

1.7 Heat and Work

Energy can cross the boundary of a closed system in two distinct forms: heat and work. It is important to distinguish between these two forms of energy.
**Heat**

- "Heat is defined as the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference." Then it follows that there cannot be any heat transfer between two systems that are at the same temperature.
- The **temperature difference** is the **driving potential** for heat transfer.
- A process during which there is no heat transfer is called an **adiabatic process**. In an adiabatic process, energy content and the temperature of a system can be changed by other processes, such as work.
- All heat interaction need not to be result in temperature changes e.g. Evaporation and Condensation.

**Work**

- "An energy interaction between a system and its surroundings during a process can be considered as work transfer, if its sole effect on everything external to the system could have been to raise a weight."
- It is also a form of energy in transit like heat.

**Sign Convention for Heat & Work**

- Heat and Work are directional quantity, and its specification requires magnitude and direction both. Universally accepted sign convetions for heat and work energy are shown in Fig. 1.10.

1. Heat transferred to a system (heat supply) and Work done by a system is considered **positive**.
2. Heat transferred from a system (heat rejection) and Work done on a system is considered **negative**.

![Fig. 1.10 Sign convention for heat & work](image)

**Comparison of Heat and Work**

**Similarities:**

1. Both are recognized at the boundaries of a system as they cross the boundaries. That is, both heat and work are boundary phenomena.
2. Systems possess energy, but not heat or work.
3. Both are associated with a process, not a state. Unlike properties, heat or work has no meaning at a state.
4. Both are path functions (i.e. their magnitudes depend on the path followed during a process as well as the end states).
Dissimilarities:
1. In heat transfer temperature difference is required.
2. In a stable system there cannot be work transfer, however, there is no restriction for the transfer of heat.
3. The sole effect external to the system could be reduced to rise of a weight but in the case of a heat transfer other effects are also observed.

Different Forms of Work Transfer
1. Electrical work
2. Mechanical work
3. Moving boundary work
4. Flow work
5. Gravitational work
6. Acceleration work
7. Shaft work
8. Spring work

Some of the important forms of work transfer are discussed here:

Mechanical Work
- In mechanics work done by a system is expressed as a product of force \( F \) and displacement \( s \)
  \[ W = F \times s \]
- If the force is not constant, the work done is obtained by adding the differential amounts of work,
  \[ W = \int F \, ds \]
- The pressure difference is the driving force for mechanical work.

Moving Boundary Work / Displacement Work / \( pdV \) - Work
- In many thermodynamic problems, mechanical work is the form of moving boundary work. The moving boundary work is associated with real engines and compressors.
- Consider the gas enclosed in a frictionless piston cylinder arrangement as shown in Fig. 1.11. Let the initial gas pressure \( p_1 \) and volume \( V_1 \). The piston is the only boundary which moves due to gas pressure. Let the piston moves out to a new final position 2, specified by pressure \( p_2 \) and volume \( V_2 \). At any intermediate point in the travel of the piston, let the pressure be \( p \), volume \( V \) and piston cross sectional area is \( A \). When the piston moves through and infinitesimal distance \( ds \) in a quasi-equilibrium manner, the force applied on piston is,
  \[ F = p \times A \]
- Then differential work transfer through a displacement of \( ds \) during this process,
  \[ \delta W = F \times ds = p \times A \times ds = p \times dV \]
When piston moves out from initial state 1 to final state 2 with volume changing from $V_1$ to $V_2$, the total boundary work done by the system will be,

$$W_{1-2} = \int_{V_1}^{V_2} p\,dV \quad (kJ)$$

or

$$W_{1-2} = \int_{V_1}^{V_2} pdv \quad (kJ/kg)$$

This work transfer during a process is equal to the area under the curve on a $p-V$ diagram as shown in Fig. 1.11 (a).

**Flow Work**

Flow energy or flow work refers to work required to push a certain mass of fluid into and out of the control volume. It is necessary for maintaining continuous flow through a control volume.

Consider a fluid element of volume $V$, entering the control volume through a cross-sectional area $A$ as shown in Fig. 1.11 (b).

If $p$ is the fluid pressure acting uniformly at the imaginary piston at the entrance of the control volume, the force applied on the fluid element by imaginary piston is,

$$F = p \times A$$

If the fluid is pushed by a distance $L$, then the flow work will be,

$$W_f = p \times A \times L = p \times V$$

- Flow work at the entrance, $W_{f1} = p_1V_1$
- Flow work at the exit, $W_{f2} = p_2V_2$
Specific Heat

- “It is defined as heat energy required to change the temperature of the unit mass of a substance by one degree.” It is designated as C and measured in kJ/kg-K.
- In general, the specific heat can be calculated as,
  \[ C = \frac{1}{m} \frac{\delta Q}{dT} = \frac{\delta q}{dT} \]
- Gases have two specific heats, \( C_p \) and \( C_v \) but for liquids and solids, the specific volume is very small and its change with pressure and temperature is negligible, thus they have only one specific heat.

Zeroth Law of Thermodynamics

☞ “It states that if two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other.”
☞ The Zeroth law was first formulated and labeled by R. H. Fowler in 1931.

1.8 References

5. Thermal Science and Engineering by D. S. Kumar, S K Kataria and Sons Publications.
7. National Programme on Technology Enhanced Learning (NPTEL), A Joint Initiate by IIT’s and IISc. (Web: http://nptel.ac.in/)

“Have no fear of perfection, You will never reach it”
FIRST LAW OF THERMODYNAMICS

Course Contents

2.1 Introduction to 1st Law of Thermodynamics
2.2 First Law Applied to Cyclic Process – Joule’s Experiment
2.3 First Law Applied to a Process
2.4 Internal Energy: A Property of the System
2.5 First Law Applied to Steady Flow Processes
2.6 SFEE Applied to Engineering Applications
2.7 Unsteady Flow Processes: Filling & Emptying Process
2.8 First Law Applied to Non Flow Processes
2.9 Solved Numerical
2.10 References
2.1 Introduction to 1st Law of Thermodynamics

- The first law of thermodynamics, also known as the conservation of energy principle. It states that “Energy can neither be created nor destroyed; it can only change its form.”
- Total energy of an isolated system in all its form remains constant.
- The first law of thermodynamics cannot be proved mathematically but no process in nature is known to have violated the first law of thermodynamics.
- It is the relation of energy balance and is applicable to any kind of system (open or closed) undergoing any kind of process.

2.2 First Law Applied to a Cyclic Process – Joule’s Experiment

- **Cyclic Process**: “A process is cyclic if the initial and final states of the system executing the process are identical.”
- A system represented by a state point 1 undergoes a process 1-a-2, and comes back to initial state following the path 2-b-1.
- All properties of the system are restored, when the initial state is reached.
- During the execution of these processes:
  
  i. Area 1-a-2-3-4-1 represents the work done by the system \( W_1 \) during expansion process 1-a-2.
  
  ii. Similarly area 2-3-4-1-b-2 gives work supplied to the system \( W_2 \) during compression process 2-b-1.
  
  iii. Area 1-a-2-b-1 represents the net work \( W_1 - W_2 \) delivered by the system.
- Since the system regains its initial state, there is no change in the energy stored by the system.
- For a cyclic process, the First Law of Thermodynamics can be stated as follows:
  “When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to net work done by the system on its surroundings.”

Mathematically,

\[
\int \delta Q = \int \delta W - - - - - - - - (2.1)
\]
**Joule’s Experiment**

- The first law can be illustrated by considering the following experiment (Fig. 2.2).

![Fig. 2.2 Joule’s paddle-wheel experiment](image)

- A known mass of water is taken into a rigid and well insulated container provided with a paddle wheel.
- The insulation is provided to prevent any heat interaction with surroundings.
- The work input to the paddle wheel is measured by the fall of weight while the corresponding temperature rise of the liquid in the insulated container is measured by the thermometer.
- Joule conducted a number of experiments involving different types of work interactions and found that the work expended was proportional to increase in thermal energy, i.e.

\[ Q \propto W \]

\[ \therefore Q = \frac{W}{J} \]

\[ \therefore W = JQ \]

Where,
- \( J \) = Joule’s equivalent or mechanical equivalent of heat
- In SI system of units, both heat and work are measured in Joules.

### 2.3 First Law Applied to a Process

- The first law of thermodynamics is often applied to a process as the system changes from one state to another.
- According to first law of thermodynamics,
\[ \Delta E = Q - W \] (2.2)

Where,
\[ \Delta E = \Delta U + \Delta KE + \Delta PE + \text{other forms of energy} = \text{Net change in total energy of the system} \]

- If a closed system undergoes a change of state during which both heat and work transfer are involved, the net energy transfer will be stored or accumulated within the system. If \( Q \) is the heat transfer to the system and \( W \) is the work transferred from the system during process, the net energy transfer \( (Q - W) \) will be stored in the system. Energy in storage is neither heat nor work and is given the name "Internal Energy" or "Stored Energy" of the system.

\[ \therefore Q - W = \Delta U \] (2.3)

- Most closed systems in practice are stationary, i.e. they do not involve kinetic energy and potential energy during the process. Thus the stationary systems are called non-flow systems and the first law of thermodynamics is reduced to equation 2.3.

- In differential form first law of thermodynamics for a process can be written as,

\[ \delta Q - \delta W = \delta E \] (2.4)

- Also for a cyclic process \( \Delta U = 0 \), as the system regains its original state hence,

\[ Q - W = 0 \]

\[ \therefore Q = W \] (2.5)

### 2.4 Internal Energy: A Property of the System

- Consider a closed system which changes from state 1 to state 2 by path A and returns back to original state 1 by one of the following path as shown in Fig.2.3:
  (i) 2-B-1 (ii) 2-C-1 (iii) 2-D-1

![Fig. 2.3 Cyclic process with different paths](image-url)
Applying the 1st law for the cyclic process 1-A-2-B-1,
\[ \oint (\delta Q - \delta W) = 0 \]
\[ \therefore \int_{1, \text{via } A}^{2} (\delta Q - \delta W) + \int_{1, \text{via } B}^{2} (\delta Q - \delta W) = 0 \quad (2.6) \]

Similarly,
Applying the 1st law for the cyclic process 1-A-2-C-1,
\[ \therefore \int_{1, \text{via } A}^{2} (\delta Q - \delta W) + \int_{1, \text{via } C}^{2} (\delta Q - \delta W) = 0 \quad (2.7) \]

And,
Applying the 1st law for the cyclic process 1-A-2-D-1,
\[ \therefore \int_{1, \text{via } A}^{2} (\delta Q - \delta W) + \int_{1, \text{via } D}^{2} (\delta Q - \delta W) = 0 \quad (2.8) \]

Comparing equations 2.6, 2.7 and 2.8, we get,
\[ \int_{2, \text{via } B}^{1} (\delta Q - \delta W) = \int_{2, \text{via } C}^{1} (\delta Q - \delta W) = \int_{2, \text{via } D}^{1} (\delta Q - \delta W) \]

Since B, C and D represents arbitrary paths between the state point 2 and state point 1, it can be concluded that the integral \( \int_{2}^{1} (\delta Q - \delta W) \)

(i) Remains the same irrespective of the path along which the system proceeds,
(ii) Is solely dependent on the initial and final states of the system; is a point function and hence property.

The integral \( \int_{2}^{1} (\delta Q - \delta w) \) is called energy of the system and is given by a symbol \( E \).

Further the energy is a property of the system; its differential is exact and is denoted by \( dE \).

Thus for a process,
\[ \delta Q - \delta W = dE \]

The energy, \( E \) is an extensive property.

The specific energy \( \left(e = \frac{E}{m}\right) \) is an intensive property.
2.5 First Law Applied to Steady Flow Processes

Conservation of Mass Principle – Continuity Equation

- Conservation of mass is one of the most fundamental principles for flow systems. “It states that the mass of a system can neither be created nor destroyed but its amount remains constant during any process. It only changes its form (phase).”

- The conservation of mass principle for a control volume (CV) can be expressed as,

\[ \text{Total mass entering CV} - \text{Total mass leaving CV} = \text{Net change in mass within CV} \]

- The amount of mass flowing through a cross-section per unit time is called the mass flow rate and it is calculated as,

\[ \dot{m} = \frac{AC}{\nu} \]

Where,
\[ \dot{m} = \text{Mass flow rate in kg/sec}, \]
\[ A = \text{Cross-sectional area of flow in m}^2, \]
\[ \nu = \text{Specific volume of fluid in m}^3/\text{kg}, \]
\[ C = \text{Fluid velocity in m/sec}. \]

Further,
\[ \text{Specific volume} = \frac{1}{\text{Density}} \]
\[ \therefore \nu = \frac{1}{\rho} \]

Equation (2.9) can be expressed as,

\[ \dot{m} = \rho AC \]

- The volume flow rate through a cross-sectional area per unit time is called fluid discharge rate (Q),

\[ Q = AC \]

- For a steady flow,

\[ \dot{m} = \text{Constant} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 \]

Steady and Un-steady Flow Process

- A flow process is said to be steady when the fluid parameters (P) at any point of the control volume remains constant with respect to time; the parameters may, however, be different at different cross-section of the flow passage.

\[ \therefore \frac{\partial P}{\partial t} = 0 \]

- A flow process is un-steady when the conditions vary with respect to time.

\[ \therefore \frac{\partial P}{\partial t} \neq 0 \]
Steady Flow Energy Equation (SFEE)

- Assumptions
  The following assumptions are made in the steady flow system analysis:
  a) The mass flow through the system remains constant.
  b) Fluid is uniform in composition.
  c) The only interaction between the system and surroundings are work and heat.
  d) The state of fluid at any point remains constant with time.
  e) In the analysis only potential, kinetic and flow energies are considered.

Fig. 2.4 Schematic flow process for an open system

- Consider a flow of fluid through an open system as shown in Fig. 2.4.
- During a small time interval \( dt \) there occurs a flow of mass and energy into the fixed control volume; entry is at section 1 and exit occurs at section 2.
- The fluid enters the control volume at section 1 with average velocity \( C_1 \), Pressure \( P_1 \), Specific volume \( v_1 \), and Specific internal energy \( u_1 \).
- The corresponding values at the exit section 2 are \( C_2, P_2, v_2 \) and \( u_2 \).
- Further during, the fluid flow between the two selected sections, heat \((Q)\) and mechanical or shaft work \((W_s)\) may also cross the control surface.
- The following species of energy are taken into account while drawing up the energy balance:
A. Internal energy stored by the fluid = $U$

B. Kinetic energy = $\frac{1}{2} mc^2$

C. Potential energy = $mgZ$

D. Flow work = $P_1V_1$

E. Heat interaction = $Q$

F. Work interaction i.e. shaft work = $W_s$

According to 1st law of thermodynamics, energy balance in the symbolic form may be written as,

$$m_1 \left( u_1 + P_1 v_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m_2 \left( u_2 + P_2 v_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s \tag{2.12}$$

Equation (2.12) is the general steady flow energy equation (SFEE) and is equally applicable to compressible and incompressible; ideal and real fluids, liquids and gases.

But according to assumption (1),

$$m = m_1 = m_2$$

Also enthalpy,

$$h = u + Pv$$

$$\therefore m \left( h_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m \left( h_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s$$

SFEE can be written on the basis of unit mass or on the basis of unit time.

SFEE on unit mass basis:

$$h_1 + \frac{C_1^2}{2} + gZ_1 + q = h_2 + \frac{C_2^2}{2} + gZ_2 + w_s \tag{2.13}$$

Here, all the terms represents energy flow per unit mass of the fluid (J/kg)

### 2.6 SFEE Applied to Engineering Applications

- The SFEE applies to flow processes in many of the engineering applications, such as Turbines, Compressors, Pumps, Heat exchangers and flows through nozzles and diffusers.

- In certain flow processes, some of the energy terms in SFEE are negligibly small and can be omitted without much error.
1. Nozzles and Diffusers

- A nozzle is a device for increasing the velocity of a steadily flowing steam at the expense of its pressure and hence enthalpy.
- A diffuser is a device that increases the pressure of a fluid by slowing it down. That is nozzles and diffusers perform opposite task.
- Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses. Fig. 2.5 shows a commonly used convergent-divergent nozzle.

![Fig. 2.5 A convergent-divergent nozzle](image)

- Applying Steady Flow Energy Equation (SFEE),

\[ m \left( h_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m \left( h_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s \]

- The characteristic features of the flow through a nozzle are:
  - No shaft work; \( W_s = 0 \)
  - If walls are thermally insulated; \( Q = 0 \)
  - Nozzle is horizontal i.e No elevation difference between inlet and exit; \( Z_1 = Z_2 \)
- Hence, the SFEE is reduced to

\[ m \left( h_1 + \frac{C_1^2}{2} \right) = m \left( h_2 + \frac{C_2^2}{2} \right) \]

\[ \therefore \frac{C_2^2}{2} - \frac{C_1^2}{2} = h_1 - h_2 \]

\[ \therefore C_2^2 - C_1^2 = 2(h_1 - h_2) \]

\[ \therefore C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)} \]

If, \( C_1 << C_2 \), then,

\[ C_2 = \sqrt{2(h_1 - h_2)} \]

- Similar way SFEE can be reduced for diffusers also.
2. **Heat Exchangers**

- Condensers and Evaporators are the main types of heat exchangers.
- These are the devices where the objective is to transfer heat energy between hot and cold fluids. Therefore the heat transfer rate cannot be taken as zero.
- These devices are widely used in refrigeration system, air conditioning system, thermal power plant and various industries.
- A **steam condenser** is also a heat exchanger in which steam losses heat as it passes over the tubes through which cold fluid is flowing.
- An **evaporator** is also a heat exchanger and is used to extract heat from the cold places or fluids.
- **Boiler** is a type of evaporator and hence heat exchanger; used for the generation of steam. Thermal energy released by combustion of fuel is transferred to water which vaporizes and gets converted into steam at the desired pressure and temperature.

![Diagram of Steam Condenser, Boiler, and Evaporator](image)

**Fig. 2.6 Schematic diagram of (a) Boiler (b) Condenser (c) Evaporator**

- Applying Steady Flow Energy Equation (SFEE),
The characteristic features of the flow through a heat exchangers are:

- No shaft work; \( W_s = 0 \)
- Heat transfer, \( Q \neq 0 \) (Compulsory)
- Change in kinetic energy is negligible (compare to change in enthalpy);
  \[ \frac{C_2^2}{2} - \frac{C_1^2}{2} = 0 \]
- Change in potential energy is negligible (i.e. No elevation difference between inlet and exit); \( Z_1 = Z_2 \)

Hence, SFEE is reduced to,

\[ m(h_1 + Q) = m(h_2) \]

\[ Q = m(h_2 - h_1) \]

For condenser and evaporator, from energy balance equation,

\[ \text{Heat lost by the steam} = \text{Heat gained by the cooling water} \]

\[ m_s(h_{si} - h_{so}) = m_w(h_{wo} - h_{wi}) \]

Where,

\( m_s = \) Mass flow of steam

\( m_w = \) Mass flow of cooling water

3. Steam or Gas Turbine

- A turbine is a device for obtaining work from a flow of fluid expanding from high pressure to low pressure.

![Schematic of steam or gas turbine](image)

**Fig. 2.7 Schematic of steam or gas turbine**

- Applying Steady Flow Energy Equation (SFEE),
The characteristic features of flow through a steam or gas turbine are:

- Shaft work produced; \( W_s = +ve \)
- Negligible velocity change in the flow of fluid; \( C_1 = C_2 \)
- Negligible potential energy change; \( Z_1 = Z_2 \)
- No transfer of heat as its walls are thermally insulated; \( Q = 0 \)

Hence, SFEE is reduced to,

\[
m(h_1) + 0 = m(h_1) + W_s
\]

\[
W_s = m(h_1 - h_2)
\]

Apparently work is done at the expense of enthalpy.

4. Hydraulic Turbine

A hydraulic turbine or water turbine is a device which takes in water from a height. The water enters into the turbine, a part of its potential energy is converted into useful work (shaft work), which is used to generate electric power in a generator.

![Schematic of hydraulic turbine](image)

Fig. 2.8 Schematic of hydraulic turbine

Applying Steady Flow Energy Equation (SFEE),

\[
m\left( u_1 + P_1 v_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m\left( u_2 + P_2 v_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s
\]

The characteristic features of flow through a hydraulic turbine are:

- Shaft work produced; \( W_s = +ve \)
Negligible change in temperature of water so,
- Heat transfer rate from turbine; \( Q = 0 \)
- Change in specific internal energy; \( \Delta u = u_2 - u_1 = 0 \)

As water is an incompressible fluid, its specific volume and hence density will remain constant; \( v_1 = v_2 = v \)

- Hence, SFEE is reduced to,

\[
W_s = m \left[ (P_1 v_1 - P_2 v_2) + \left( \frac{C_1^2}{2} - \frac{C_2^2}{2} \right) + g(Z_1 - Z_2) \right]
\]

5. **Centrifugal Water Pump**

- A centrifugal water pump is a device that transfers the mechanical energy of a motor or an engine into the pressure energy of incompressible fluid like water.

---

**Fig. 2.9 Schematic of centrifugal water pump**

- Applying Steady Flow Energy Equation (SFEE),

\[
m \left( u_1 + P_1 v_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m \left( u_2 + P_2 v_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s
\]

- The characteristic features of flow through a **centrifugal water pump** are:
Shaft work required; \( W_s = -ve \)

Negligible change in temperature of water so,
- Heat transfer rate from turbine; \( Q = 0 \)
- Change in specific internal energy; \( \Delta u = u_2 - u_1 = 0 \)

As water is incompressible fluid, its specific volume and hence density will remain constant; \( v_1 = v_2 = v \)

- Hence, SFEE is reduced to,

\[
m \left( P_1 v_1 + \frac{C_1^2}{2} + gZ_1 \right) = m \left( P_2 v_2 + \frac{C_2^2}{2} + gZ_2 \right) - W_s
\]

\[
W_s = m \left[ (P_2 v_2 - P_1 v_1) + \left( \frac{C_2^2}{2} - \frac{C_1^2}{2} \right) + g(Z_2 - Z_1) \right]
\]

6. Reciprocating Compressor

- A reciprocating compressor is used for increasing the pressure of a fluid and has a piston cylinder mechanism as the primary element.
- The unit sucks in definite quantity of fluid, compresses through a required pressure ratio and then delivers the compressed air/gas to a receiver.
- Reciprocating compressors are used when small quantity of fluid with high pressure is required.

![Fig. 2.10 Schematic of reciprocating compressor](image_url)

- Applying Steady Flow Energy Equation (SFEE),

\[
m \left( h_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m \left( h_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s
\]

- The characteristic features of flow through a **reciprocating compressor** are:
Shaft work required; $W_s = -ve$

Negligible velocity change in the flow of fluid; $C_1 = C_2$

Negligible potential energy change; $Z_1 = Z_2$

Appreciable amount of heat transfer is involved; heat is lost from the system as it gets sufficient time to interact with surrounding because of low speed; $Q \neq 0$ and $Q = -ve$

\[ m h_1 - Q = m h_2 - W_s \]
\[ \therefore W_s = Q + m(h_2 - h_1) \]

7. **Rotary Compressor**

- Rotary compressors are used for increasing the pressure of a fluid and have a rotor as the primary element.
- Rotary compressors are employed where high efficiency, medium pressure rise and large flow rate are the primary considerations.

![Schematic of rotary compressor](image)

Applying Steady Flow Energy Equation (SFEE),
\[ m \left( h_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m \left( h_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s \]

The characteristic features of flow through a **rotary compressor** are:
- Shaft work required; $W_s = -ve$
- Negligible velocity change in the flow of fluid; $C_1 = C_2$
- Negligible potential energy changes; $Z_1 = Z_2$
- Flow process is treated as adiabatic due to vary high flow rates; $Q = 0$

Hence, SFEE is reduced to,
8. Throttling Process

- Throttling is the expansion of fluid from high pressure to low pressure. This process occurs when fluid passes through an obstruction (partially opened valve, porous plug or a small orifice) placed in the fluid flow passage.

- The throttling process is commonly used for the following purposes:
  
  I. For determining the condition of steam (dryness fraction).
  
  II. For controlling the speed of the turbine.
  
  III. Used in refrigeration plant for reducing the pressure of the refrigerant before entry into the evaporator.

- Fig. 2.12 shows the schematic of porous plug experiment performed by Joule and Thomson in 1852. A stream of incompressible fluid is made to pass steadily through a porous plug placed in an insulated and horizontal pipe.

\[
mh_1 = mh_2 - W_s
\]
\[
\therefore W_s = m(h_2 - h_1)
\]

\[\text{Fig. 2.12 The Joule – Thomson porous plug experiment}\]

\[\text{Fig. 2.13 Throttling devices (a) An adjustable valve (b) A porous plug}\]

- Applying Steady Flow Energy Equation (SFEE),
\[ m \left( h_1 + \frac{C_1^2}{2} + gZ_1 \right) + Q = m \left( h_2 + \frac{C_2^2}{2} + gZ_2 \right) + W_s \]

- The characteristic features of a throttling process are:
  - No shaft work required; \( W_s = 0 \)
  - No heat interaction as pipe is thermally insulated; \( Q = 0 \)
  - Negligible velocity change in the flow of fluid; \( C_1 = C_2 \)
  - Negligible potential energy changes as the pipe is placed horizontally; \( Z_1 = Z_2 \)

- Hence steady flow energy equation reduced to,
  \[ h_1 = h_2 \]

- Enthalpy of fluid remains constant during throttling process. Thus the throttling expansion process is an isenthalpic process.

- For a perfect gas,
  \[ C_p T_1 = C_p T_2 \]

- \( \therefore T_1 = T_2 \)

Thus for a perfect gas, the temperature before and after throttling is always same.

### 2.7 Unsteady Flow Processes: Filling and Emptying Process

- In engineering practice, the variable flow process applications are as common as the steady flow process. The rate of energy and mass transfer into and out of the control volume are not same in the case of unstable (or variable or transient or unsteady) flow process.

- Following two cases only will be discussed:
  2. Tank Emptying Process or Tank Discharge Process

#### 1. Tank Filling Process

- The tank/bottle initially contains fluid having mass \( m_i \), at the state \( P_i, v_i \) and \( T_i \). The corresponding values after the filling operation are \( m_f, P_f, v_f \) and \( T_f \).

- In addition there may be heat and work interactions with the surroundings.

- The work interaction is possible by change in volume of the bottle or by internal electrical or mechanical devices.
- Mass of fluid entering = \( m_f - m_i \)
- Energy of entering fluid = \( (m_f - m_i)e \)
- Energy balance gives,

\[
(m_f - m_i)e + Q = (m_f u_f - m_i u_i) + W
\]

Where,
\( u_f \) & \( u_i \) = Final & Initial sp. Internal energy of fluid mass

- In a filling process of a tank from a large reservoir (pipeline), the properties of the entering fluid stream are essentially constant and so will be the energy \( e_p \) accompanying unit mass of fluid at entrance to control volume.

\[
e_p = u_p + P_p v_p + \frac{C_p^2}{2} = h_p + \frac{C_p^2}{2}
\]

\[
\therefore (m_f - m_i) \left( h_p + \frac{C_p^2}{2} \right) + Q = (m_f u_f - m_i u_i) + W
\]

The suffix 'p' refers to state of fluid in the pipe line.

- In the absence of any work interaction \( (W = 0) \) and when the tank is thermally insulated \( (Q = 0) \).

\[
\therefore (m_f - m_i) \left( h_p + \frac{C_p^2}{2} \right) = (m_f u_f - m_i u_i)
\]

- Neglecting kinetic energy of the incoming fluid,

\[
(m_f - m_i)(h_p) = (m_f u_f - m_i u_i)
\]

- If the tank is initially empty,

\[
m_f h_p = m_f u_f
\]

\[
\therefore h_p = u_f
\]

- Thus the specific internal energy of fluid charged into empty insulated tank is equal to the specific enthalpy of the filling fluid in the charging pipe line.

- If the fluid is an ideal gas, then temperature of gas in the tank after it is charged is given by,

\[
C_p T_p = C_v T_2
\]

\[
\therefore T_2 = \gamma T_p
\]

2. Tank Emptying Process

- The tank emptying process is the reverse of filling process, i.e. there is flow of fluid from the tank to the surrounding.

- The surroundings are much larger than the tank being emptied and so the energy \( e_p = h_p + \frac{C_p^2}{2} \) accompanying unit mass of fluid at exit from the control volume will be constant.
- Energy balance gives,

\[(m_i - m_f) \left( h_p + \frac{C_p^2}{2} \right) + Q = (m_i u_i - m_f u_f)\]

- For no heat transfer and negligible exit velocity,

\[(m_i - m_f)(h_p) = (m_i u_i - m_f u_f)\]

- Further if the tank is to be fully emptied \((m_f = 0)\)

\[\therefore m_i h_p = m_i u_i\]

\[\therefore h_p = u_i\]

### 2.8 First Law Applied to Non Flow Processes

- Following are the important non-flow processes, which are commonly used in engineering applications:

  A. Constant Volume Process (Isochoric)
  B. Constant Pressure Process (Isobaric)
  C. Constant Temperature Process (Isothermal)
  D. Adiabatic Process \((Q = 0)\) or Isentropic Process (Reversible Adiabatic; \(S = C\))
  E. Polytropic Process

- Fig. 2.15 to 2.19 shows schematic and P-v diagram for all the processes listed above.

![Fig. 2.15 Constant volume process (Isochoric)](image)
Fig. 2.16 Constant pressure process (Isobaric)

Fig. 2.17 Constant temperature process (Isothermal)

Fig. 2.18 Reversible Adiabatic Process (Isentropic process)
In a Polytropic process, the index \( n \) depends only on the heat and work quantities during the process. The various processes considered earlier are special cases of Polytropic process for a perfect gas. This is illustrated on P-v diagram in Fig. 2.19.

Fig. 2.19 Polytropic process for different values on index ‘\( n \)’

For Air (Perfect Gas)

\[
R = 0.287 \text{ KJ/kg} - k
\]

\[
C_p = 1.005 \text{ KJ/kg} - k
\]

\[
C_v = 0.718 \text{ KJ/kg} - k
\]

\[
\gamma = 1.4
\]

Relationship between \( R, C_p, C_v \), and \( \gamma \)

\[
R = C_p - C_v
\]

\[
\gamma = \frac{C_p}{C_v}
\]
2.9 Solved Numerical

Ex 2.1. [GTU; Jun-2014; 7 Marks]

In steam power plant 1 kg of water per second is supplied to the boiler. The enthalpy and velocity of water entering the boiler are 800 kJ/kg and 5 m/s. The water receives 2200 kJ/kg of heat in the boiler at constant pressure. The steam after passing through the turbine comes out with a velocity of 50 m/s, and its enthalpy is 2520 kJ/kg. The inlet is 4 m above the turbine exit. Assuming the heat losses from the boiler and the turbine to the surroundings are 20 kJ/sec. Calculate the power developed by the turbine. Consider the boiler and turbine as single system.

Solution:

Given Data:
- \( m_w = 1 \text{ kg/sec} \)
- \( h_1 = 800 \text{ kJ/kg} \)
- \( C_1 = 5 \text{ m/s} \)
- \( q_s = 2200 \text{ kJ/kg} \)
- \( C_2 = 50 \text{ m/s} \)
- \( h_2 = 2520 \text{ kJ/kg} \)
- \( Z_1 - Z_2 = 4 \text{ m} \)
- \( Q_r = -20 \text{ kJ/sec} \)

To be Calculated: \( P = ? \)

\[ q = \frac{Q_r}{m_w} = \frac{-20}{1} \]
\[ q = -20 \text{ kJ/kg} \]

Net Heat Transfer to the System,
\[ q_{net} = q_s - q_r \]
\[ q_{net} = 2200 - 20 \]
\[ q_{net} = 2180 \text{ kJ/kg} \]

Apply Steady Flow Energy Equation,
\[ h_1 + \frac{C_1^2}{2} + gZ_1 + q_{net} = h_2 + \frac{C_2^2}{2} + gZ_2 + w_{net} \]
\[ w_{net} = (h_1 - h_2) + \frac{C_1^2}{2} - \frac{C_2^2}{2} + g(Z_1 - Z_2) + q_{net} \]
\[ w_{net} = (800 - 2520) \times 10^3 + \frac{(5^2)}{2} - \frac{(50^2)}{2} + 9.81(4) + (2180 \times 10^3) \]
\[ w_{net} = 458801.74 \text{ kJ/kg} \]
\[ w_{net} = 458.801 \text{ kJ/kg} \]

Power Developed by the Turbine:
\[ P = m_w \times w_{net} \]
\[ P = 1 \times 458.801 \]
\[ P = 458.801 \text{ kW} \]
Ex 2.2. [GTU; Jun-2010; 7 Marks]
Air at a temperature of 15°C passes through a heat exchanger at velocity of 30 m/s, where temperature is raised to 800°C. It then enters a turbine with same velocity of 30 m/s and expands until temperature falls to 650°C. On leaving the turbine the air is taken at velocity of 60 m/s to a nozzle where it expands until the temperature has fallen to 500°C. If the air flow rate is 2 kg/s, calculate (a) rate of heat transfer to air in the heat exchanger, (b) power output from turbine assuming no heat loss and (c) velocity at exit from the nozzle. Assuming no heat loss.

Solution:

Given Data:
\( \dot{m}_a = 2 \text{ kg/sec} \)
\( C_1 = C_2 = 30 \text{ m/s} \)
\( T_1 = 15 \text{ °C} \)
\( T_2 = 800 \text{ °C} \)

To be Calculated:
\( a) q_s = ? \)
\( b) P = ? \)
\( c) C_4 = ? \)

[1] Heat Exchanger
⇒ For H.E.
\[ w_{net} = 0 \]
\[ Z_1 = Z_2 \text{ (Assume)} \]
Also,
\[ C_1 = C_2 \text{ (Given)} \]
⇒ Apply Steady Flow Energy Equation to Heat Exchanger (1–2),
\[ h_1 + \frac{C_1^2}{2} + gZ_1 + q_{net} = h_2 + \frac{C_2^2}{2} + gZ_2 + w_{net} \]
\[ \therefore q_{net} = (h_2 - h_1) \]
\[ \therefore q_{net} = C_p(T_2 - T_1) \]
\[ \therefore q_{net} = 1.005(800 - 15) \]
\[ \therefore q_{net} = 788.925 \text{ kJ/kg} \]
⇒ Rate of Heat transfer:
\[ \dot{q}_{net} = \dot{m}_a \times q_{net} \]
\[ \dot{q}_{net} = 2 \times 788.925 \]
\[ \dot{q}_{net} = 1577.85 \text{ kW} \]
[2] Turbine

\( q_{net} = 0 \) (No heat loss)

\[ Z_2 = Z_3 \text{ (Assume)} \]

\( Z_1 = Z_2 \) (Assume that nozzle is horizontal)

\( q_{net} = 0 \) (No heat loss)

Apply Steady Flow Energy Equation to Turbine (2–3),

\[ h_2 + \frac{C_2^2}{2} + gZ_2 + q_{net} = h_3 + \frac{C_3^2}{2} + gZ_3 + w_{net} \]

\[ \therefore w_{net} = (h_2 - h_3) + \left( \frac{C_2^2}{2} - \frac{C_3^2}{2} \right) \quad \frac{C_2^2}{2} - \frac{C_3^2}{2} \]

\[ \therefore w_{net} = 1.005 \times 10^3 \times (800 - 650) + \left( \frac{30^2}{2} - \frac{60^2}{2} \right) \]

\[ \therefore w_{net} = 149400 \text{ J/kg} \]

Power Output from Turbine:

\[ P = \dot{m}_a \times w_{net} \]

\[ P = 2 \times 149400 \]

\[ P = 298800 \text{ W} \]

[3] Nozzle

\( w_{net} = 0 \)

\[ Z_1 = Z_2 \text{ (Assume that nozzle is horizontal)} \]

\( q_{net} = 0 \) (No heat loss)

Apply Steady Flow Energy Equation to Nozzle (3–4),

\[ h_3 + \frac{C_3^2}{2} + gZ_3 + q_{net} = h_4 + \frac{C_4^2}{2} + gZ_4 + w_{net} \]

\[ \therefore \frac{C_4^2}{2} = (h_3 - h_4) + \frac{C_3^2}{2} \]

\[ \therefore \frac{C_4^2}{2} = C_p(T_3 - T_4) + \frac{C_3^2}{2} \]

\[ \therefore \frac{C_4^2}{2} = 1.005 \times 10^3 \times (650 - 500) + \frac{60^2}{2} \]

\[ \therefore \frac{C_4^2}{2} = 152550 \]

\[ \therefore C_4 = 552.358 \text{ m/sec} \]

2.10 References

4. Thermal Science and Engineering by D. S. Kumar, S K Kataria and Sons Publications.
SECOND LAW OF THERMODYNAMICS

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3.9 Thermodynamic Temperature Scale
3.1 Introduction

- First law of thermodynamics deals with conservation and conversion of energy. It stipulates that when a thermodynamic process is carried out, energy is neither gained nor lost. Energy only transforms from one form into another and the energy balance is maintained. The law, however, fails to state the condition under which energy conversions are possible. The law presumes that any change of a thermodynamic state can take place in either direction.

- However, this is not true; particularly in the inter-conversion of heat and work. Processes proceed spontaneously in certain directions but not in opposite directions, even though the reversal of processes does not violate the first law.

3.2 Examples of First law

- It is common experience that a cup of hot coffee left in a cooler room eventually cools off. This process satisfies the first law of thermodynamics since the amount of energy lost by the coffee is equal to the amount gained by the surrounding air. Now let us consider the reverse process—the hot coffee getting even hotter in a cooler room as a result of heat transfer from the room air. We all know that this process never takes place. Yet, doing so would not violate the first law as long as the amount of energy lost by the air is equal to the amount gained by the coffee.

![Figure 1 coffee in a cold room](image1.png)

- As another familiar example, consider the heating of a room by the passage of electric current through a resistor. Again, the first law says that the amount of electric energy supplied to the resistance wires be equal to the amount of energy transferred to the room air as heat. Now let us attempt to reverse this process. It will come as no surprise that transferring some heat to the wires does not cause an equivalent amount of electric energy to be generated in the wires.

![Figure 2 Transferring heat to a wire will not generate electricity](image2.png)
Consider a paddle-wheel mechanism that is operated by the fall of a mass. The paddle wheel rotates as the mass falls and stirs a fluid within an insulated container. As a result, the potential energy of the mass decreases, and the internal energy of the fluid increases in accordance with the conservation of energy principle. However, the reverse process, raising the mass by transferring heat from the fluid to the paddle wheel, does not occur in nature, although doing so would not violate the first law of thermodynamics.

Consider a running automobile vehicle stopped by applying brakes, and the process changes the kinetic energy of the vehicle into heat and the brakes get heated up. Thus increase in internal energy of brakes in accordance with the first law. Now cooling of brakes to their initial state never puts the vehicle in to motion. Heat in the brake cannot convert to mechanical work even though that would not violate the principle of energy conversion.

When a block slides down a rough plane, it warmer. However, the reverse process where the block slides up the plane and becomes cooler is not true even though the first law will still hold good.

Water flows from a higher level to a lower level, and reverse is not automatically possible. A mechanical energy from an external source would be required to pump the water back from the lower level to higher level.

Fuels (coals, diesel, and petrol) burns with air to form the products of combustion. Fuels once burnt cannot be restored back to original form.

When hydrogen and oxygen are kept in an isolated system, they produce water on chemical reaction. But the water never dissociates into hydrogen and oxygen again.

It is clear from these above arguments that processes proceed in a certain direction and not in the reverse direction.
3. Second Law of Thermodynamics

A process cannot take place unless it satisfies both the first and second laws of thermodynamics.

![Figure 5 A processes must satisfy both the first and second laws of thermodynamics](image)

Therefore, it is reasonable to conclude that a process must satisfy the first law to occur. However, as explained here, satisfying the first law alone does not ensure that the process will actually take place.

3.2.1 Limitations of First Law of Thermodynamics

- First law does not help to predict whether the certain process is possible or not.
- A spontaneous process can proceed in a particular direction only, but first law does not give information about direction.
- First law not provides sufficient condition for a certain process to take place.
- First law establishes equivalence between the amount of heat used and mechanical work, but does not specify the conditions under which conversion of heat into work is possible, neither the direction in which heat transfer can take place.

3.3 Basic Definitions

3.3.1 Thermal Energy Reservoir

- “It is defined as sufficiently large system in stable equilibrium that can supply or absorb finite amount of heat without any change in its temperature.”
- A thermal reservoir is thus characterized by its temperature which remains constant.
- In practice, large bodies of water such as oceans, lakes, rivers, and atmospheric air can be considered thermal energy reservoirs.

![Figure 6 Thermal energy reservoirs](image)
Heat Source

- “It is defined as the thermal reservoir which is at high temperature and supplies heat is called a heat source.” i.e. boiler furnace, combustion chamber etc.

Heat Sink

- “It is defined as the thermal reservoir which is at low temperature and to which heat is transferred is called heat sink”. i.e. atmospheric air, ocean, rivers etc.

3.3.2 Heat Engine

- “It is defined as thermodynamic device used for continuous production of work from heat when operating in a cyclic process is called heat engine”.

Characteristics of Heat Engine:

- It receives heat from a high-temperature source at temperature $T_1$ (furnace, nuclear reactor, solar energy etc.)
- It converts the part of this heat to work (mostly in the form of a rotating shaft).

- It rejects the remaining waste heat to a low-temperature sink (the atmosphere, rivers etc.).
- It operates on complete thermodynamic cycle.

Thermal Efficiency

- “It is defined as the ratio of the desired net work output to the required heat input is called thermal efficiency.”
3. Second Law of Thermodynamics

Thus thermal efficiency of a heat engine can be expressed as,

\[
\eta_{th} = \frac{\text{desired work output}}{\text{required heat input}} = \frac{W_{net}}{Q_{in}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}
\]

Where \(Q_1\) = Heat supplied to system, kJ
\(Q_2\) = Heat rejected from system, kJ
\(W\) = Net work done by a system, kJ

3.3.3 Refrigerator

- “It is defined as the mechanical device that used for the transfer of heat from a low-temperature medium to a high-temperature medium is called refrigerator.”
- The objective of a refrigerator is to maintain the refrigerated space at a low temperature by absorbing heat from it and reject to higher-temperature medium.

![Figure 9 Refrigerator](image)

**Coefficient of Performance of Refrigerator**

- “The COP of a refrigerator can be expressed as the ratio of refrigerating effect to the work input.”
- Mathematically,

\[
COP_R = \frac{\text{desired output}}{\text{required input}} = \frac{\text{refrigerating effect}}{\text{work input}} = \frac{Q_2}{W_{net,in}}
\]

- The conservation of energy principle for a cyclic device requires that,

\[
W_{net,in} = Q_1 - Q_2
\]

\[
COP_R = \frac{Q_2}{Q_1 - Q_2}
\]

3.3.4 Heat Pump

- “It is defined as the mechanical device that transfers heat from a low-temperature medium to a high-temperature is called heat pump.”
The objective of heat pump is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low-temperature source and reject to higher temperature source.

**Coefficient of Performance of Heat Pump**
- “The COP of a heat pump can be expressed as the ratio of heating effect to the work input”.
- Mathematically,

\[
COP_{HP} = \frac{\text{desired output}}{\text{required input}} = \frac{\text{heating effect}}{\text{work input}} = \frac{Q_1}{W_{net,in}}
\]

![Figure 10 Heat pump](image)

- The conservation of energy principle for a cyclic device requires that,

\[
W_{net,in} = Q_1 - Q_2
\]

\[
COP_{HP} = \frac{Q_1}{Q_1 - Q_2}
\]

**3.3.5 Perpetual-Motion Machines (PMM)**
- “It is defined as the device that violates either law (first or second) is called a perpetual-motion machine.”
- **PMM1**: “A device that violates the first law of thermodynamics is called a perpetual-motion machine of the first kind (PMM1).”
- **PMM2**: “A device that violates the second law of thermodynamics is called a perpetual-motion machine of the second kind (PMM2).”

![Figure 11 Perpetual motion machine of the second kind](image)
3. Second Law of Thermodynamics

\[ \eta_{th} = \frac{W_{net}}{Q_i} = \frac{Q_1 - Q_2}{Q_i} = 1 - \frac{Q_2}{Q_i} \]

- In above equation, if \( Q_2 = 0 \), then \( W_{net} = Q_1 \) and \( \eta_{th} = 100\% \). That is, if the engine exchanges heat only with one thermal reservoir, then the entire heat supplied to it gets converted into an equivalent amount of work and the efficiency becomes 100\%. Such a heat engine is called a PMM2. The PMM2 is follow first law, but it violates the Kelvin-Planck statement of second law.

3.4 The Statements of Second Law of Thermodynamics

3.4.1 Kelvin–Planck Statement

- "It is impossible to construct a device that operates in thermodynamic cycle produce no effect other than work output and exchange heat with a single reservoir". (a)

3.4.2 Clausius Statement

- "It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature reservoir to a higher-temperature reservoir."

\[ \text{OR} \]

- "It is impossible for any system to operate in such a way that the sole result would be an energy transfer by heat from a cooler to a hotter body."

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3.4.3 Equivalency of the Two Statements
(a) Violation of Clausius statement leading to violation of Kelvin-Planck statement.
- As shown in Fig. (a) a refrigerator \( R \) that operates in a cycle and transfers \( Q_2 \) amount of heat from low temperature reservoir at \( T_2 \) to a high temperature reservoir at \( T_1 \) without any work input. This is in violation of the Clausius statement.

\[ \begin{align*}
&\text{Reservoir at } T_1 \\
&\downarrow Q_2 \\
&\downarrow W = 0 \\
&\uparrow Q_1 \\
&\text{Reservoir at } T_2
\end{align*} \]

(b) Violation of Kelvin-Planck statement leading to violation of Clausius statement.
- Along with this heat engine \( E \), that also operates in a cycle, takes \( Q_1 \) amount of heat from the high temperature reservoir, delivers \( Q_1 - Q_2 \) amount of work to the surroundings and rejects the remaining \( Q_2 \) amount of heat to the low temperature reservoir.
- As shown in Fig. (b) the composite system constitutes a device that receives \( Q_1 - Q_2 \) amount of heat from the high temperature reservoir and converts it completely into an equivalent amount of work \( W = Q_1 - Q_2 \) without rejecting any heat to the low temperature reservoir. This is violation of the Kelvin-Planck statement.

\[ \begin{align*}
&\text{Reservoir at } T_1 \\
&\downarrow Q_1 - Q_2 \\
&\downarrow W = Q_1 - Q_2 \\
&\text{Composite system} \\
&\uparrow W = Q_1 - Q_2 \\
&\text{Reservoir at } T_2
\end{align*} \]
This composite system constitutes a device which transfers heat from the low temperature reservoir to the high temperature reservoir without any work input. This is in violation of the Clausius statement. Thus violation of Kelvin-Planck statement leads to violation of Clausius statement also.

![Diagram](image)

*Figure 15 Proof of the violation of the Kelvin–Planck statement leads to the violation of the Clausius statement*

Therefore, the Clausius and the Kelvin–Planck statements are two equivalent expressions of the second law of thermodynamics.

### 3.5 Reversible and Irreversible Process

#### 3.5.1 Reversible Process

**Definition:** “A reversible process is defined as a process that can be reversed without leaving any trace on the surroundings and both the system and the surroundings are restored to their respective initial states by reversing the direction of the process”.

**Conditions of Reversible Process**

- The process must proceed in a series of equilibrium states.
- Heat transfer should not take place with finite temperature difference.
- The process should be quasi-static and it should proceed at infinitely slow speed.
- The process should not involve friction of any kind (mechanical and intermolecular)

**Salient Features**

- It is quasi-static process which can be carried out in the reverse direction along the same path. It can be proceed in either direction without violating the second law of thermodynamics.
- The energy transfer as heat and work during the forward process should be identically equal to energy transfer as heat and work during the reversal of the process.
- It is possible only if the net heat and net work exchange between the system and the surroundings is zero for the combined (original and reverse) process or it leaves no trace or evidence of its occurrence in the system and surroundings.
Reversible processes can be viewed as theoretical limits for the corresponding irreversible ones.

- The more closely we approximate a reversible process, the more work delivered by a work-producing device or the less work required by a work-consuming device.
- It leads to the definition of the second law efficiency for actual processes, which is the degree of approximation to the corresponding reversible processes. This enables us to compare the performance of different devices that are designed to do the same task on the basis of their efficiencies.
- It is idealized process actually do not occur in nature.
- There should be no free or unrestricted expansion and no mixing of the fluids.
- Work done during reversible process is represented by area under process curve on p-v diagram, and is equal to \( \int_{1}^{2} p\,dv \)

Some Notable Examples of ideal reversible processes are:

1. Motion without friction.
2. Frictionless adiabatic and isothermal expansion or compression.
3. Restricted and controlled expansion or compression.
4. Elastic stretching of a solid.
5. Restrained discharge of the battery.
7. Polarisation, magnetisation effects and electrolysis.
8. Condensation and boiling of liquids.

3.5.2 Irreversible Process

- **Definition**: “An irreversible process is defined as a process that can be reversed with permanent leaving any trace on the surroundings and both the system and the surroundings are not restored to their respective initial states by reversing the direction of the process”.

- These processes that occurred in a certain direction, once having taken place, these processes cannot reverse themselves spontaneously and restore the system to its initial state.

- *For example*, once a cup of hot coffee cools, it will not heat up by retrieving the heat it lost from the surroundings. If it could, the surroundings, as well as the system (coffee), would be restored to their original condition, and this would be a reversible process.

- It should be pointed out that a system can be restored to its initial state following a process, regardless of whether the process is reversible or irreversible. But for reversible processes, this restoration is made without leaving any net change on the surroundings, whereas for irreversible processes, the surroundings usually do some work on the system and therefore does not return to their original state.

**Salient Features**

1. It can be carried out in one direction.
2. It occurs at a finite rate.
3. It cannot be reversed without permanent change in surroundings.
4. The system is in never in equilibrium state at any instant during an irreversible process.

**Some Notable Examples of an irreversible process are:**

1. Spontaneous chemical reaction.
2. Viscous flow, fluid flow with friction.
3. Inelastic deformation and hysteresis effects.
4. Electric circuit with resistance.
6. Heat transfer takes place with finite temperature difference.
7. Free expansion and throttling process.
8. Friction—sliding friction as well as friction in the flow of fluids.
3.5.3 Irreversibilities

- **Definition:** “It is defined as the factors that cause a process to be irreversible are called irreversibilities.”

(A) Causes of Irreversibilities

- They include friction, unrestrained expansion, mixing of two fluids, and heat transfer across a finite temperature difference, electric resistance, inelastic deformation of solids, and chemical reactions. The presence of any of these effects renders a process irreversible. A reversible process involves none of these. Some of the frequently encountered irreversibilities are discussed briefly below.

1. Friction:

- When two bodies in contact are forced to move relative to each other (a piston in a cylinder, for example, as shown in Fig. a friction force that opposes the motion develops at the interface of these two bodies, and some work is needed to overcome this friction force. The energy supplied as work is eventually converted to heat during the process and is transferred to the bodies in contact, as evidenced by a temperature rise at the interface.

![Figure 17 Friction renders a process irreversible](image)

- When the direction of the motion is reversed, the bodies are restored to their original position, but the interface does not cool, and heat is not converted back to work. Instead, more of the work is converted to heat while overcoming the friction forces that also oppose the reverse motion. Since the system (the moving bodies) and the surroundings cannot be returned to their original states, this process is irreversible. Therefore, any process that involves friction is irreversible.

2. Unrestrained expansion:

- Unrestrained expansion of a gas separated from a vacuum by a membrane, as shown in Fig. When the membrane is ruptured, the gas fills the entire tank. The only way to restore the system to its original state is to compress it to its initial volume, while transferring heat from the gas until it reaches its initial temperature. From the conservation of energy considerations, it can easily be shown that the amount of heat transferred from the gas equals the amount of work done on the gas by the surroundings.
3. Second Law of Thermodynamics

The restoration of the surroundings involves conversion of this heat completely to work, which would violate the second law. Therefore, unrestrained expansion of a gas is an irreversible process.

3. Heat transfer through a finite temperature difference:

- Consider a can of cold soda left in a warm room Fig. Heat is transferred from the warmer room air to the cooler soda. The only way this process can be reversed and the soda restored to its original temperature is to provide refrigeration, which requires some work input. At the end of the reverse process, the soda will be restored to its initial state, but the surroundings will not be. The internal energy of the surroundings will increase by an amount equal in magnitude to the work supplied to the refrigerator. The restoration of the surroundings to the initial state can be done only by converting this excess internal energy completely to work, which is impossible to do without violating the second law.

- Since only the system, not both the system and the surroundings, can be restored to its initial condition, heat transfer through a finite temperature difference is an irreversible process.
(B) Types of Irreversibilities
- **Internally Irreversibilities**: These are associated with dissipative effects within working fluid itself.
- **Externally Irreversibilities**: These are associated with dissipative effects outside the working fluid or boundaries of the system. i.e. Mechanical friction occurring during process.

![Diagram](image1)

*Figure 20 A reversible processes involves no internal and external irreversibilities (Totally reversible)*

- As shown in Fig. Both processes are internally reversible, since both take place isothermally and both pass through exactly the same equilibrium states.
- The first process shown is externally reversible also, since heat transfer for this process takes place through an infinitesimal temperature difference dT. The second process, however, is externally irreversible, since it involves heat transfer through a finite temperature difference dT.

![Diagram](image2)

*Figure 21 Totally and internally reversible heat transfer processes*
− **Mechanical Irreversibilities**: These are associated with fluid friction (intermolecular friction) between the molecules and mechanical friction between the molecules and mechanical parts and friction between molecules and atmosphere.

− **Thermal Irreversibilities**: These are associated with energy transfer as heat due to a finite temperature difference between parts of system or between system and its environment.

### 3.6 The Carnot Cycle (Carnot Heat engine)

**Assumptions for Carnot cycle**

1. The piston moving in a cylinder does not develop any friction during motion.
2. The walls of piston and cylinder are considered as perfect insulators of heat.
3. The cylinder head is so arranged that it can be a perfect heat conductor or perfect heat insulator.
4. The transfer of heat does not affect the temperature of source or sink.
5. Working medium is a perfect gas and has constant specific heat.
6. Compression and expansion are reversible.

− The Carnot cycle is composed of four reversible processes—two isothermal and two adiabatic. Consider a closed system that consists of a gas contained in an adiabatic piston–cylinder device, as shown in Fig. 3.19 the insulation of the cylinder head is such that it may be removed to bring the cylinder into contact with reservoirs to provide heat transfer.
3. Second Law of Thermodynamics

- **Reversible Isothermal Expansion** (process 1-2, $T_H = \text{constant}$): In this process, high temperature energy source is put in contact with cylinder cover and $Q_1$ amount of heat is supplied while the gas expands isothermally at temperature $T_H$. The amount of heat transferred to the gas during this process is given by,

\[ Q_1 = W_{1-2} = P_i V_i \ln \frac{V_2}{V_1} = m R T_H \ln \frac{V_2}{V_1} \]

![Figure 23 P-v diagram of the Carnot cycle](image)

- **Reversible Adiabatic Expansion** (process 2-3): In this process the adiabatic cover is put in contact on the cylinder head, and the gas is expanded adiabatically, thus the temperature decreases from $T_H$ to $T_L$.

- **Reversible Isothermal Compression** (process 3-4, $T_L = \text{constant}$): In this process, low temperature energy sink is put in contact with cylinder head cover and $Q_2$ amount of heat is rejected while the gas compressed isothermally at temperature $T_L$. The amount of total heat transferred from the gas during this process is given by,

\[ Q_2 = W_{3-4} = P_i V_i \ln \frac{V_3}{V_4} = m R T_L \ln \frac{V_3}{V_4} \]

- Reversible Adiabatic Compression (process 4-1): In this process the adiabatic cover is put in contact on the cylinder head, and the gas is compressed adiabatically, thus temperature increases from $T_L$ to $T_H$ and returns to its initial state 1 to complete the cycle.

- Thermal efficiency of Carnot cycle is given by,

\[ \eta_{th} = \frac{W_{net}}{Q_i} \]

- As there is not heat interaction along the reversible adiabatic processes 2-3 and 4-1, and application of first law of thermodynamics for the complete cycle gives,
3. Second Law of Thermodynamics

\[ \delta W = \delta Q \]

\[ W_{net} = Q_1 - Q_2 = mRT_H \ln \frac{V_2}{V_1} - mRT_L \ln \frac{V_3}{V_4} \]

- Substituting the values of \( W_{net} \) in above equation we get,

\[ \eta_{th,Carnot} = \frac{mRT_H \ln \frac{V_2}{V_1} - mRT_L \ln \frac{V_3}{V_4}}{mRT_H \ln \frac{V_2}{V_1}} \]

\[ \eta_{th,Carnot} = 1 - \frac{T_L}{T_H} \frac{\ln \frac{V_3}{V_4}}{\ln \frac{V_2}{V_1}} \]

- For the adiabatic expansion and compression process 2-3 and 4-1,

\[ \frac{T_2}{T_3} = \frac{T_H}{T_L} = \left( \frac{V_3}{V_2} \right)^{\gamma - 1} \text{ and } \frac{T_4}{T_1} = \frac{T_H}{T_L} = \left( \frac{V_4}{V_1} \right)^{\gamma - 1} \]

\[ \frac{T_H}{T_L} = \left( \frac{V_3}{V_2} \right)^{\gamma - 1} = \left( \frac{V_4}{V_1} \right)^{\gamma - 1} \]

\[ \left( \frac{V_3}{V_2} \right) = \left( \frac{V_4}{V_1} \right) \text{ or } \left( \frac{V_3}{V_4} \right) = \left( \frac{V_2}{V_1} \right) \]

- Substitute the values in above equation, we get,

\[ \eta_{th,Carnot} = 1 - \frac{T_L}{T_H} \]

**Conclusions from Carnot heat engine are:**

1. The efficiency is independent of the working fluid and depends upon the temperature of source and sink. Being a reversible cycle, the Carnot cycle is the most efficient cycle operating between two specified temperature limits.

2. If \( T_L = 0 \), the engine will have an efficiency of 100%. However that means absence of heat sink which is violation of Kelvin-Plank statement of the second law.

3. The efficiency is directly proportional with the Temperature difference \( T_H - T_L \) between the source and sink. Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system. If \( T_H = T_L \), no work will be done and efficiency will be zero.

- Even though the Carnot cycle cannot be achieved in reality, the efficiency of actual cycles can be improved by attempting to approximate the Carnot cycle more closely.
The Carnot cycle is impracticable because of the following reasons:

- All the four processes have to be reversible. This necessitates that working fluid must have no internal friction between the fluid particle and no mechanical friction between the piston and cylinder wall. It is impossible to perform a frictionless process.
- The heat absorption and rejection take place with infinitesimal temperature difference. Accordingly the rate of energy transfer will be very low and the engine will deliver only infinitesimal power. It is impossible to transfer the heat without temperature potential.
- Isothermal process can be achieved only if the piston moves very slowly to allow heat transfer so that the temperature remains constant. Also Reversible isothermal heat transfer is very difficult to achieve in reality because it would require very large heat exchangers and it would take a very long time (a power cycle in a typical engine is completed in a fraction of a second). Therefore, it is not practical to build an engine that would operate on a cycle that closely approximates the Carnot cycle.
- Adiabatic process can be achieved only if the piston moves as fast as possible so that the heat transfer is negligible due to very short time available. The isothermal and adiabatic processes take place during the same stroke therefore the piston has to move very slowly for part of the stroke and it has to move very fast during remaining stroke. This variation of motion of the piston during the same stroke is not possible.
- The source and sink temperatures that can be used in practice are not without limits, however. The highest temperature in the cycle is limited by the maximum temperature that the components of the heat engine, such as the piston or the turbine blades, can withstand. The lowest temperature is limited by the temperature of the cooling medium utilized in the cycle such as a lake, a river, or the atmospheric air.
- There is insignificant difference in the slopes of isothermal and adiabatic lines. Consequently the p-v plot is greatly extended both in horizontal and vertical directions. The cylinder involves great pressure and volumes, and thus becomes bulky and heavy.

3.7 The Reversed Carnot Cycle (Carnot Refrigerator or Carnot heat pump)

- The Carnot heat-engine cycle just described is a totally reversible cycle. Therefore, all the processes that comprise it can be reversed, in which case it becomes the Carnot refrigeration cycle. Refrigerator and heat pump are reversed heat engines.
- This time, the cycle remains exactly the same, except that the directions of any heat and work interactions are reversed: Heat in the amount of $Q_2$ is absorbed from the low-temperature reservoir, heat in the amount of $Q_1$ is rejected to a high-temperature reservoir, and a work input of $W_{net.in}$ is required to accomplish all this. The P-V diagram of the reversed Carnot cycle is the same as the one given for the Carnot cycle, except that the directions of the processes are reversed, as shown in Fig.
- **Process 1-2:** Isentropic expansion of the working fluid in the clearance space of the cylinder. The temperature falls from $T_H$ to $T_L$. 
3. Second Law of Thermodynamics

- **Process 2-3**: Isothermal expansion during which heat $Q_2$ is absorbed at temperature $T_L$ from the space being cooled.
- **Process 3-4**: Isothermal compression of working fluid. The temperature rises from $T_L$ to $T_H$.
- **Process 4-1**: Adiabatic compression of working fluid during which heat $Q_1$ is rejected to a system at higher temperature.

![Figure 24 P-V diagram of the reversed Carnot cycle](image)

- By using equations outlines in Carnot heat engine,
- For Carnot heat pump,

$$COP_{hp} = \frac{Q_1}{Q_2} = \frac{mRT_H \ln \frac{V_2}{V_1}}{mRT_H \ln \frac{V_2}{V_1} - mRT_L \ln \frac{V_1}{V_4}}$$

$$COP_{hp} = \frac{T_1}{T_1 - T_2}$$

- For Carnot refrigerator,

$$COP_r = \frac{Q_2}{Q_1} = \frac{mRT_L \ln \frac{V_2}{V_1}}{mRT_H \ln \frac{V_2}{V_1} - mRT_L \ln \frac{V_3}{V_4}}$$

$$COP_r = \frac{T_2}{T_1 - T_2}$$

3.8 The Carnot Theorem and Carnot Corollaries

3.8.1 Carnot Theorem and Its Proof

- “The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs.”
- Thus if $\eta_{A(Rev)} > \eta_{B(Rev)}$
Proof of Carnot Theorem

- Consider a reversible engine $E_A$ and an irreversible engine $E_B$ operating between the same thermal reservoirs at temperatures $T_1$ and $T_2$ as shown in Fig. For the same quantity of heat $Q$ withdrawn from the high temperature source, the work output from these engines is $W_A$ and $W_B$ respectively. As such the heat rejected is given by the reversible engine $E_A$ is $Q - W_A$ and that from irreversible engine is $Q - W_B$.

- Let us assume that $\eta_{B_{(rev)}} > \eta_{A_{(rev)}}$ ; $W_B > W_A$ ; $Q - W_B < Q - W_A$ (violation of Carnot theorem). Thus if irreversible engine $E_B$ delivered $W_B - W_A$ more amount of work than the first reversible engine $E_A$ and $W_A$ is utilized to run reversible refrigerator $R_A$ by reversing the reversible engine $E_A$ then composite system as shown in Fig. is an engine that produces a net amount of work while exchanging heat with a single reservoir which is the violation of Kelvin-Plank statement (PMM-2).
- Therefore, we conclude that no irreversible heat engine can be more efficient than a reversible one operating between the same two reservoirs, thus our assumption $\eta_{B_{(rev)}} > \eta_{A_{(rev)}}$ is wrong, because $\eta_{A_{(rev)}} > \eta_{B_{(rev)}}$ is only true to satisfy Carnot theorem.

3.8.2 Carnot Corollaries

Corollary-1

- “All reversible heat engines operating between the two thermal reservoirs with fixed temperature have same efficiencies.”
- Thus $\eta_{A_{(rev)}} = \eta_{B_{(rev)}}$
- Consider a reversible engine $E_A$ and reversible engine $E_B$ operating between the same thermal reservoirs at temperatures $T_1$ and $T_2$ as shown in Fig. For the same quantity of heat $Q$ withdrawn from the high temperature source, the work output from these
The efficiency of any reversible heat engine operating between two thermal reservoirs is independent of the nature of working fluid and depends only on the temperature of thermal reservoirs.”

- As shown in Fig. The efficiency is same because both are reversible engines and work on the Carnot cycle. Efficiency depends only upon the temperature of the reservoirs. So, work is produced by engine E equal to work is required to heat pump R.

- Now assume that, efficiency of engine E be increased by changing nature of working substance. It is as shown in fig. means that the engine E produces more work and rejects less heat to sink.
- However engine E receives Q amount of heat from source and pump R delivers same amount of heat to source. Therefore, we can eliminate high temperature source and combined system as shown in Fig. receives $W_E - W_R$ amount of heat from sink and produced same amount of work.
- This violates second law of the thermodynamics. Therefore it is concluded that efficiency does not depend on any properties of working fluid other than temperature of reservoirs.

### 3.9 Thermodynamic Temperature Scale

- **Definition:** "A temperature scale that is independent of the properties of the thermometric substance that are used to measure temperature is called a thermodynamic temperature scale."

- A thermodynamic temperature scale is established based on fact that the thermal efficiency of reversible heat engines is a function of the reservoir temperatures only. That is,

$$
\eta_{th,rev} = \phi(t_1, t_2)
$$

- Where $\phi$ signify the form of function that connects the temperature with temperature scale and it independent of the properties of the working fluid. The nature of $\phi$ need to be determine to give thermodynamic temperature scale.

- Consider two reversible engines $E_1$ is supplied with $Q_1$ amount of heat from the high temperature reservoir at $t_1$ and rejects $Q_2$ amount of heat at low temperature reservoir $t_2$ which is directly receives by reversible heat engine $E_2$ which further rejects $Q_3$ to the low temperature reservoir at $t_3$ as shown in Fig.
The amounts of heat rejected by engines $E_1$ and $E_2$ must be the same since engines $E_1$ and $E_2$ and can be combined into one reversible engine operating between the same reservoirs as engine $E_3$ and thus the combined engine will have the same efficiency as engine $E_3$. Thus we can write for each reversible engine,

$$\eta_{rev1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{1}{Q_2/Q_1} = 1 - \frac{1}{f(t_1, t_2)}$$

$$\eta_{rev2} = 1 - \frac{Q_2}{Q_3} = 1 - \frac{1}{Q_2/Q_3} = 1 - \frac{1}{f(t_2, t_3)}$$

Thus we can write for combine reversible engine,

$$\eta_{rev3} = 1 - \frac{Q_3}{Q_1} = 1 - \frac{1}{Q_3/Q_1} = 1 - \frac{1}{f(t_1, t_3)}$$

Thus we can write,

$$\frac{Q_1}{Q_2} = f(t_1, t_2); \frac{Q_2}{Q_3} = f(t_2, t_3); \frac{Q_1}{Q_3} = f(t_1, t_3)$$

Now consider the identity,

$$\frac{Q_1}{Q_2} = \frac{Q_2}{Q_3}; f(t_1, t_2) = \frac{f(t_1, t_3)}{f(t_2, t_3)}$$
Above equation reveals that the left-hand side is a function of $t_1$ and $t_2$, and therefore the right-hand side must also be a function of $t_1$ and $t_2$ only. That is, the value of the product on the right-hand side of this equation is independent of the value of $t_2$. This condition will be satisfied only if the function $f$ has the following form:

$$\frac{Q_1}{Q_2} = f(t_1, t_2) = \frac{\psi(t_1)}{\psi(t_2)}$$

Where $\psi$ is another function of $t$. The choice of function $\psi$ depends upon chosen scale of temperature and has infinite variety of forms. If the single form is selected it may written as,

$$\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

This temperature scale is called the Kelvin scale, and the temperatures on this scale are called absolute temperatures. On the Kelvin scale, the temperature ratios depend on the ratios of heat transfer between a reversible heat engine and the reservoirs and are independent of the physical properties of any substance.

**Second Law Summary**

*In the remainder of this chapter, we apply the Kelvin–Planck statement of the second law to draw conclusions about systems undergoing thermodynamic cycles. The chapter concludes with a discussion of the Clausius inequality, which provides the basis for developing the entropy concept. This is a traditional approach to the second law in engineering thermodynamics. However, the order can be reversed—namely; the entropy statement can be adopted as the starting point for study of the second law aspects of systems.*

**Reference Books:**

1. Thermal Science and Engineering by D. S. Kumar
2. Fundamental of Engineering Thermodynamics by Michael J. Moran
3. Engineering Thermodynamics by R. K. Rajput
4. Engineering Thermodynamics by P. K. Nag
5. Thermodynamics an Engineering approach by Yunus A. Cengel

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"Work is the only thing that gives substance to life."

– Albert Einstein
**Course Contents**

4.1 Introduction
4.2 Characteristics of entropy
4.3 Clausius Theorem
4.4 Entropy is a property
4.5 Clausius Inequality
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4.11 Third Law of Thermodynamics (Nernst Law)
4.1 Introduction

- The entropy is a thermodynamics property of a working substance and serves as a valuable tool in the second law analysis of engineering devices. We know that all heat is not equally valuable for converting into work.
- Entropy is a function of a quantity of heat which shows the possibility of conversion of that heat into work.
- The increase in entropy is lower when heat is added at a high temperature and the increase in entropy is higher when heat is added at a low temperature.
- The maximum entropy means, there is minimum availability for conversion into work and the minimum entropy means, there is maximum availability for conversion into work.

4.2 Characteristics of Entropy

- The characteristics of entropy in a summarised form are given below:
- Entropy is property of system.
- For reversible process between state 1 and 2, the change in entropy is given by,
  \[ S_2 - S_1 = \oint \left( \frac{\delta Q}{T} \right)_{\text{Rev}} \]
- The change in entropy for the system may be positive, negative or zero (depending on the heat absorption, rejection or absence)
- Entropy is point function, independent on path of the process.
- For a reversible process, the change in entropy for the surrounding is equal in magnitude but opposite in sign to the change in entropy for the system. so total or net change for system plus surrounding is equal to zero.
- The increase of entropy during the process is a measure of the loss of availability of the energy of the system.
- The entropy of a pure substance approaches zero at the absolute zero temperature.
- From the molecular point of view, entropy can also be considered to be a measure of microscopic disorder.
- The entropy is a measure the lack of information about a system.
- The entropy of system is changed due to main three causes
  a) Heat transfer - heat transfer to system increases the entropy of system, and heat transfer from system decreases the entropy of that system
  b) Mass flow
  c) Irreversibilities - such as friction, heat transfer due to finite temperature difference and fast expansion or compression.
\section*{4.3 Clausius Theorem}

- The cyclic integration of $\frac{\delta Q}{T}$ is equal to zero.
- For any reversible process,
  \[ \oint \frac{\delta Q}{T} = 0 \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{p-v_diagram.png}
\caption{p-v diagram}
\end{figure}

- Consider a closed curve ABCDA representing a reversible cycle as shown in Fig. Let. closed cycle ABCDA is divided into a number of small Carnot cycles by drawing adiabatic and isothermal lines in such a way that, Algebraic sum of heat transferred during the sets of isothermal processes of Carnot cycles is equal to heat transferred during the original reversible cycle (ABCDA).
- Thus the original reversible cycle is replaced by number of Carnot cycles. If the adiabatic processes of Carnot cycles are close to one another (i.e. process 2-3 is adiabatic expansion while process 5-6 adiabatic compression, so process 2-3 is cancelled out) and the number of Carnot cycles is large, the saw toothed zig-zag line will coincide with the original cycle.
- For the elemental Carnot cycle 1-2-3-4 receives heat $\delta Q_{H1}$ at temperature $T_{H1}$ during reversible isothermal process 1-2 and rejects heat $\delta Q_{L1}$ at temperature $T_{L1}$ during reversible isothermal process 3-4,
- From kelvin temperature scale,
  \[ \frac{\delta Q_{H1}}{T_{H1}} = \frac{\delta Q_{L1}}{T_{L1}} \]
- If heat supplied is taken as positive and heat rejected as negative.
  \[ \left( \frac{\delta Q_{H1}}{T_{H1}} \right) + \left( \frac{\delta Q_{L1}}{T_{L1}} \right) = 0 \]
- Similarly, for the elemental Carnot cycle 5-6-7-8-5,
  \[ \left( \frac{\delta Q_{H2}}{T_{H2}} \right) + \left( \frac{\delta Q_{L2}}{T_{L2}} \right) = 0 \]
If similar equations are written for all the elemental Carnot cycles, then for the whole original cycle.

\[
\left( \frac{\delta Q_{H1}}{T_{H1}} \right) + \left( \frac{\delta Q_{L1}}{T_{L1}} \right) + \cdots = 0 \\
\oint \frac{\delta Q}{T} = 0
\]

4.4 Entropy is a Property

Let us consider a system undergoing a reversible process from state 1 to state 2 along path A and then from state 2 to the original state 1 along path B as shown in Fig. 2. Applying the Clausius theorem to this reversible cyclic process, we have

\[
\oint \frac{\delta Q}{T} = 0
\]

Hence when the system passes through the cycle 1-A-2-B-1, we have

\[
\int_{1}^{2} (A) \frac{\delta Q}{T} + \int_{2}^{1} (B) \frac{\delta Q}{T} = 0
\]

Now consider another reversible cycle in which the system changes from state 1 to state 2 along path A, but returns from state 2 to the original state 1 along a different path C. For this reversible cyclic process, we have

\[
\int_{1}^{2} (A) \frac{\delta Q}{T} + \int_{2}^{1} (C) \frac{\delta Q}{T} = 0
\]

From above equation we have,

\[
\int_{1}^{2} (B) \frac{\delta Q}{T} = \int_{2}^{1} (C) \frac{\delta Q}{T}
\]

Above equation indicates that no restriction is imposed on paths, except that they must be reversible, the quantity \( \frac{\delta Q}{T} \) is a function of the initial and final states of the.
system and is independent of the path of the process. Hence it represents a property of the system.

4.5 Clausius Inequality

- When a system undergoes a complete cyclic process, the integral of \( \frac{\delta Q}{T} \) around the cycle is less than zero or equal to zero.
- Mathematically,
  \[
  \oint \left( \frac{\delta Q}{T} \right) \leq 0
  \]
- Consider a reversible engine R and irreversible engine I working between two thermal reservoir at temperature \( T_H \) and \( T_L \).
- Efficiency of reversible engine,
  \[
  \eta_R = 1 - \frac{\delta Q_L}{\delta Q_H} = 1 - \frac{T_L}{T_H}
  \]
- Efficiency of irreversible engine,
  \[
  \eta_I = 1 - \frac{\delta Q_L}{\delta Q_H} \neq 1 - \frac{T_L}{T_H}
  \]
- We know that efficiency of reversible engine is more than irreversible engine under same temperature limit.

\[
\therefore \eta_R > \eta_I
\]
\[
\therefore \left( 1 - \frac{\delta Q_L}{\delta Q_H} \right)_R > \left( 1 - \frac{\delta Q_L}{\delta Q_H} \right)_I
\]
\[
\therefore \left( 1 - \frac{T_L}{T_H} \right)_R > \left( 1 - \frac{\delta Q_L}{\delta Q_H} \right)_I
\]
\[
\therefore \frac{T_L}{T_H} < \left( \frac{\delta Q_L}{\delta Q_H} \right)_I
\]
\[
\therefore \frac{\delta Q_H}{T_H} < \frac{T_L}{T_L}
\]
\[
\therefore \frac{\delta Q_H}{T_H} - \frac{\delta Q_L}{T_L} < 0
\]
- We know heat added should be positive and heat rejected should be negative.

\[
\therefore \frac{\delta Q_H}{T_H} - \left( \frac{-\delta Q_L}{T_L} \right) < 0
\]
\[
\therefore \frac{\delta Q_H}{T_H} + \frac{\delta Q_L}{T_L} < 0
\]
- Considering complete original irreversible cycle,
  \[
  \therefore \left[ \left( \frac{\delta Q_{H1}}{T_{H1}} \right) + \left( \frac{\delta Q_{L1}}{T_{L1}} \right) + \left( \frac{\delta Q_{H2}}{T_{H2}} \right) + \left( \frac{\delta Q_{L2}}{T_{L2}} \right) + \cdots \right] \quad \oint \left( \frac{\delta Q}{T} \right)_I < 0
  \]
According to Clausius theorem $\oint (\delta Q/T) = 0$ for reversible cycle. Combing results for reversible and irreversible cycle,

$$\oint \left( \frac{\delta Q}{T} \right) \leq 0$$

This expression known as Clausius inequality.

$$\oint \left( \frac{\delta Q}{T} \right)_{\text{Reversible}} = 0$$
$$\oint \left( \frac{\delta Q}{T} \right)_{\text{Irreversible}} < 0$$
$$\oint \left( \frac{\delta Q}{T} \right)_{\text{Impossible}} > 0$$

4.6 Principle of increase of entropy or change in entropy of the universe

- The entropy of an isolated system during a process always increases or in the limiting case of reversible process remains constant.
- Mathematically,

$$\Delta S_{\text{isolated}} \geq 0$$

- Now, consider any system and its surrounding within a single boundary as shown in fig. The combination of system and surrounding within a single boundary is called the universe.

![Figure 3 Entropy change of universe](image)

- Applying the principle of increase in entropy.

$$dS_{\text{Universe}} \geq 0$$

Where,

$$dS_{\text{Universe}} = dS_{\text{System}} + dS_{\text{Surrounding}}$$

- In the combined closed system, the heat $\delta Q$ transferred from the system to surrounding.
– For reversible process,

\[ dS_{\text{system}} = \frac{-\delta Q}{T_{\text{sys}}} \]

\[ dS_{\text{surr}} = \frac{\delta Q}{T_{\text{surr}}} \]

– Total change in entropy for the combined system,

\[ dS_{\text{system}} + dS_{\text{surr}} = \delta Q \left( \frac{1}{T_{\text{sys}}} + \frac{1}{T_{\text{surr}}} \right) \]

\[ \therefore dS_{\text{Universe}} = \delta Q \left( \frac{1}{T_{\text{surr}}} - \frac{1}{T_{\text{sys}}} \right) \]

– But, \( T_{\text{sys}} > T_{\text{surr}} \),

\[ \therefore \delta Q \left( \frac{1}{T_{\text{surr}}} - \frac{1}{T_{\text{sys}}} \right) > 0 \]

\[ \therefore dS_{\text{Universe}} \geq 0 \]

– Above equation states that the process involving the integration of a system and the surroundings, net entropy of universe increases or remains constant.

– Since all natural processes are irreversible, the entropy is increasing continuously. The entropy becomes maximum value when the system reaches a stable equilibrium state from non-equilibrium state.

### 4.7 Applications of entropy principle

– Mixing of two fluid

– Heat transfer through a finite temperature difference

– Maximum work obtained from two finite bodies

– Minimum work required for refrigerator operates between two finite bodies

– Isothermal dissipation of work

– Adiabatic dissipation of work

### 4.8 Change of Entropy in a Reversible Process

– Let,

– \( S_1 \) = Entropy at the initial state 1, and

– \( S_2 \) = Entropy at the final state 2.

– Then, the change in entropy of a system, as it undergoes a change from state 1 to 2, becomes,

\[ S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right) \]
Lastly, if the two equilibrium states 1 and 2 are infinitesimal near to each other, the integral sign may be omitted and $S_2 - S_1$ becomes equal to $dS$.

Hence equation may be written as

$$dS = \int_1^2 \left( \frac{\delta Q}{T} \right)$$

Where $dS$ is an exact differential.

Thus, from equation, we find that the change of entropy in a reversible process is equal to $\delta Q/T$. This is the mathematical formulation of the second law of thermodynamics.

Equation indicates that when an inexact differential $\delta Q$ is divided by an integrating factor $T$ during a reversible process, it becomes an exact differential.

### 4.9 Entropy and Irreversibility

We know that change in entropy in a reversible process is equal to $\frac{\delta Q}{T}$. Let us now find the change in entropy in an irreversible process.

Consider a closed system undergoing a change from state 1 to state 2 by a reversible process 1-L-2 and returns from state 2 to the initial state 1 by an irreversible process 2-M-1 as shown in Fig. on the thermodynamic coordinates, pressure and volume.
Since entropy is a thermodynamic property, we can write

\[ \int dS = \int_{2(L)}^{2} (dS)_R + \int_{2(M)}^{1} (dS)_I = 0 \]

Now for a reversible process, from equation, we have

\[ \int_{2(L)}^{2} (dS)_R = \int_{1(L)}^{2} \left( \frac{\delta Q}{T} \right)_R \]

From above equation,

\[ \int_{1(L)}^{2} \left( \frac{\delta Q}{T} \right)_R + \int_{2(M)}^{1} (dS)_I = 0 \]

Again, since in above equation the processes 1-L-2 and 2-M-1 together form an irreversible cycle, applying Clausius equality to this expression, we get

\[ \int \left( \frac{\delta Q}{T} \right) = \int_{1(L)}^{2} \left( \frac{\delta Q}{T} \right)_R + \int_{2(M)}^{1} \left( \frac{\delta Q}{T} \right)_I < 0 \]

Now subtracting above equation,

\[ \int_{2(M)}^{1} (dS)_I > \int_{2(M)}^{1} \left( \frac{\delta Q}{T} \right)_I \]

This for infinitesimal changes in states can be written as

\[ (dS)_I > \int \left( \frac{\delta Q}{T} \right)_I \]

Combining equation, we can write the equation in the general form as,

\[ dS \geq \frac{\delta Q}{T} \]

Where equality sign stands for the reversible process and inequality sign stands for the irreversible process. It may be noted here that the effect of irreversibility is always to increase the entropy of the system.

**4.10 Entropy Changes For a Closed System**

1) **Heating of a Gas at Constant Volume Process**

Let 1 kg of gas be heated at constant volume and let the change in entropy and absolute temperature be from s1 to s2 and T1 to T2 respectively as shown in Fig.

\[ \delta q = C_v dT \]

Divided by T both the sides,

\[ \frac{\delta q}{T} = C_v \frac{dT}{T} \]
4. Entropy

Engineering Thermodynamics (2131905)

Figure 6 T-S diagram for constant volume process

- Integrating both the sides, we obtain

\[ \int_{1}^{2} ds = C_v \frac{dT}{T} \]

\[ s_2 - s_1 = C_v \ln \frac{T_2}{T_1} = C_v \ln \frac{P_2}{P_1} \]

2) Heating of a Gas at Constant Pressure Process

- Let 1 kg of gas be heated at constant pressure and let the change in entropy and absolute temperature be from \( s_1 \) to \( s_2 \) and \( T_1 \) to \( T_2 \) respectively as shown in Fig.

\[ \delta q = C_p dT \]

Figure 7 T-S diagram for constant pressure process

- Divided by \( T \) both the sides,

\[ \frac{\delta q}{T} = C_p \frac{dT}{T} \]
4. Entropy

- Integrating both the sides, we obtain

\[ ds = C_p \frac{dT}{T} \]

\[ s_2 - s_1 = C_p \ln \frac{T_2}{T_1} = C_p \ln \frac{P_2}{P_1} \]

3) Heating of Gas at Isothermal Process

- An isothermal expansion 1-2 at constant temperature \( T \) is shown in Fig. Entropy changes from \( s_1 \) to \( s_2 \) when gas absorbs heat during expansion. The heat taken by the gas is given by the area under the line 1-2 which also represents the work done during expansion.

![Figure 8 T-S diagram for isothermal process](image)

- For unit mass of substance, \( \delta q = du + \delta w = C_idT + \delta w \)
- If the process is reversible, \( \delta w = pdv \)
- Since the temperature remains constant, \( du = 0 \) and therefore, \( \delta q = pdv \)
- Divided by \( T \) both the sides,

\[ \frac{\delta q}{T} = \frac{pdv}{T} \]

but \( pv = RT \) for unit mass of the gas

\[ \frac{\delta q}{T} = \frac{pdv}{T} \]

\[ ds = \frac{Rdv}{v} \]

- Integrating both the sides, we obtain

\[ \int_{1}^{2} ds = R \frac{dv}{v} \]
4. Entropy

\[
s_2 - s_1 = R \ln \frac{v_2}{v_1} = R \ln \frac{P_2}{P_1}
\]

4) **Reversible Adiabatic Process (Isentropic Process)**

- During an adiabatic process as heat is neither supplied nor rejected by the system, \( \delta q = 0 \)
- Divided by \( T \) both the sides then,
  \[
  \frac{\delta q}{T} = 0 \\
  ds = 0
  \]

![Figure 9: T-S diagram for adiabatic process](image)

- This shows that there is no change in entropy and hence it is known as *isentropic process*. Fig. represents an adiabatic process. It is a vertical line 1-2 and therefore area under this line is nil; hence heat supplied or rejected and entropy change is zero.

5) **Polytropic Process**

(a) **Entropy change in terms of temperature and volume:**

- for unit mass of substance, \( \delta q = du + \delta w = C_v dT + \delta w \)
  \[
  \delta q = C_v dT + pdv
  \]
- Divided by \( T \) both the sides,
  \[
  \frac{\delta q}{T} = C_v \frac{dT}{T} + p \frac{dv}{v} \quad (pv = RT)
  \]
  \[
  ds = C_v \frac{dT}{T} + \frac{R}{v} dv
  \]
- Integrating both the sides between initial and final states, we obtain
  \[
  \int_{s_1}^{s_2} ds = C_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v}
  \]
4. Entropy

(b) Entropy change in terms of pressure and volume:

- Apply combined gas equation,

\[
\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}
\]

- \[
\frac{T_2}{T_1} = \frac{p_2 v_2}{p_1 v_1}
\]

- Substitute the value of \(\frac{T_2}{T_1}\) in above equation,

\[
s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}
\]

(c) Entropy change in terms of temperature and pressure:

- Apply combined gas equation,

\[
\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}
\]

\[
\frac{v_2}{v_1} = \frac{T_2 p_1}{T_1 p_2}
\]

- Substitute the value of \(\frac{v_2}{v_1}\) in equation,

\[
s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{T_2 p_1}{T_1 p_2}
\]

4.11 Third Law of Thermodynamics (Nernst Law)

- From a statistical point of view, entropy is a measure of molecular randomness (the uncertainty about the positions of molecules at any instant). At absolute zero temperature molecules become completely motionless. Therefore, the entropy of a pure crystalline substance at absolute zero temperature is zero since there is no uncertainty about the state of the molecules at that instant.
This is third law of thermodynamics which states that

“The entropy of all perfect crystalline substance (solid) is zero at absolute zero temperature.’

If a system exists in its equilibrium crystalline state, its atoms are arranged in a pattern that represents the maximum degree of order, and if it also at absolute zero temperature, there must be a minimum of disordering thermal motion. Thus, it seems logical to assign zero absolute entropy to any system exhibiting its maximum state of order and minimum thermal motion.

The third law of thermodynamics provides an absolute reference point for the determination of entropy. The entropy determined relative to this reference point is called absolute entropy. Application of Third law of thermodynamics

(1) Provides an absolute reference point for the determination of entropy
(2) Explaining the behavior of solids at very low temperature.
(3) Measurement of action of chemical forces of the reacting substances.
(4) Analyzing the chemical and phase equilibrium.

Entropy summary
In this chapter, we have introduced the property entropy and illustrated its use for thermodynamic analysis. Like mass and energy, entropy is an extensive property that can be transferred across system boundaries. Entropy transfer accompanies both heat transfer and mass flow. Unlike mass and energy, entropy is not conserved but is produced within systems whenever internal irreversibilities are present.

Reference Books:
1. Thermal Science and Engineering by D. S. Kumar
2. Engineering Thermodynamics by R. K. Rajput
3. Fundamental of Engineering Thermodynamics by Michael J. Moran

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Never say ‘No’, never say ‘I cannot’, for you are infinite. All the power is within you. You can do anything.
– Swami Vivekananda
Course Contents

5.1 Introduction
5.2 Basic Definitions
5.3 Available Energy Referred to a Cycle
5.4 Exergy (availability) of the closed system
5.5 Availability of the Open System (Steady Flow System)
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5.7 Reversibility and Irreversibility
5.8 Cause of irreversibility
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5. Energy

5.1 Introduction

- Exergy is “the maximum useful work that could be obtained from the system at a given state in a specified environment.” In other words the exergy of a system is the maximum useful work possible during a process that brings the system into equilibrium with a heat reservoir. When the surroundings are the reservoir, exergy is the potential of a system to cause change as it achieves equilibrium with its environment.
- Exergy is then the energy that is available to be used. Therefore, it is also called the availability or available energy. After the system and surroundings reach equilibrium the exergy becomes zero.
- Anergy is the “energy which is not utilizable and rejected to the surroundings.” It is also called unavailable energy.
- For any thermodynamics system,
  
  \[
  \text{Energy supplied} = \text{Work done} + \text{Heat rejected} = \text{Available energy} + \text{Unavailable energy} = \text{Exergy} + \text{Anergy}
  \]

5.2 Basic Definitions

1) Available Energy

- It is maximum portion of energy which can be converted into work by reversible processes which reduce the system to a dead state.

2) Unavailable Energy

- It is that portion of energy which cannot be converted into work even by reversible process which reduces system in a state of equilibrium. That energy is rejected to sink.

3) Dead State

- When system comes to complete equilibrium with its environment there is no energy difference exists to promote further work is called dead state.

![Figure 1 Representation of dead state and availability](image-url)
5.3. Exergy (Available Energy) referred to a cycle

1) Exergy referred to infinite heat source
   - As shown in Fig. represents a reversible engine that operates between a constant temperature reservoir at temperature \( T \) and a sink at temperature \( T_0 \).
   - Heat \( Q \) supplied by the reservoir and the available work \( W_{\text{max}} \).

   ![Figure 2 Available and unavailable energy: heat withdrawn from an infinite reservoir](image)

   - Efficiency of the reversible heat engine,
     \[
     \eta_{\text{rev}} = \frac{T - T_0}{T} = \frac{W_{\text{max}}}{Q}
     \]

     \[
     W_{\text{max}} = Q \left( \frac{T - T_0}{T} \right) = Q \left( 1 - \frac{T_0}{T} \right) = Q - \frac{Q}{T} T_0 = Q - T_0 dS
     \]
     \[
     Q = W_{\text{max}} + T_0 dS
     \]
   - But, we know that Heat supplied = available energy + Unavailable energy
     \[
     Q = W_{\text{max}} + UE
     \]
   - From above equation, Unavailable energy \( UE = T_0 dS \)
   - \( W_{\text{max}} \) is the availability and on \( T-S \) diagram it is given by the area 1-2-3-4. The area 3-4-5-6 represents the unavailable portion of the heat supplied to the engine.
   - Unavailable energy is the energy rejected from the engine, and hence represents the portion of heat supplied that cannot be converted into work. The unavailable energy equals the product of the lowest temperature of heat rejection and the change of entropy of the system during the process of heat supply (unavailable energy = \( T_0 dS \)).

2) Exergy referred to finite heat source/ Lost work-Exergy destruction in heat transfer process
   - Consider certain quantity of heat \( Q \) transfer from a system at constant temperature \( T_1 \) to another system at constant temperature \( T_0 \) \( (T_1 > T_0) \) as shown in Fig. Before heat is transfer, the energy \( Q \) is available at \( T_1 \) and ambient temperature is \( T_0 \).
Figure 3 Decrease in available energy due to heat transfer through a finite temperature difference

- Initial available energy is given by,

\[(AE)_1 = Q \left(1 - \frac{T_0}{T_1}\right)\]

- After heat transfer, the energy \(Q\) is available at \(T_2\) and again the ambient temperature is \(T_0\).

- Final available energy is given by,

\[(AE)_2 = Q \left(1 - \frac{T_0}{T_2}\right)\]

- Change in available energy \(= (AE)_1 - (AE)_2\)

\[= Q \left(1 - \frac{T_0}{T_1}\right) - Q \left(1 - \frac{T_0}{T_2}\right)\]

\[= T_0 \left(- \frac{Q}{T_1} + \frac{Q}{T_2}\right)\]

\[= T_0 \left(dS_1 + dS_2\right)\]

- Change in available energy \(= T_0 dS_{\text{net}}\)

- Where, \(dS_{\text{net}}\) is the net change in the entropy of the combination of the two interacting systems. This total entropy change is called entropy of universe or entropy production. Since the heat transfer has been through a finite temperature difference, the process is irreversible, i.e., \(dS_{\text{net}} > 0\) and hence there is loss or decrease of available energy.

Description

- \(abcd\) is the power cycle when heat is available at \(T_1\). Area under \(cd\) then represents the unavailable portion of energy.
\- \textit{efgd} is the power cycle when heat is available at $T_2$. Area under $dg$ then represents the unavailable energy.

\- Increase in unavailable energy due to irreversible heat transfer is then represented by the shaded area under $eg$. The increase in unavailable energy equals the decrease in available energy, and is given by the product of the ambient temperature and the net increase in the entropy of the interacting systems.

**Conclusion**

\- Whenever heat is transferred through a finite temperature difference, there is always a loss of available energy.

\- Greater the temperature difference $T_1 - T_2$, the more net increase in entropy and, therefore, loss of available energy.

\- The available energy of a system at a higher temperature is more than at a lower temperature, and decreases progressively as the temperature falls. Stated in other words, quality of energy of a fluid at higher temperature is superior to that at lower temperature; a high temperature fluid has capacity to do more work under the same identical conditions.

**5.4 Exergy (availability) of the closed system (Non Flow System)**

\- Consider a piston-cylinder arrangement (closed system) in which the fluid expanding reversibly from initial state of $p_1$, $V_1$, $T_1$ to final atmospheric state of $p_0$, $V_0$, $T_0$. During this process, fluid produces work $W_{\text{fluid}}$ and rejects heat $Q$ to atmosphere. To find the maximum work done, we will assume that the heat $Q$ rejected by the system is utilized in reversible Carnot engine producing work $W_{\text{engine}}$ and rejecting part of heat $Q_0$ to atmosphere at temperature $T_0$, as shown in Fig. practically this would be possible by having infinite number of reversible engines arranged in parallel, each one receiving heat at a different constant temperature but each one rejecting heat at atmospheric temperature $T_0$.

\- Following are the heat and work interactions take place in given system:

\- **Expansion work** ($W_{\text{exp}}$):

\- The fluid expands and expansion work $W_{\text{exp}}$ is obtained. From the principal of energy conservation,

$$\delta Q = \delta W + du$$

$$-Q = W_{\text{exp}} + (u_0 - u_1)$$

\- Negative sign indicate that heat leaves the system

$$W_{\text{exp}} = (u_1 - u_0) - Q$$
Engine work ($W_{\text{eng}}$):

Heat rejected by piston cylinder assembly may be utilized to run reversible heat engine which receives. The work done by the engine is given by,

$$W_{\text{eng}} = Q \left(1 - \frac{T_0}{T_1}\right) = Q - T_0 \left(S_1 - S_0\right)$$

Maximum work ($W_{\text{max}}$):

The summation of expansion work $W_{\text{exp}}$ and the engine work $W_{\text{eng}}$ gives maximum obtainable from the given arrangement,

$$W_{\text{max}} = (u_1 - u_0) - Q - T_0 \left(S_1 - S_0\right)$$

Surrounding work ($W_{\text{surr}}$):

When the piston moving outwards has to spend a work in pushing the atmosphere against its own pressure. This work, which may be called as the surroundings work is simply dissipated, and such is not useful. It is given by,

$$W_{\text{surr}} = p_0 \left(V_0 - V_1\right)$$
Figure 5 As a closed system expands, some work needs to be done to push the atmospheric air out of the way.

- **Net work** ($W_{net}$):
  - The energy available for work transfer less work absorbed in moving the environment is called the useful work or net work.
  - Maximum available useful work or net work is given by,
    \[
    W_{net} = W_{max} - W_{surr}
    \]
    \[
    W_{net} = (u_1 - u_0) - T_0 (S_1 - S_0) - p_0 (V_0 - V_1)
    \]
    \[
    W_{net} = (u_1 + p_0 V_1 - T_0 S_1) - (u_0 + p_0 V_0 - T_0 S_0)
    \]
    \[
    W_{max} = A_1 - A_0
    \]
  - Where $A = u + p_0 V - T_0 S$ is known as non-flow availability function. It is composite property of the system and surroundings as it consists of three extensive properties of the system ($u, V, S$) and two intensive properties of the surroundings ($p_0, T_0$).
  - When system undergoes change from state 1 to state 2 without reaching the dead state, then
    \[
    (W_a)_{max} = W_{net} = (A_1 - A_0) - (A_2 - A_0) = A_1 - A_2
    \]

5.5 Exergy (availability) of steady flow open system

- Consider a flow of fluid through a open system as shown in fig. The working fluid enter the system at section 1 ($p_1, \nu_1, T_1, U_1, C_1$) and leave the system at section ($p_0, \nu_0, T_0, U_0, C_0$) and passing at a steady rate. Let the system rejects heat $Q$, which for getting maximum work should be passed through a reversible engine.
  - Steady flow energy equation may written as,
    \[
    U_1 + p_1 V_1 + m C_1^2 \over 2 + m g z_1 - Q = U_0 + p_0 V_0 + m C_0^2 \over 2 + m g z_0 - W_{system}
    \]
  - Neglecting kinetic and potential energy changes,
    \[
    u_1 + p_1 V_1 - Q = u_0 + p_0 V_0 + W_s
    \]
    \[
    H_1 - Q = H_0 + W_s
    \]
    Shaft work, $W_s = (H_1 - H_0) - Q$
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5.6 Second law efficiency or effectiveness

- The heat $Q$ rejected by the system may be utilized to run a reversible heat engine. The work output from this engine is,

$$W_{\text{eng}} = Q \left(1 - \frac{T_0}{T_i}\right) = Q - T_0 \left(S_i - S_0\right)$$

- Maximum available useful work or net work is given by,

$$W_{\text{net}} = W_s + W_{\text{eng}}$$

$$W_{\text{net}} = (H_i - H_0) - Q + Q - T_0 \left(S_i - S_0\right)$$

$$W_{\text{net}} = (H_i - T_0 S_i) - (H_0 - T_0 S_0)$$

$$W_{\text{net}} = B_1 - B_0$$

- Where $B = H - T_0 S$ is known as steady flow availability function.

Figure 6 Availability of open flow system

- Normally, performance of a process or device is measured with help of thermal efficiency or coefficient of performance. They are defined on basis of first law and hence it is referred to as the first law efficiency. But, this first law efficiency is not a measure of the fraction of maximum work actually utilized. Therefore, there is need to define a second law efficiency as the ratio of the actual thermal efficiency to the maximum possible thermal efficiency under same condition.

- Consider two heat engines, both having a thermal efficiency of 30%, as shown in Fig. One of the engines (engine A) is supplied with heat from a source at 600 K, and the other one (engine B) from a source at 1000 K. Both engines reject heat to a medium at 300 K. At first glance, both engines seem to convert to work the same fraction of heat that they receive; thus they are performing equally well. When we take a second look at these engines in light of the second law of thermodynamics, however, we see a totally different picture. These engines, at best, can perform as reversible engines, in which case their efficiencies would be,
Now it is becoming apparent that engine B has a greater work potential available to it (70% of the heat supplied as compared to 50% for engine A), and thus should do a lot better than engine A. Therefore, we can say that engine B is performing poorly relative to engine A even though both have the same thermal efficiency. It is obvious from this example that the first-law efficiency alone is not a realistic measure of performance of engineering devices. To overcome this deficiency, we define second-law efficiency.

Second-law efficiency is defined as the ratio of the actual thermal efficiency to the maximum possible (reversible) thermal efficiency under the same conditions.

\[ \eta_{1} = \frac{\eta_{th}}{\eta_{th,rev}} \] (heat engines)

Based on this definition, the second-law efficiencies of the two heat engines discussed above are,

\[ \varepsilon = \frac{\eta_{th}}{\eta_{rev}} \]

<table>
<thead>
<tr>
<th>( \eta_{th} )</th>
<th>( \eta_{rev} )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>0.30</td>
<td>0.70</td>
<td>0.4285</td>
</tr>
</tbody>
</table>

It means that engine 1 is converting 60% of the available work potential to useful work. And for engine 2 is 42.85%.
5.7 Reversibility and Irreversibility

- The concept of maximum work, useful to make some standard to which the actual performance of engineering devices can be compared. The maximum work is possible when the processes are totally reversible and satisfy the conditions of reversibility.

**Conditions of reversibility:**

- *The criterion for reversibility are as follows:*
  
  a) The process must be free from internal (molecular friction) and mechanical friction.
  
  b) The energy transfer as heat and work during the forward process should be identically equal to energy transfer as heat and work during the reversal of the process.
  
  c) The process should be quasi-static, i.e. the process must proceed in a series of equilibrium state, it should proceed at infinitely slow speed. For this pressure difference between system and surroundings must be infinitely small.
  
  d) The heat transfer to or from system, if any, should be only through infinitely small temperature difference.
  
  e) There should be no free expansion.
  
  f) There should be no mixing of the fluids.

- In fact, all the processes in nature are irreversible. Therefore, the actual work done by system always less than theoretical idealized work done (reversible work done) due to the presence of Irreversibilities with processes. The irreversibility represents the loss of work due to the fact that all real processes are irreversible. Therefore, the irreversibility of the process is defined as “the difference of reversible maximum work (idealized work) and the actual work”.

- Mathematically,

\[ I = W_{\text{max}} - W_{\text{actual}} \]

5.8 Cause of irreversibility

- The irreversibility of a process may be due to either one or both of the following,

  (a) **Lack of equilibrium (mechanical, chemical or thermal equilibrium) during the process**

  (i) *Heat transfer through a finite temperature difference:*

  - We know that the reversible process in which heat is transferred through an infinitesimal temperature difference would require an infinite amount of time or infinite area. But, all actual heat transfer process are taken place through a finite temperature difference and so irreversible. Hence heat transfer take place greater temperature difference, the greater is the irreversibility.

  (ii) *Lack of pressure equilibrium within the system or between system and surroundings:*
When pressure difference exists within the system or between system and surroundings, then both the system and its surrounding or only system will undergo a change of state. The reverse process of this process is not possible without producing any other effect.

(iii) Free expansion:

The process of free expansion is irreversible.

(b) Irreversibility due to dissipative effects

Some time the work is converted into molecular internal energy through the agency of such phenomena as friction, viscosity, inelasticity, electric resistance and magnetic hysteresis. These effects are called dissipative effects, and this effects increases irreversibility of a process.

(i) Friction:

In case of brake, kinetic energy of flywheel is converted into molecular internal energy due to friction. However, the conversion of this increase in molecular internal energy into kinetic energy within the system to cause the wheel to rotate is not possible.

(ii) Stirring work:

When paddle wheel rotates in the insulated container, work may be transferred into a system. The work transferred is dissipated adiabatically into an increase in the molecular internal energy of (i.e. increase in the temperature of fluid). The same amount of work produce by the system at the expense of its molecular internal energy and temperature of the system goes down, is not possible. So stirring work is irreversible process.

(iii) Transfer of electricity through a resistor:

The flow of electric current through a wire represents work transfer. The part of the work transfer is stored as an increase in the internal energy of the wire and remainder leaves the system as heat. The reverse process, the conversion of heat into electric work is not possible.

Irreversibilities are two types as

(1) External irreversibilities:

These are associated with dissipating effects outside the working fluid, i.e. Mechanical friction occurring during a process due to some external source.

Internal irreversibilities:

These are associated with dissipating effects within the working fluid

(2) Mechanical irreversibilities

It is generally associated with friction between fluid molecules, friction between fluid and mechanical parts, friction between mechanical parts and atmosphere.

(3) Thermal irreversibilities

It is associated with heat transfer due to finite temperature difference between the parts of system or between system and surrounding.
5.9 Irreversibility and gouy-stodala theorem

a) Irreversibility of closed system
- It is defined as the difference between the maximum work output from the system and the expansion work. That is,

\[ I = \left[ (u_1 - T_0 S_1) - (u_2 - T_0 S_2) \right] - \left[ -Q - (u_2 - u_1) \right] \]

\[ I = T_0 \left( S_2 - S_1 \right) + Q \]

\[ I = T_0 \left( dS \right)_{sys} + Q \]

- Change in entropy of environment due to addition of heat \( Q \) at constant atmospheric temperature \( T_0 \),

\[ dS_{surr} = \frac{Q}{T_0} \; ; \; Q = T_0 dS_{surr} \]

\[ I = T_0 \left[ (dS)_{sys} + (dS)_{surr} \right] = T_0 (dS)_{net} \]

\[ I = T_0 \left( dS \right)_{univ} \]

b) Irreversibility refers to steady flow system:

\[ I = \left[ (h_1 - T_0 S_1) - (h_2 - T_0 S_2) \right] - \left[ (h_1 - h_2) - Q \right] \]

\[ I = T_0 \left( S_2 - S_1 \right) + Q \]

\[ I = T_0 \left( dS \right)_{sys} + T_0 \left( dS \right)_{surr} = T_0 \left( dS \right)_{net} \]

\[ I = T_0 \left( dS \right)_{univ} \]

Gouy-stodala Theorem:
- The rate of loss of exergy (available energy) in a process is proportional to the rate to the rate of entropy generation.

\[ I = W_{lost} = T_0 (\Delta S_{universe}) = T_0 S_{gen} \]

Application of Gouy-stodala theorem:
- Heat transfer through a finite temperature difference
- Flow with friction
- Mixing of two fluid

1) Heat transfer through a finite temperature difference
- Consider two thermal reservoir having finite temperature difference \( T_1 - T_2 \) as shown in fig.

- The heat \( Q \) from hot reservoir at \( T_1 \) to cold reservoir at \( T_2 \). Now consider reversible heat engine working between these two reservoirs.

- The engine absorbs \( Q_1 \) heat from source and reject \( Q_2 \) heat to heat sink.
In this chapter, we have introduced the property exergy and illustrated its use for thermodynamic analysis. Like mass, energy, and entropy, exergy is an extensive property that can be transferred across system boundaries. Exergy transfer accompanies heat transfer, work, and mass flow. Like entropy, exergy is not conserved. Exergy is destroyed within systems whenever internal irreversibilities are present. Entropy production corresponds to exergy destruction.
Reference Books:
1. Thermal Science and Engineering by D. S. Kumar
2. Engineering Thermodynamics by R. K. Rajput

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You have to grow from the inside out. None can teach you, none can make you spiritual. There is other teacher but your own soul.
6

VAPOR POWER CYCLES

Course Contents

6.1 Performance Parameters of Vapor Power Cycles
6.2 The Carnot Vapor Cycle
6.3 The Rankine Cycle
6.4 Differences Between Carnot Vapor and Rankine Cycle
6.5 Effect of Operating Variables on Rankine Cycle Performance
6.6 Reheat Rankine Cycle
6.7 Regenerative Rankine Cycle (With Open Feed Water Heater)
6.8 Solved Numerical
6.9 References
6.1 Performance Parameters of Vapor Power Cycle

a) Thermal Efficiency

Thermal efficiency is given by,

\[ \eta = \frac{\text{Net work done in the cycle}}{\text{Heat supplied in the cycle}} = \frac{w_{\text{net}}}{q_1} \]

b) Work Ratio

The work ratio for a power plant is defined as ratio of the net work output of the cycle to the work developed by the turbine. It is expressed as,

\[ \text{Work ratio} = \frac{w_{\text{net}}}{w_t} \]

\[ \therefore \text{Work ratio} = \frac{w_t - w_p}{w_t} = 1 - \frac{w_p}{w_t} \]

A low work ratio implies large pump work. Larger the pump work, lower the work ratio.

c) Steam Rate or Specific Steam Consumption (SSC)

It is defined as the flow rate of steam per unit of power developed (kWh).

\[ SSC = \frac{\text{Steam flow rate}}{\text{Power output}} \]

\[ SSC = \frac{3600}{w_{\text{net}}}; \text{kg/kWh} \]

Where, \( w_{\text{net}} \) is in kJ/kg.

d) Heat Rate

Thermal efficiency can be expressed as heat rate, which is a measure of the rate of heat input \( q_1 \) required to produce unit work output (1kW).

\[ \text{Heat rate} = \frac{3600 \times q_1}{w_{\text{net}}} \]

\[ \therefore \text{Heat rate} = \frac{3600}{\eta_{\text{th}}}; \text{kJ/kWh} \]

6.2 The Carnot Vapor Cycle

- The Carnot cycle is the most efficient cycle operating between two specified temperature limits. The Carnot cycle is an ideal cycle for vapor power plants. However, the Carnot cycle is not a suitable model for power cycles, as there are so many practical difficulties are associated with it (discussed later).
Fig. 6.1 shows P-v, T-S, h-S and schematic diagram of the Carnot cycle when steam is used as the working substance.

Consider 1 kg of steam as a working substance for the analysis. The cycle is composed of two isentropic and two isothermal processes. Since the working fluid is vapor, the isothermal processes are also the constant pressure processes. These processes are discussed below:

- **Process 4 – 1: Reversible isothermal heat addition in the boiler**

  Heat is supplied to the feed water (saturated water) in a boiler. Latent heat is supplied and isothermal evaporation of water takes place at constant pressure until the water gets converted into dry & saturated steam.

  Heat added isothermally at temperature $T_1$ is given by,

  \[ q_1 = h_1 - h_4 \]

- **Process 1 – 2: Isentropic expansion in the turbine**
The dry & saturated steam undergoes frictionless adiabatic expansion in the steam turbine and work is done by the system\( (+ve) \).

Turbine work is given by,
\[ w_t = h_1 - h_2 \]

- **Process 2 – 3: Controlled condensation in the condenser**

Steam leaving the turbine is condensed isothermally at constant pressure.

Heat rejected isothermally at temperature \( T_2 \) is given by,
\[ q_2 = h_2 - h_3 \]

- **Process 3 – 4: Isentropic compression in the pump (Pumping process)**

The wet steam is compressed isentropically and it is restored to initial state point 4 and work is done on the steam\( (-ve) \).

Compressor work is given by,
\[ w_c = h_4 - h_3 \]

**Thermal Efficiency of Carnot Vapor Cycle**

- Thermal efficiency,
\[ \eta = \frac{\text{Net Work}}{\text{Heat Supplied}} \]
\[ \eta = \frac{w_{net}}{q_1} \]
\[ \eta = \frac{w_{net}}{q_1} \]

- **Heat supplied** to the working fluid in a boiler,
\[ q_1 = \text{Area under the line } 4 - 1 \text{ in } T - s \text{ diagram} \]
\[ q_1 = T_1 dS = T_1 (S_1 - S_4) \]

As \( S_1 = S_2 \) and \( S_3 = S_4 \)
\[ \therefore q_1 = T_1(S_2 - S_3) \]

- **Heat rejected** during condensation process,
\[ q_2 = \text{Area under the line } 2 - 3 \text{ in } T - s \text{ diagram} \]
\[ q_2 = T_2 dS = T_2 (S_2 - S_3) \]

- The **net work done** of the cycle,
\[ w_{net} = q_1 - q_2 \]

From equation 6.2 and 6.3,
\[ \therefore w_{net} = T_1(S_2 - S_3) - T_2(S_2 - S_3) \]
\[ \therefore w_{net} = (S_2 - S_3)(T_1 - T_2) \]
- From equation 6.1,
Thermal efficiency,
\[ \eta_{\text{Carnot}} = \frac{(S_2 - S_3)(T_1 - T_2)}{T_1(S_2 - S_3)} \]
\[ \therefore \eta_{\text{Carnot}} = \frac{T_1 - T_2}{T_1} \]
\[ \therefore \eta_{\text{Carnot}} = 1 - \frac{T_2}{T_1} \quad (6.5) \]

- Equation 6.5 of Carnot vapor cycle efficiency is similar to the Carnot gas power cycle. It shows that, the efficiency of Carnot cycle is depends upon the limit of temperatures and is independent of the nature of working substance.

**Practical Difficulties Associated with Carnot Vapor Power Cycle**

I. The cycle is more difficult to operate in practice with superheated steam due to the necessity of supplying the superheat at constant temperature. So maximum possible temperature is limited.

II. In the turbine, the dry and saturated steam expands isentropically. The quality of steam decreases during expansion. The presence of high moisture content in steam will lead to erosion and wear of the turbine blades.

III. It is difficult to control the condensation at state 3, before reaching to saturated liquid state.

IV. It is difficult to compress a wet vapor (water + steam) isentropically, as required by the process 3-4. Because of large specific volume of vapor than liquid, the compressor size and work input will have to be large and this higher compression work will reduce the thermal efficiency of the plant.

- These practical difficulties limit the use of Carnot cycle as a suitable model for design of steam power plants.

### 6.3 The Rankine Cycle

- Many practical difficulties associated with the Carnot vapor cycle can be eliminated in Rankine cycle. The steam coming out of the boiler is usually in superheated state, and expands in the turbine. After expanding in the turbine, the steam is condensed completely in the condenser.

- The Rankine cycle is the ideal cycle for vapor power plants. The ideal Rankine cycle is shown schematically and on a P-v, T-s & h-s diagrams in Fig. 6.2. The liquid, vapor and wet regions are also indicated with the help of saturation curve.
The ideal Rankine cycle consists of the following four processes:

- **Process 4 – 1: Constant pressure heat addition in the boiler**
  
  Heat is supplied to the feed water (compressed liquid) in a boiler. Sensible heat and latent heat is supplied to the compressed liquid until the liquid gets converted into superheated steam. 
  
  Heat supplied is given by,
  
  \[ q_1 = h_1 - h_4 \]

- **Process 1 – 2: Isentropic expansion in the turbine**
  
  The superheated steam undergoes frictionless adiabatic expansion in the steam turbine and work is done by the system (+ve). Pressure of steam is reduced to condenser pressure.
Turbine work is given by,

\[ w_t = h_1 - h_2 \]

- **Process 2 – 3: Constant pressure heat rejection in the condenser**

Steam leaving the turbine (normally wet steam) is condensed at constant pressure in the condenser till the fluid reaches the saturated liquid state 3.

Heat rejected is given by,

\[ q_2 = h_2 - h_3 \]

- **Process 3 – 4: Isentropic compression in the pump (Pumping process)**

The saturated liquid is compressed isentropically in the pump to the boiler pressure at the state 4 and work is done on the liquid (\(-ve\)).

Pump work is given by,

\[ w_p = h_4 - h_3 \]

**Thermal Efficiency of Rankine Cycle**

- Thermal efficiency,

\[ \eta = \frac{Net\ Work}{Heat\ Supplied} \]

\[ \eta = \frac{w_{net}}{q_1} \] \hspace{1cm} (6.6)

- **Heat supplied** to the working fluid in a boiler during process 4 – 1,

\[ q_1 = h_1 - h_4 \] \hspace{1cm} (6.7)

- **Turbine work** during process 1 – 2,

\[ w_t = h_1 - h_2 \] \hspace{1cm} (6.8)

- **Heat rejected** during condensation process 2 – 3,

\[ q_2 = h_2 - h_3 \] \hspace{1cm} (6.9)

- **Pump work** during process 3 – 4,

\[ w_p = h_4 - h_3 \] \hspace{1cm} (6.10)

Where,

\[ h_3 = h_{f3} \] = Enthalpy of saturated liquid at condenser pressure \(P_2\)

\[ h_4 \] = Enthalpy of compressed liquid at boiler pressure \(P_1\) and can be calculated as,

\[ h_4 = h_3 + w_p \] \hspace{1cm} (6.11)

Isentropic compression work for the flow process can be obtained as,

\[ w_p = \int_{P_2}^{P_1} v dP = v_{f3}(P_1 - P_2) \] \hspace{1cm} (6.12)
Where,

\[ v_f^3 = \text{Specific volume of liquid at condenser pressure } P_2 \]

- The **net work done** of the cycle,

\[ w_{\text{net}} = w_t - w_p \]

From equation 6.8 and 6.12,

\[ w_{\text{net}} = (h_1 - h_2) - v_f^3(P_1 - P_2) - (6.13) \]

- From equation 6.6, 6.7 and 6.13,

Thermal efficiency,

\[ \eta_{\text{Rankine}} = \frac{w_{\text{net}}}{q_1} = \frac{(h_1 - h_2) - v_f^3(P_1 - P_2)}{(h_1 - h_4)} - (6.14) \]

- Since feed pump work is very small compared to turbine work, \( w_p \) is usually neglected.

\[ \therefore \eta_{\text{Rankine}} = \frac{(h_1 - h_2)}{(h_1 - h_4)} - (6.15) \]

### 6.4 Differences Between Carnot and Rankine Cycles

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Carnot Vapor Cycle</th>
<th>Rankine Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>It is a reversible cycle.</td>
<td>It is an irreversible cycle.</td>
</tr>
<tr>
<td>2</td>
<td>It has theoretically maximum efficiency.</td>
<td>It has less thermal efficiency than that of the Carnot vapor cycle.</td>
</tr>
<tr>
<td>3</td>
<td>Heat is added at constant temperature.</td>
<td>Heat is added at constant pressure.</td>
</tr>
<tr>
<td>4</td>
<td>Use of superheated steam is practically difficult.</td>
<td>It uses superheated steam and performs better.</td>
</tr>
<tr>
<td>5</td>
<td>Controlled condensation is required before being saturated liquid.</td>
<td>Complete condensation of steam takes place.</td>
</tr>
<tr>
<td>6</td>
<td>Mixture of water and steam exist after condensation.</td>
<td>Only saturated water exists after condensation.</td>
</tr>
<tr>
<td>7</td>
<td>It requires a large pump work to handle the two phase mixture.</td>
<td>It requires negligible pump work to handle the liquid water only.</td>
</tr>
<tr>
<td>8</td>
<td>Since it uses saturated steam, the moisture content at the end of expansion is much higher which can lead to blade erosion.</td>
<td>It uses superheated steam in the cycle, at the end of expansion; the quality of steam is not objectionable.</td>
</tr>
<tr>
<td>9</td>
<td>It is a theoretical cycle and cannot be used in practice.</td>
<td>Almost all thermal power plants operate on Rankine cycle.</td>
</tr>
</tbody>
</table>
6.5 Effect of Operating Variables on Rankine Cycle Performance

Steam power plants are responsible for the production of most electric power in the world, and even small increases in thermal efficiency can mean large savings from the fuel requirements. Therefore, every effort is made to improve the efficiency of the cycle on which steam power plants operate.

The basic idea behind all the modifications to increase the thermal efficiency of a power cycle is the same: increase the average temperature at which heat is transferred to the working fluid in the boiler, or decrease the average temperature at which heat is rejected from the working fluid in the condenser. That is, the average fluid temperature should be as high as possible during heat addition and as low as possible during heat rejection. It can be seen from equation 6.16.

Mean Temperature of Heat Addition

From the T-s diagram of Rankine cycle we can derive efficiency of Rankine cycle as below:

\[
\eta_{\text{Rankine}} = \frac{W_{\text{net}}}{q_1} = \frac{q_1 - q_2}{q_1}
\]

\[
\therefore \eta_{\text{Rankine}} = 1 - \frac{q_2}{q_1}
\]

\[
\therefore \eta_{\text{Rankine}} = 1 - \frac{T_L(s_1 - s_4)}{T_m(s_1 - s_4)}
\]

\[
\therefore \eta_{\text{Rankine}} = 1 - \frac{T_L}{T_m}-(6.16)
\]

Where,

\[T_L = \text{Temperature of heat rejection}\]

\[T_m = \text{Mean temperature of heat addition}\]

The effect of operating variables i.e. boiler pressure, condenser pressure and superheating are discussed below:

1) Effect of Boiler Pressure

The effect of increasing the boiler pressure on the performance of Rankine cycle is illustrated on a T-s diagram in Fig. 6.3.

By increasing the boiler pressure, the mean temperature of heat addition increases, and thus raises the thermal efficiency of the cycle.
Fig. 6.3 Effect of boiler pressure on Rankine cycle performance

- Notice that for a fixed turbine inlet temperature and condenser pressure, the cycle shifts to the left and the moisture content of steam at the turbine exit increases. This undesirable side effect can be corrected, however, by reheating the steam (discussed later).
- The net work done by the cycle remains almost same, thus, the Rankine cycle efficiency increases, with an increase in maximum pressure or boiler pressure.

2) Effect of Condenser Pressure

- The effect of lowering the condenser pressure on the Rankine cycle efficiency is illustrated on a T-s diagram in Fig. 6.4.
The steam enters the condenser as a saturated mixture of vapor and moisture at the saturation temperature corresponding to the pressure inside the condenser. *If this condenser pressure lowered*, the saturation temperature of exhausted steam is decreases, and thus, the amount of heat rejection in the condenser also decreases, which will increase the efficiency of Rankine cycle.

For comparison purposes, the turbine inlet state is maintained the same. The colored area on this diagram represents the increase in net work output as a result of lowering the condenser pressure from $P_4$ to $P_4'$. The heat input requirements also increase (represented by the area under curve 2 to 2’), but this increase is very small. Thus the overall effect of lowering the condenser pressure is an increase in the thermal efficiency of the cycle.

However there are limitations to this method. These are

- Lowering the condenser pressure causes an increase in moisture content of the steam leaving the turbine. It is an unfavorable effect and may lead to turbine blade erosion.
- To maintain the high vacuum, the air extraction pump will run continuously and its work input will increase, thus reducing the useful work.

### 3) Effect of Superheating

The effect of superheating on the performance of Rankine cycle is illustrated on a T-s diagram in Fig. 6.5.

![Fig. 6.5 Effect of superheating on Rankine cycle performance](image)

- The colored area on this diagram represents the increase in the net work. The total area under the process curve 3-3’ represents the increase in the heat input. Thus both the net work and heat input increase as a result of superheating the steam to a higher temperature.
- The overall effect is an increase in thermal efficiency, however, since the average temperature at which heat added is increases.
6. Vapor Power Cycles

Superheating the steam to higher temperatures has another very desirable effect: It decreases the moisture content of the steam at the turbine exit, as can be seen from the T-s diagram.

However the metallurgical considerations restrict the superheating of steam to a very high temperature.

Quick Review:

- We can conclude that the efficiency of Rankine cycle can be increased by lowering the condenser pressure, by increasing the boiler pressure and by superheating the steam.
- The quality of steam leaving the turbine decreases by lowering the condenser pressure and by increasing the boiler pressure, while it improves by superheating.

6.6 Reheat Rankine Cycle

Increasing the boiler pressure increases the thermal efficiency of the Rankine cycle, but it also increases the moisture content of the steam to unacceptable levels. This side effect can be corrected by reheating the steam.

Reheating is a practical solution to the excessive moisture problem in turbines, and it is commonly used in modern steam power plants.

Reheating is to be done by allowing the steam to expand in the turbine in two or more stages and reheat it in between. In other words, modify the simple ideal Rankine cycle with a reheat process.

Due to reheating, the work output of the turbine increases, thus improving the thermal efficiency.

The T-s diagram of the ideal reheat Rankine cycle and the schematic of the power plant operating on this cycle are shown in Fig. 6.6.

The ideal reheat Rankine cycle differs from the simple ideal Rankine cycle in that the expansion process takes place in two stages. In the first stage (the high pressure turbine), steam is expanded isentropically to an intermediate pressure and sent back to the boiler where it is reheated at constant pressure, usually to the inlet temperature of the first turbine stage. Steam then expands isentropically in the second stage (low-pressure turbine) to the condenser pressure.
Fig. 6.6 Schematic, T-s and h-s diagram of Reheat Rankine cycle

- **Turbine work** per unit mass of steam,
  \[ w_t = WD \text{ in the HP turbine} + WD \text{ in the LP turbine} \]
  \[ w_t = (h_1 - h_2) + (h_3 - h_4) \]

- **Pump work** per unit mass of steam,
  \[ w_p = (h_6 - h_5) \]

- **Net work done** per unit mass of steam,
  \[ w_{net} = w_t - w_p \]
  \[ \therefore w_{net} = (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5) \]

- **Heat supplied** per unit mass of steam,
  \[ q_s = \text{Heat supplied in boiler} + \text{Heat supplied in reheater} \]
  \[ \therefore q_s = (h_1 - h_6) + (h_3 - h_2) \]
6. Reheat cycle efficiency,

\[ \eta_{\text{reheat}} = \frac{\text{Net work done}}{\text{Heat supplied}} \]

\[ \therefore \eta_{\text{reheat}} = \frac{w_{\text{net}}}{q_s} \]

\[ \therefore \eta_{\text{reheat}} = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)} \]

- The reheat cycle reduces the moisture content at low pressure turbine and accordingly the erosion and corrosion problems in the turbine are eliminated.
- It is evident from the T-s diagram that there is very less gain in the thermal efficiency by reheating the steam. However, the mean temperature of heat addition can be increased by including the number of expansion and reheating processes. Thus, the thermal efficiency of the cycle would further increase.

**Advantages of Reheating:**

a) There is an increased output of the turbine.

b) Erosion and corrosion problems in the steam turbine are eliminated/avoided.

c) There is an improvement in the thermal efficiency of the turbines.

d) Final dryness fraction of steam is improved.

e) There is an increase in the nozzle and blade efficiencies.

**Disadvantages of Reheating:**

a) Reheating requires more maintenance.

b) The increase in thermal efficiency is not appreciable in comparison to the expenditure incurred in reheating.

6.7 Regenerative Rankine Cycle (With Open Feed-Water Heater)

- In a simple Rankine cycle, a significant amount of heat is added for sensible heating of compressed liquid coming out of the pump. The mean temperature at which sensible heat added is much lower than the source temperature.

- The mean temperature of heat addition in the Rankine cycle can be improved by decreasing the amount of heat supplied at lower temperatures and it is to be done by preheating the feed water before it enters the boiler.

- For preheating, part of the steam is extracted at some intermediate stage during expansion in the turbine, which is also known as “bleeding” and it is mixed with feed water coming from the condenser. The rest of the steam expands in the turbine to the condenser pressure.
- The device where the feed water is heated by regeneration is called a regenerator, or a feed water heater (FWH).

- Fig. 6.7 shows the schematic. T-s and h-s diagram of a Regenerative Rankine cycle employing open feed water heater.

- Let, 1 kg of steam be leaving the boiler and entering the turbine. \( m_1 \) kg of steam per kg is extracted at the state 2 from the turbine at intermediate pressure \( p_2 \). \((1 - m_1)\) kg of steam per kg flow through the remaining part of the turbine during expansion from 2-3, condensation from 3-4 and pumping from 4-5.

- \((1 - m_1)\) kg of steam enters in open feed water heater and mixed with \( m_1 \) kg of steam extracted from the turbine at the state 2.

- After mixing the mass of saturated liquid becomes 1 kg at the state 6 and it is pumped to boiler pressure at the state 7.

![Fig.6.7 Schematic, T-s and h-s diagram of Regenerative cycle with open feed-water heater](image-url)
- **Heat supplied** in the boiler,
  \[ q_s = (h_1 - h_7) \]

- **Heat rejected** in the condenser,
  \[ q_r = (1 - m_1)(h_3 - h_4) \]

- **Turbine work,**
  \[ w_t = 1 \times (h_1 - h_2) + (1 - m_1)(h_2 - h_3) \]

- **Pump work,**
  \[ w_p = \text{Work input to pump 1 and pump 2} = w_{p1} + w_{p2} \]
  \[ w_p = 1 \times (h_7 - h_6) + (1 - m_1)(h_5 - h_4) \]

- **Thermal efficiency** of the regenerative cycle,
  \[ \eta_{reg} = \frac{\text{Net work}}{\text{Heat supplied}} \]
  \[ \therefore \eta_{reg} = \frac{w_t - w_p}{q_s} \]

**Advantages of Regenerative cycle over Simple Rankine cycle:**

a) It raises the temperature of feed water to saturation temperature, and thus the amount of heat addition in the boiler reduces.

b) With an infinite number of heaters, the heating process in the boiler tends to become reversible.

c) The thermal stresses set up in the boiler are minimized. This is due to the fact that temperature ranges in the boiler are reduced.

d) The thermal efficiency is improved because the average temperature of heat addition to the cycle is increased.

e) The blade height is less due to the reduced amount of steam passed through the low pressure stages.

f) A small size condenser is required.

**Disadvantages of Regeneration:**

a) The plant becomes more complicated and less flexible.

b) Because of addition of heaters greater maintenance is required.

c) For given power a large capacity boiler is required.

d) The heaters are costly and the gain in thermal efficiency is not much in comparison to the heavier costs.
6.8 Solved Numerical

Ex 6.1. [GTU; Jun-2010; 7 Marks]
A Carnot cycle works on steam between the pressures limits of 7 MPa and 7 kPa. Determine (a) The thermal efficiency, (b) Turbine work and (c) Compression work per kg of steam.

Solution:

\[ p_1 = 7 \text{MPa} = 70 \text{bar} \]
\[ p_2 = 7 \text{kPa} = 0.07 \text{bar} \]

To be Calculated:

\[ a) \eta_{th} = ? \]
\[ b) w_t = ? \]
\[ c) w_c = ? \]

From Steam Table:
@ \( p_1 = 70 \text{bar} \)
\[ h_g = 2773.5 \frac{kJ}{kg}, s_g = 5.8162 \frac{kJ}{kg-K}, h_f = 1267.42 \frac{kJ}{kg}, s_f = 3.1219 \frac{kJ}{kg-K} \]

@ \( p_2 = 0.07 \text{bar} \)
\[ h_f = 163.38 \frac{kJ}{kg}, s_f = 0.5591 \frac{kJ}{kg-K}, h_{fg} = 2409.2 \frac{kJ}{kg}, s_{fg} = 7.7176 \frac{kJ}{kg-K} \]

\[ a) \eta_{th} = \frac{h_1 - h_2}{h_0 - h_2} \]

\[ h_1 = (h_g)_{70 \text{bar}} \]
\[ h_2 = (h_f + x_2 h_{fg})_{0.07 \text{bar}} \]
\[ s_1 = s_2 \]
\[ (s_g)_{70 \text{bar}} = (s_f + x_2 s_{fg})_{0.07 \text{bar}} \]
\[ 5.8162 = 0.5591 + x_2 7.7176 \]
\[ x_2 = 0.6811 \]
\[ h_2 = 163.38 + 0.6811 \times 2409.2 \]
\[ h_2 = 1804.286 \frac{kJ}{kg} \]

\[ b) w_t = h_1 - h_2 \]
[Diagram showing the Carnot cycle with points 1, 2, 3, and 4]

\[ c) w_c = h_0 - h_2 \]

\[ s_3 = s_4 \]
\[ (s_f + x_3 s_{fg})_{0.07 \text{bar}} = (s_f)_{70 \text{bar}} \]
Ex 6.2. [GTU; Jan 2015; 7 Marks]

In a Rankine cycle, the steam at inlet to the turbine is saturated at pressure of 35bar and exhaust pressure is 0.2bar. Determine: (a) The pump work, (b) The turbine work, (c) The Rankine efficiency, (d) The quality of steam at the end of expansion. Assume flow rate of 9.5kg/sec. Use of steam table is permitted.

**Solution:**

<table>
<thead>
<tr>
<th>Given Data</th>
<th>To be Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 = 35 \text{ bar} )</td>
<td>a) ( w_p = ? )</td>
</tr>
<tr>
<td>( p_2 = 0.2 \text{ bar} )</td>
<td>b) ( w_t = ? )</td>
</tr>
<tr>
<td>( m = 9.5 \text{ kg/sec} )</td>
<td>c) ( \eta_{th} = ? )</td>
</tr>
</tbody>
</table>

![Diagram of the Rankine cycle process](image)
From Steam Table:

@ \( p_1 = 35 \text{ bar} \)

\[ h_g = 2802 \frac{kJ}{kg}, s_g = 6.1228 \frac{kJ}{kg - K} \]

@ \( p_2 = 0.2 \text{ bar} \)

\[ h_f = 251.45 \frac{kJ}{kg}, h_{fg} = 2358.4 \frac{kJ}{kg}, s_f = 0.8321 \frac{kJ}{kg - K}, s_{fg} = 7.0773 \frac{kJ}{kg - K} \]

\[ v_f = 0.0010172 \text{ m}^3/\text{kg} \]

\( \Rightarrow \) Enthalpy at point 1,

\[ h_1 = (h_{g1})_{@35 \text{ bar}} \]

\[ \therefore h_1 = 2802 \text{ kJ/kg} \]

\( \Rightarrow \) Enthalpy at point 2,

\[ h_2 = (h_f + x_2 h_{fg})_{@0.2 \text{ bar}} \]

But,

\[ s_1 = s_2 \]

\[ (s_{g1})_{@35 \text{ bar}} = (s_f + x_2 s_{fg})_{@0.2 \text{ bar}} \]

\[ 6.1228 = 0.8321 + x_2 \times 7.0773 \]

\[ \therefore x_2 = 0.7475 \]

\[ \therefore h_2 = h_f + x_2 h_{fg} \]

\[ \therefore h_2 = 251.45 + 0.7475 \times 2358.4 \]

\[ \therefore h_2 = 2014.49 \text{ kJ/kg} \]

\( \Rightarrow \) Enthalpy at point 3,

\[ h_3 = (h_{f3})_{@0.2 \text{ bar}} \]

\[ \therefore h_3 = 251.45 \text{ kJ/kg} \]

\( \Rightarrow \) Pump Work,

\[ w_p = \int_{p_1}^{p_2} v dP = (v_{f3})_{@0.2 \text{ bar}}(P_1 - P_2) \]

\[ \therefore w_p = 0.0010172 \times (35 \times 10^2 - 0.2 \times 10^2) \]

\[ \therefore w_p = 3.5398 \text{ kJ/kg} \]

\( \Rightarrow \) Pump Power:

\[ W_p = m \times w_p \]

\[ W_p = 9.5 \times 3.5398 \]

\[ W_p = 33.6281 \text{ kW} \]

\( \Rightarrow \) Enthalpy at point 4,

\[ w_p = h_4 - h_3 \]

\[ \therefore h_4 = 3.5398 + 251.45 \]

\[ \therefore h_4 = 254.9898 \text{ kJ/kg} \]

\( \Rightarrow \) Heat Supplied,

\[ q_s = h_1 - h_4 \]

\[ \therefore q_s = 2802 - 254.9898 \]

\[ \therefore q_s = 2547.01 \text{ kJ/kg} \]

\( \Rightarrow \) The Turbine Power:

\[ W_t = m(h_1 - h_2) \]

\[ \therefore W_t = 9.5 \times (2802 - 2014.49) \]
\[ W_t = 7481.345 kW \]

\[ \eta = \frac{W_{net}}{Q_s} = \frac{W_t - W_p}{\dot{m}(h_1 - h_4)} \]

\[ \eta = \frac{7481.345 - 33.6281}{9.5(2802 - 3.5398)} \]

\[ \therefore \eta = 0.2801 = 28.01\% \]

\[ x_2 = 0.7475 \]

### 6.9 References


7

GAS POWER CYCLES

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7.1 Terminology Used in Gas Power Cycles

a) Cycle: “A Cycle is defined as a repeated series of operations occurring in a certain order.”

b) Air standard cycle: “The thermodynamics cycle with air as the working fluid is called an air standard cycle.”

c) Compression ratio ($r$):

$$r = \frac{Total \ cylinder \ volume}{Clearance \ volume}$$
$$r = \frac{V_C + V_S}{V_C}$$

Higher the compression ratio better will be the performance of an engine.

d) Piston Speed: “The distance travelled by the piston in one minute is called piston speed.”

$$Piston \ Speed = \frac{2LN \ m}{60 \ sec}$$

(e) Mechanical Efficiency: It is defined as the ratio of the brake power and the indicated power. Mechanical efficiency is indicator of losses due to friction.

$$\eta_{mech} = \frac{B. P.}{I. P.}$$

(f) Thermal Efficiency: “It is the ratio of work done to heat supplied by fuel.”

$$\eta_{th} = \frac{\text{Work output}}{\text{Heat input}} = \frac{Q_1 - Q_2}{Q_1}$$

Where,

$Q_1$ = Heat addition

$Q_2$ = Heat rejection

[Assuming no friction & heat losses, so $W = Q_1 - Q_2$]

i. Indicated thermal efficiency = Indicated Power/ Heat supplied by fuel

$$\eta_{ith} = \frac{I. P.}{m_f \times CV}$$

Where, $m_f$ = mass of fuel supplied, Kg/sec and CV = calorific value of fuel, J/kg

ii. Brake thermal efficiency = Brake Power/ Heat supplied by fuel

$$\eta_{bth} = \frac{B. P.}{m_f \times CV}$$
Also

\[ \eta_{mech} = \frac{\eta_{bth}}{\eta_{it}} \quad (7.7) \]

**g) Air standard efficiency:** The efficiency of the engine using air as the working medium is known as “Air standard efficiency” or “Ideal efficiency”.

- The actual efficiency of a cycle is always less than the air standard efficiency of that cycle under ideal conditions.
- This is taken into account by introducing a new term “Relative efficiency”.

\[ \eta_{relative} = \frac{\text{Actual thermal efficiency}}{\text{Air standard efficiency}} \quad (7.8) \]

- The analysis of all air standard cycles is based upon the following assumptions.

**Assumptions:**

1. The gas in the engine cylinder is a perfect gas i.e. it obeys the gas laws and has constant specific heat.
2. The compression and expansion processes are adiabatic and they take place without internal friction i.e. these processes are isentropic.
3. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with the cylinder at appropriate points during the process.
4. The engine operates in a closed cycle. The cylinder is filled with constant amount of working medium and the same fluid is used repeatedly.

**The approach and concept of ideal air cycle helps to…….**

1. Indicate the ultimate performance i.e. to determine the maximum ideal efficiency of a specific thermodynamics cycle.
2. Study qualitatively the influence of different variables on the performance of an actual engine.
3. Evaluate one engine relative to another.

### 7.2 Mean Effective Pressure

- The pressure variation versus volume inside the cylinder of a reciprocating engine is plotted with the help of an engine indicator. The resulting contour is closed one and is referred to as indicator diagram as shown in Fig. 7.1.
The area enclosed by the contour is a measure of the work output per cycle from the engine.

Mean effective pressure is defined as the average pressure acting on the piston which will produce the same output as is done by the varying pressure during a cycle.

Therefore

\[
\text{Area of indicator loop} = \text{Area of rectangle abcd}
\]

The height of the rectangle than represents the mean effective pressure.

\[
\text{mep} = \frac{\text{work done per cycle}}{\text{swept volume}} = \frac{\text{Area of indicator loop}}{\text{length of loop}}
\]

Unit: bar or KN/m²

Mean effective pressure is used as a parameter to compare the performance of reciprocating engines of equal size.

An engine that has a large volume of mep will deliver more net work and will thus perform better.

### 7.3 The Carnot Gas Power Cycle

A Carnot cycle is a hypothetical cycle consisting four different processes: two reversible isothermal processes and two reversible adiabatic (isentropic) processes.

According to Carnot theorem \textit{“No cycle can be more efficient than a reversible cycle operating between the same temperature limits.”}

Assumptions made in the working of the Carnot cycle

a) Working fluid is a perfect gas.

b) Piston cylinder arrangement is weightless and does not produce friction during motion.

c) The walls of cylinder and piston are considered as perfectly insulated.

d) Compression and expansion are reversible.

e) The transfer of heat does not change the temperature of sources or sink.
Fig. 7.2 shows essential elements for a Carnot cycle, P-v and T-S diagrams.

![Diagram of Carnot cycle](image)

**Fig. 7.2 P-v, T-S and schematic diagram of Carnot gas power cycle**

- This cycle has the highest possible efficiency and it consists four simple operations as below:
  
  a) **Isothermal Expansion (1 → 2)**
  
  The source of heat (H) is applied to the end of the cylinder and isothermal reversible expansion occurs at temperature $T_1$. During this process $q_1$ heat is supplied to the system.

  b) **Adiabatic Expansion (2 → 3)**
  
  Non-conducting cover (C) is applied to the end of the cylinder and the cylinder becomes perfect. Adiabatic cover is brought in contact with the cylinder head. Hence no heat transfer takes place. The fluid expands adiabatically and reversibly. The temperature falls from $T_1$ to $T_2$.

  c) **Isothermal Compression (3 → 4)**
  
  Adiabatic cover is removed and sink (S) is applied to the end of the cylinder. The heat, $q_2$ is transferred reversibly and isothermally at temperature $T_2$ from the system to the sink (S).
d) Adiabatic compression (4 – 1):

Adiabatic cover is brought in contact with cylinder head. This completes the cycle and system is returned to its original state at 1. During the process, the temperature of system is raised from $T_2$ to $T_1$.

**Efficiency of Carnot Gas Cycle:**

- Consider 1 kg of working substance (air) is enclosed in the cylinder.
- **Heat supplied** during isothermal process (1 – 2):
  \[ q_1 = p_1 V_1 \ln \frac{V_2}{V_1} \]
  \[ \therefore q_1 = RT_1 \ln \frac{V_2}{V_1} \]
- **Heat rejected** during isothermal compression (3 – 4):
  \[ q_2 = p_3 V_3 \ln \frac{V_4}{V_3} \]
  \[ \therefore q_2 = RT_2 \ln \frac{V_4}{V_3} \]

- During adiabatic expansion (2 – 3) and adiabatic compression (4 – 1), the heat transfer from or to the system is zero.

- **Work done**, 
  \[ W = q_1 - q_2 \]
  \[ \therefore W = RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_4}{V_3} \]
  \[ \quad (7.10) \]
- Let $r$ = ratio of expansion for process (1 – 2) = $\frac{V_2}{V_1}$
  = ratio of compression for process (3 – 4) = $\frac{V_4}{V_3}$
- by substituting the value of $r$ in equation 7.10, we get,
  \[ W = RT_1 \ln r - RT_2 \ln r \]
  \[ \quad (7.11) \]
- **Thermal efficiency**, 
  \[ \eta = \frac{\text{Work done}}{\text{Heat supplied}} \]
  \[ \therefore \eta = \frac{RT_1 \ln r - RT_2 \ln r}{RT_1 \ln \frac{V_2}{V_1}} = \frac{RT_1 \ln r - RT_2 \ln r}{RT_1 \ln r} \]
  \[ \therefore \eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \]
  \[ \quad (7.12) \]
Where,
\( T_1 \) = Maximum temperature of the cycle (K)
\( T_2 \) = Minimum temperature of cycle (K)

- In equation 7.12, if temperature \( T_2 \) decreases, efficiency increases and it becomes 100% if temperature \( T_2 \) becomes absolute zero; which is impossible to attain.

**Limitations of Carnot Gas Cycle:**

- The Carnot cycle is hypothetical.
- The thermal efficiency of Carnot cycle depends upon absolute temperature of heat source \( T_1 \) and heat sink \( T_2 \) only, and independent of the working substance.
- Practically it is not possible to neglect friction between piston and cylinder. It can be minimized but cannot be eliminated.
- It is impossible to construct cylinder walls which are perfect insulator. Some amount of heat will always be transferred. Hence perfect adiabatic process cannot be achieved.
- The isothermal and adiabatic processes take place during the same stroke. Therefore the piston has to move very slowly for isothermal process and it has to move very fast during remaining stroke for adiabatic process which is practically not possible.
- The output obtained per cycle is very small. This work may not be able to overcome the friction of the reciprocating parts.

### 7.4 The Otto Cycle OR Constant Volume Cycle (Isochoric)

- The cycle was successfully applied by a German scientist Nicolous A. Otto to produce a successful 4 – stroke cycle engine in 1876.

![Fig. 7.3 p-V and T-s diagrams of Otto cycle](image-url)
The thermodynamic cycle is operated with isochoric (constant volume) heat addition and consists of two adiabatic processes and two constant volume changes.

Fig. 7.3 shows the Otto cycle plotted on $p-V$ and $T-s$ diagram.

**Adiabatic Compression Process (1 – 2):**
- At pt. 1 cylinder is full of air with volume $V_1$, pressure $P_1$, and temp. $T_1$.
- Piston moves from BDC to TDC and an ideal gas (air) is compressed isentropically to state point 2 through compression ratio,

$$r = \frac{V_1}{V_2}$$

**Constant Volume Heat Addition Process (2 – 3):**
- Heat is added at constant volume from an external heat source.
- The pressure rises and the ratio $r_p \text{ or } \alpha = \frac{P_3}{P_2}$ is called expansion ratio or pressure ratio.

**Adiabatic Expansion Process (3 – 4):**
- The increased high pressure exerts a greater amount of force on the piston and pushes it towards the BDC.
- Expansion of working fluid takes place isentropically and work done by the system.
- The volume ratio $\frac{V_4}{V_3}$ is called isentropic expansion ratio.

**Constant Volume Heat Rejection Process (4 – 1):**
- Heat is rejected to the external sink at constant volume. This process is so controlled that ultimately the working fluid comes to its initial state 1 and the cycle is repeated.
- Many petrol and gas engines work on a cycle which is a slight modification of the Otto cycle.
- This cycle is called constant volume cycle because the heat is supplied to air at constant volume.

**Thermal Efficiency of an Otto Cycle:**
- Consider a unit mass of air undergoing a cyclic change.
- **Heat supplied** during the process 2 – 3,

$$q_1 = C_V(T_3 - T_2)$$

- **Heat rejected** during process 4 – 1,

$$q_2 = C_V(T_4 - T_1)$$
Work done,

\[ \therefore W = q_1 - q_2 \]
\[ \therefore W = C_V(T_3 - T_2) - C_V(T_4 - T_1) \]

Thermal efficiency,

\[ \eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W}{q_1} \]
\[ = \frac{C_V(T_3 - T_2) - C_V(T_4 - T_1)}{C_V(T_3 - T_2)} \]
\[ = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \quad \text{(7.13)} \]

For Adiabatic compression process (1 – 2),

\[ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma^{-1}} = r^{\gamma^{-1}} \]
\[ \therefore T_2 = T_1 r^{\gamma^{-1}} \quad \text{-- -- -- -- -- (7.14)} \]

For Isentropic expansion process (3 – 4),

\[ \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma^{-1}} \]
\[ \therefore T_3 = T_4 \left(\frac{V_4}{V_3}\right)^{\gamma^{-1}} \]
\[ \therefore T_3 = T_4 \left(\frac{V_1}{V_2}\right)^{\gamma^{-1}} \quad (\because V_1 = V_4, V_2 = V_3) \]
\[ \therefore T_3 = T_4 (r)^{\gamma^{-1}} \quad \text{-- -- -- -- -- (7.15)} \]

From equation 7.13, 7.14 & 7.15, we get,

\[ \eta_{otto} = 1 - \frac{(T_4 - T_1)}{T_4 r^{\gamma^{-1}} - T_1 r^{\gamma^{-1}}} \]
\[ \therefore \ \eta_{otto} = 1 - \frac{(T_4 - T_1)}{r^{\gamma^{-1}}(T_4 - T_1)} \]
\[ \therefore \ \eta_{otto} = 1 - \frac{1}{r^{\gamma-1}} \quad \text{-- -- -- -- -- (7.16)} \]

Expression 7.16 is known as the air standard efficiency of the Otto cycle.

It is clear from the above expression that efficiency increases with the increase in the value of \( r \) (as \( \gamma \) is constant).

We can have maximum efficiency by increasing \( r \) to a considerable extent, but due to practical difficulties its value is limited to 8.
− In actual engines working on Otto cycle, the compression ratio varies from 5 to 8 depending upon the quality of fuel.

− At compression ratios higher than this, the temperature after combustion becomes high and that may lead to spontaneous and uncontrolled combustion of fuel in the cylinder.

− The phenomenon of uncontrolled combustion in petrol engine is called **detonation** and it leads to poor engine efficiency and in structural damage of engine parts.

− Fig. 7.4 shows the variation of air standard efficiency of Otto cycle with compression ratio.

![Fig. 7.4 Variation of Otto cycle efficiency with compression ratio](image)

**Mean Effective Pressure:**

− **Net work done** per unit mass of air,

\[
W_{\text{net}} = C_v(T_3 - T_2) - C_v(T_4 - T_1)
\]

− **Swept volume,**

\[
\text{Swept volume} = V_1 - V_2 = V_1 \left(1 - \frac{V_2}{V_1}\right) = \frac{RT_1}{P_1} \left(1 - \frac{1}{r}\right)
\]

\[
= \frac{RT_1}{P_1 r} (r - 1)
\]

− **Mean effective pressure,**

\[
m_{\text{ep}} = \frac{\text{Work done per cycle}}{\text{swept volume}} = \frac{C_v(T_3 - T_2) - C_v(T_4 - T_1)}{\frac{RT_1}{P_1 r} (r - 1)}
\]

\[
= \frac{C_v}{R} \frac{P_1 r}{(r - 1)} \left[\frac{(T_3 - T_2) - (T_4 - T_1)}{T_1}\right]
\]
- For process 1–2,

\[
\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}
\]

\[
T_2 = T_1 r^{\gamma-1}
\]

- Process 2–3,

\[
\frac{T_3}{T_2} = \frac{P_3}{P_2} (\because V_2 = V_3)
\]

\[
\therefore T_3 = T_2 \alpha \quad (\alpha = \text{explosion pressure ratio})
\]

\[
\therefore T_3 = T_1 \alpha r^{\gamma-1}
\]

- Process 3–4,

\[
T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1}
\]

\[
\therefore T_4 = T_1 \alpha r^{\gamma-1} \left(\frac{V_2}{V_1}\right)^{\gamma-1}
\]

\[
\therefore T_4 = T_1 \alpha r^{\gamma-1} \times \frac{1}{r^{\gamma-1}}
\]

\[
\therefore T_4 = T_1 \cdot \alpha
\]

Substituting all these temperature values in equation 7.19, We get,

\[
\text{mep} = \frac{C_V}{R} \frac{P_1 r}{(r-1)} \left[ (T_1 \alpha r^{\gamma-1} - T_1 r^{\gamma-1}) - (T_1 \alpha - T_1) \right] T_1
\]

\[
\therefore \text{mep} = \frac{C_V}{R} \frac{P_1 r}{(r-1)} \left[ T_1 r^{\gamma-1} (\alpha - 1) - T_1 (\alpha - 1) \right] T_1
\]

\[
\therefore \text{mep} = \frac{C_V}{R} \frac{P_1 r}{(r-1)} [(r^{\gamma-1} - 1)(\alpha - 1)]
\]

\[
\therefore \text{mep} = \frac{P_1 r}{(r-1)(\gamma-1)} [(r^{\gamma-1} - 1)(\alpha - 1)]
\]

(\because \frac{C_p}{C_V} = \frac{1}{\gamma - 1})

\[
\begin{bmatrix}
    C_p \\
    C_V \\
    C_p - C_V = R, \\
    C_V (\frac{C_p}{C_V} - 1) = R, \\
    \frac{C_V}{R} = \frac{1}{\gamma - 1}
\end{bmatrix}
\]
**Condition for Maximum Work:**

- For unit mass of air,
  \[ W = q_1 - q_2 \]
  \[ \therefore W = C_v (T_3 - T_2) - C_v (T_4 - T_1) \]
  \[ \therefore \frac{W}{C_v} = T_3 - T_2 - T_4 + T_1 \quad \ldots \quad (7.21) \]

- We know that,
  \[ T_2 = T_1r^{\gamma-1} \]
  \[ T_4 = T_3 \frac{1}{r^{\gamma-1}} = T_3 \frac{T_1}{T_2} \quad (\because r^{\gamma-1} = \frac{T_2}{T_1}) \]

So
  \[ \frac{W}{C_v} = T_3 - T_2 - T_3 \frac{T_1}{T_2} + T_1 \quad \ldots \quad (7.22) \]

- The intermediate temperature \( T_2 \) for maximum work output can be obtained by differentiating the above equation with respect to \( T_2 \) & setting the derivatives equal to zero.
  \[ \therefore \frac{1}{C_v} \frac{dW}{dT_2} = -1 + \frac{T_1 T_3}{T_2^2} = 0 \quad (for \ max \ work) \]
  \[ \therefore T_2^2 = T_1 T_3 \]
  \[ \therefore T_2 = \sqrt{T_1 T_3} \quad \ldots \quad (7.23) \]

- Similarly for temperature \( T_4 \)
  \[ \frac{W}{C_v} = T_3 - \frac{T_1 T_3}{T_4} - T_4 + T_1 \]
  \[ \therefore \frac{1}{C_v} \frac{dW}{dT_4} = \frac{T_1 T_3}{T_4^2} - 1 = 0 \quad (for \ max \ work) \]
  \[ \therefore T_4 = \sqrt{T_1 T_3} \quad \ldots \quad (7.24) \]

- Thus for maximum work,
  \[ T_2 = T_4 = \sqrt{T_1 T_3} \quad \ldots \quad (7.25) \]
  i.e. the intermediate temperature \( T_2 \) & \( T_4 \) must be equal for maximum work.

**Maximum work,**

\[ W_{\text{max}} = C_v(T_3 - T_2 - T_4 + T_1) \]
\[ \therefore W_{\text{max}} = C_v(T_3 - \sqrt{T_1 T_3} - \sqrt{T_1 T_3} + T_1) \]
\[ \therefore W_{\text{max}} = C_v(T_3 + T_1 - 2\sqrt{T_1 T_3}) \quad \ldots \quad (7.26) \]
7.5 The Diesel Cycle OR Constant Pressure Cycle (Isobaric)

- This cycle was discovered by a German engineer Dr. Rudolph Diesel. Diesel cycle is also known as *constant pressure heat addition cycle*.

![Fig. 7.5 p-V and T-s diagrams of Diesel cycle](image.png)

Adiabatic Compression Process (1 – 2):
- Isentropic (Reversible adiabatic) compression with \( \frac{V_1}{V_2} \).

Constant Pressure Heat Addition Process (2 – 3):
- The heat supply is stopped at point 3 which is called the cut-off point and the volume ratio \( \frac{V_3}{V_2} \) is called *cut off ratio* or Isobaric expansion ratio.

Adiabatic Expansion Process (3 – 4):
- Isentropic expansion of air \( \frac{V_4}{V_3} \) = isentropic expansion ratio.

Constant Volume Heat Rejection Process (4 – 1):
- In this process heat is rejected at constant volume.

This thermodynamics cycle is called constant pressure cycle because heat is supplied to the air at constant pressure.

**Thermal Efficiency for Diesel Cycle:**
- Consider unit mass of air.
- **Heat supplied** during process 2 – 3,
  \[ q_1 = C_p(T_3 - T_2) \]
- **Heat rejected** during process 4 – 1,
  \[ q_2 = C_v(T_4 - T_1) \]
- **Work done**, 
  \[ W = q_1 - q_2 \]
  \[ W = C_p(T_3 - T_2) - C_v(T_4 - T_1) \]

- **Thermal efficiency**, 
  \[ \eta = \frac{Work\ done}{Heat\ supplied} \]
  \[ \therefore \eta = \frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)} \]
  \[ \therefore \eta = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} \]
  \[ \therefore \eta = 1 - \frac{1}{\gamma} \frac{(T_4 - T_1)}{(T_3 - T_2)} \]

- For adiabatic compression process (1 - 2),
  \[ r = \frac{V_1}{V_2} - - - - - - - - (a) \]
  \[ \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma \]
  \[ P_2 = P_1 r^\gamma - - - - - - - - (b) \]
  \[ T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 r^{\gamma-1} - - - - - - - - (c) \]

- For constant pressure heat addition process (2 - 3)
  \[ P_3 = P_2 = P_1 r^\gamma - - - - - - - - (d) \]
  \[ \rho = \frac{V_3}{V_2} \text{ (Cutoff ratio)} - - - - - - - - (e) \]
  \[ T_3 = T_2 \frac{V_3}{V_2} \]
  \[ = T_2 \rho \]
  \[ \therefore T_3 = T_1 r^{\gamma-1} \rho - - - - - - - - (f) \]

- For adiabatic expansion process (3 - 4),
  \[ P_4 = P_3(V_3/V_4)^\gamma = P_3(V_3/V_1)^\gamma \]
  \[ \therefore P_4 = P_3 \left(\frac{V_3}{V_2}\right)^\gamma = P_3(\rho/r)^\gamma - - - - - - - - (g) \]
  \[ T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3 \left(\frac{\rho}{r}\right)^{\gamma-1} \]
\[ T_4 = \frac{T_1 r^{\gamma-1} \rho^{\gamma-1}}{r^{\gamma-1}} \]
\[ \therefore T_4 = T_1 \rho^{\gamma -} \quad (h) \]

From equation 7.27,
\[ \eta = 1 - \frac{(T_4 - T_1)}{\gamma (T_3 - T_2)} \]
\[ \therefore \eta = 1 - \frac{1}{\gamma} \frac{(T_1 \rho^{\gamma} - T_1)}{r^{\gamma-1} \rho - T_1^{r^{\gamma-1}}} \]
\[ \therefore \eta = 1 - \frac{1}{\gamma} \frac{(\rho^{\gamma} - 1)}{(r^{\gamma-1} \rho - r^{\gamma-1})} \]
\[ \therefore \eta = 1 - \frac{1}{r^{\gamma-1} \frac{(\rho^{\gamma} - 1)}{\gamma (\rho - 1)}} \quad (7.28) \]

- Apparently the efficiency of diesel cycle depends upon the compression ratio \((r)\) and cutoff ratio \((\rho)\) and hence upon the quantity of heat supplied.

- Fig. 7.6 shows the air standard efficiency of diesel cycle for various cut off ratio.

![Fig. 7.6 Efficiency of Diesel cycle for various cut-off ratio](image)

- Further,
\[ K = \frac{\rho^{\gamma} - 1}{\gamma (\rho - 1)} \]

reveals that with an increase in the cut-off ratio \((\rho)\) the value of factor \(K\) increases.

That implies that for a diesel engine at constant compression ratio, the efficiency would increase with decrease in \(\rho\) and in the limit \(\rho \to 1\), the efficiency would become
\[ 1 - \frac{1}{r^{\gamma-1}} \]

- Since the factor \(K = \frac{\rho^{\gamma-1}}{\gamma (\rho - 1)}\) is always greater than unity, the Diesel cycle is always less efficient than a corresponding Otto cycle having the same compression ratio.
However Diesel engine operates on much higher compression ratio (14 to 18) compared to those for S.I. Engines operating on Otto cycle.

High compression ratios for Diesel engines are must not only for high efficiency but also to prevent diesel knock; a phenomenon which leads to uncontrolled and rapid combustion in diesel engines.

**Mean Effective Pressure:**

- **Net work done** per unit mass of air,

\[
W_{\text{net}} = C_p (T_3 - T_2) - C_v (T_4 - T_1)
\]  

- **Swept volume,**

\[
\text{Swept volume} = V_1 - V_2 = V_1 \left(1 - \frac{V_2}{V_1}\right) = \frac{RT_1}{P_1} \left(1 - \frac{1}{r}\right)
\]  

- **Mean effective pressure,**

\[
\text{mep} = \frac{\text{work done per cycle}}{\text{swept volume}}
\]

\[
\therefore \text{mep} = \frac{C_p (T_3 - T_2) - C_v (T_4 - T_1)}{\frac{RT_1}{P_1} (r - 1)}
\]

\[
\therefore \text{mep} = \frac{C_v}{R} \frac{P_1 r}{(r - 1)} \left[\frac{\gamma (T_3 - T_2) - (T_4 - T_1)}{T_1}\right] - - - - - - - - (7.31)
\]

- From equation (c), (f) and (h),

\[
T_2 = T_1 r^{\gamma - 1}
\]

\[
T_3 = T_1 r^{\gamma - 1} \rho
\]

\[
T_4 = T_1 \rho^{\gamma - 1}
\]

\[
\therefore \text{mep} = \frac{C_v}{R} \frac{P_1 r}{(r - 1)} \left[\frac{\gamma (T_1 r^{\gamma - 1} \rho - T_1 r^{\gamma - 1} - (T_1 \rho^{\gamma - 1} - T_1))}{T_1}\right]
\]

\[
\therefore \text{mep} = \frac{P_1 r}{(\gamma - 1)(r - 1)} \left[\gamma r^{\gamma - 1} (\rho - 1) - (\rho^{\gamma - 1} - 1)\right] - - - - - - - - (7.32)
\]

### 7.6 The Dual Combustion Cycle OR The Limited Pressure Cycle

- This is a cycle in which the addition of heat is partly at constant volume and partly at constant pressure.
**Fig. 7.7 p-V and T-s diagrams of Diesel cycle**

**Adiabatic Compression Process (1 – 2):**
- Isentropic (Reversible adiabatic) compression with $\frac{V_1}{V_2}$.

**Constant Volume Heat Addition Process (2 – 3):**
- The heat is supplied at constant volume with explosion ratio or pressure ratio $\alpha = \frac{P_3}{P_2}$.

**Constant Pressure Heat Addition Process (3 – 4):**
- The heat supply is stopped at point 4 which is called the cut-off point and the volume ratio $\rho = \frac{V_4}{V_3}$ is called cut off ratio.

**Adiabatic Expansion Process (4 – 5):**
- Isentropic expansion of air with $\frac{V_5}{V_4}$ isentropic expansion ratio.

**Constant Volume Heat Rejection Process (5 – 1):**
- In this process heat is rejected at constant volume.

The high speed Diesel engines work on a cycle which is slight modification of the Dual cycle.

**Thermal Efficiency for Dual Cycle:**
- Consider unit mass of air undergoing the cyclic change.
- **Heat supplied**, $q_1 = q_{2-3} + q_{3-4}$

$$q_1 = C_V(T_3 - T_2) + C_P(T_4 - T_3)$$

- **Heat rejected** during process 5 – 1,

$$q_2 = C_V(T_5 - T_1)$$
- **Work done**, 

\[ W = q_1 - q_2 \]

\[ W = C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1) \]

- **Thermal efficiency**, 

\[ \eta = \frac{\text{Work done}}{\text{Heat supplied}} \]

\[ \therefore \eta = \frac{C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)} \]

\[ \therefore \eta = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \quad \text{(7.33)} \]

- For adiabatic compression process (1–2),

\[ r = \frac{V_1}{V_2} \quad \text{--- (a)} \]

\[ \frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^\gamma \]

\[ P_2 = P_1 r^\gamma \quad \text{--- (b)} \]

\[ T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{y-1} = T_1 r^{y-1} \quad \text{--- (c)} \]

- For constant volume heat addition process (2–3)

\[ V_3 = V_2 = \frac{V_1}{r} \]

\[ \alpha = \frac{P_3}{P_2} \quad \text{(Pressure ratio)} \quad \text{--- (d)} \]

\[ \therefore P_3 = P_2 \alpha = P_1 r^\gamma \alpha \]

\[ T_3 = T_2 \frac{P_3}{P_2} = T_2 \alpha \]

\[ \therefore T_3 = T_1 r^{y-1} \alpha \quad \text{--- (e)} \]

- For constant pressure heat addition process (3–4)

\[ P_3 = P_4 = P_1 r^\gamma \alpha \quad \text{--- (f)} \]

\[ \rho = \frac{V_4}{V_3} \quad \text{(Cutoff ratio)} \quad \text{--- (g)} \]

\[ T_4 = T_3 \frac{V_4}{V_3} \]
\[
\therefore T_4 = T_3 \rho \\
\therefore T_4 = T_1 r^{\gamma - 1} \rho \alpha - \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (h)
\]

- For adiabatic expansion process \((4 \rightarrow 5)\),

\[
P_4 V_4^\gamma = P_5 V_5^\gamma
\]

\[P_5 = P_4 \left(\frac{V_4}{V_5}\right)^\gamma = P_3 \left(\frac{V_4}{V_3}\right)^\gamma \quad (\because V_1 = V_5 \text{ & } P_3 = P_4)
\]

\[P_5 = P_3 \left(\frac{V_4}{V_3}\right)^\gamma = P_3 \left(\frac{V_4}{V_1}\right)^\gamma \quad (\because V_3 = V_2)
\]

\[\therefore P_5 = P_3 \left(\frac{V_4}{V_1}\right)^\gamma = P_3 (\rho/r)^\gamma - \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (i)
\]

And

\[T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{\gamma - 1}
\]

\[\therefore T_5 = T_4 \left(\frac{\rho}{r}\right)^{\gamma - 1}
\]

\[\therefore T_5 = \frac{T_1 r^{\gamma - 1} \rho \alpha \rho^{\gamma - 1}}{r^{\gamma - 1}}
\]

\[\therefore T_5 = T_1 \alpha \rho^{\gamma} - \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (j)
\]

From equation 7.33,

\[\eta = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma (T_4 - T_3)}
\]

\[\therefore \eta = 1 - \frac{(T_1 \alpha \rho^{\gamma} - T_1)}{(T_1 r^{\gamma - 1} \alpha - T_1 r^{\gamma - 1}) + \gamma (T_1 r^{\gamma - 1} \alpha - T_1 r^{\gamma - 1})}
\]

\[\therefore \eta = 1 - \frac{(\rho^{\gamma} - 1)}{[r^{\gamma - 1} ((\alpha - 1) + \gamma \alpha (\rho - 1))]}
\]

\[\therefore \eta = 1 - \frac{1}{r^{\gamma - 1}} \left[\frac{(\alpha \rho^{\gamma} - 1)}{((\alpha - 1) + \gamma \alpha (\rho - 1))}\right] - \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7.34)
\]

- It can be seen from the equation 7.34 that the thermal efficiency of a Dual cycle can be increased by supplying a greater portion of heat at constant volume (high value of \(\alpha\)) and smaller portion at constant pressure (low value of \(\rho\)).

- In the actual high speed Diesel engines operating on this cycle, it is achieved by early fuel injection and an early cut-off.

- It is to be noted that Otto and Diesel cycles are special cases of the Dual cycle.

- If \(\rho = 1 \ (V_3 = V_4)\)

Hence, there is no addition of heat at constant pressure. Consequently the entire heat is supplied at constant volume and the cycle becomes the Otto cycle.
By substituting $\rho = 1$ in equation 7.34, we get,

$$\eta = 1 - \frac{1}{r^{(r-1)}} = Efficiency \ of \ Otto \ cycle$$

- Similarly if $\alpha = 1$, the heat addition is only at constant pressure and cycle becomes Diesel cycle.

By substituting $\alpha = 1$ in equation 7.34, we get,

$$\eta = 1 - \frac{1}{r^{r-1}} \left[ \frac{(\rho^r - 1)}{\gamma (\rho - 1)} \right] = Efficiency \ of \ Diesel \ cycle$$

**Mean Effective Pressure:**

- **Net work done** per unit mass of air,

$$W_{\text{net}} = C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1)$$

- **Swept volume,**

$$\text{Swept volume} = V_1 - V_2 = V_1 \left(1 - \frac{V_2}{V_1}\right) = \frac{RT_1}{P_1} \left(1 - \frac{1}{r}\right)$$

$$= \frac{RT_1}{P_1 r} (r - 1)$$

- **Mean effective pressure,**

$$mep = \frac{\text{Work done per cycle}}{\text{swept volume}}$$

$$\therefore mep = \frac{C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1)}{\frac{RT_1}{P_1} (r - 1)}$$

$$\therefore mep = \frac{C_v}{R} \frac{P_1 r}{(r - 1)} \left[ \frac{(T_3 - T_2) + \gamma(T_4 - T_3) - (T_5 - T_1)}{T_1} \right]$$

- From equation (c), (e), (h) and (j),

$$T_2 = T_1 r^{y-1}$$

$$T_3 = T_1 r^{y-1} \alpha$$

$$T_4 = T_1 r^{y-1} \alpha \rho$$

$$T_5 = T_1 \alpha \rho^y$$

$$\therefore mep$$

$$= \frac{C_v}{R} \frac{P_1 r}{(r - 1)} \left[ \frac{\gamma (T_1 r^{y-1} \alpha - T_1 r^{y-1}) + \gamma (T_1 r^{y-1} \alpha \rho - T_1 r^{y-1} \alpha) - (T_1 \alpha \rho^y - T_1)}{T_1} \right]$$

$$\therefore mep = \frac{P_1 r}{(y - 1)(r - 1)} [(\alpha - 1)r^{y-1} + \gamma \alpha r^{y-1}(\rho - 1) - (\alpha \rho^y - 1)]$$

$$- - - - - - - - (7.37)$$
7.7 Comparison of Otto, Diesel and Dual Cycles

- Following are the important variable factors which are used as a basis for comparison of the cycles:
  - Compression ratio
  - Maximum pressure
  - Heat supplied
  - Heat rejected
  - Net work.

A. For the Same Compression Ratio and the Same Heat Input

![Diagram showing P-V and T-S diagrams for Otto, Diesel, and Dual cycles]

We know that,
\[ \eta = 1 - \frac{\text{Heat Rejected}}{\text{Heat Supplied}} = 1 - \frac{q_2}{q_1} \quad \text{(7.38)} \]

- The quantity of heat rejected from each cycle is represented by the appropriate area under the line 4–1 on the T–S diagram.
- From equation 7.38; it is clear that the cycle which has the least heat rejected will have the highest efficiency.

\[ \therefore \eta_{\text{Otto}} > \eta_{\text{Dual}} > \eta_{\text{Diesel}} \]

B. Same Maximum Pressure and Temperature

- When pressure is the limiting factor in engine design, it becomes necessary to compare the air standard cycles on the basis of same maximum pressure & temperature.
Here the Otto cycle must be limited to low compression ratio to fulfill the condition that point 3 (same maximum pressure & temperature) is to be a common state for all the three cycles.

From Fig. 7.9 it is clear that the heat rejected is same for all the three cycles. Hence with the same heat rejected, the cycle with greater heat addition is more efficient.

We know that,

\[ \eta = 1 - \frac{\text{Heat Rejected}}{\text{Heat Supplied}} = 1 - \frac{q_2}{q_1} \quad \text{(7.39)} \]

From Fig. 7.9,

\[ \therefore \eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}} \]

C. For Constant Maximum Pressure and Heat Input

Fig. 7.10 shows the Otto and Diesel cycles on P-V and T-S diagrams for constant maximum pressure and heat input respectively.

\[ Fig. 7.10 (a) P-V diagram and (b) T-S diagram \]
For the constant maximum pressure, points 3 and 3’ must lie on the constant pressure line.

Also for the same heat input the areas $a - 2 - 3 - b$ and $a - 2' - 3' - c$ on the T-S plot must be equal.

Now,

$$\eta = 1 - \frac{\text{Heat Rejected}}{\text{Heat Supplied}} = 1 - \frac{q_2}{q_1} = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7.40)$$

Hence for the same amount of heat supplied the cycle with less heat rejected has a higher value of thermal efficiency.

From Fig. 7.10,

$$\therefore \eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$$

### 7.8 The Brayton Cycle OR The Joule Cycle

The Brayton cycle is a constant pressure cycle for a perfect gas. It is also called Joule cycle.

It is a theoretical cycle on which constant pressure gas turbine works.

The various operations are as follows:

- **Isentropic Compression (1 – 2):**
  The air is compressed isentropically from the lower pressure $p_1$ to the upper pressure $p_2$, the temperature rising from $T_1$ to $T_2$. No heat flow occurs.

- **Constant Pressure Heat Addition (2 – 3):**
  The compressed air is passed through a heat exchanger, where heat is externally supplied to it at constant pressure. Heat flows into the system increasing the volume from $V_2$ to $V_3$ and temperature from $T_2$ to $T_3$ whilst the pressure remains constant at $p_2$.

- **Isentropic Expansion (3 – 4):**
  Isentropic expansion of high pressure & high temperature air takes place in the turbine during which the work is done by the system. The air is expanded isentropically from $p_2$ to $p_1$, the temperature falling from $T_3$ to $T_4$. No heat flow occurs.

- **Constant Pressure Heat Rejection (4 – 1):**
  The air at state point 4 is passed through a heat exchanger and heat is rejected at constant pressure. The volume decreases from $V_4$ to $V_1$ and the temperature from $T_4$ to $T_1$ whilst the pressure remains constant at $p_1$. 

The closed Brayton cycle is shown in the Fig. 7.11 (a) and it is represented on p-v and T-s diagrams as shown in Figs. 7.11 (b) and (c) respectively.

**Fig. 7.11 The P-v, T-s and Schematic diagram of Brayton cycle**

**Thermal Efficiency for Closed Brayton Cycle:**

- For unit mass of air,
  - **Heat supplied** during process $2 \rightarrow 3$,
    \[
    q_1 = C_P(T_3 - T_2)
    \]
  - **Heat rejected** during process $4 \rightarrow 1$,
    \[
    q_2 = C_P(T_4 - T_1)
    \]
  - **Work done**,
    \[
    W = q_1 - q_2
    \]
\[ W = C_p(T_3 - T_2) - C_p(T_4 - T_1) \]

- **Thermal efficiency**, 
\[ \eta = \frac{\text{Work done}}{\text{Heat supplied}} \]
\[ \therefore \eta = \frac{C_p(T_3 - T_2) - C_p(T_4 - T_1)}{C_p(T_3 - T_2)} \]
\[ \therefore \eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \] (7.41)

- Take pressure ratio,
\[ r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4} \] (7.41a)

- For isentropic compression process (1 – 2),
\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} = (r_p)^{\frac{1}{\gamma}} \] (7.41b)

- For isentropic expansion process (3 – 4),
\[ \frac{T_3}{T_4} = \left( \frac{P_3}{P_4} \right)^{\frac{1}{\gamma}} = (r_p)^{\frac{1}{\gamma}} \] (7.41c)

- Thus from equation (7.41b) and (7.41c),
\[ \frac{T_2}{T_1} = \frac{T_3}{T_4} \]
\[ \therefore \frac{T_4}{T_1} = \frac{T_3}{T_2} \] (7.41d)

- From equation 7.41,
\[ \eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \]
\[ \therefore \eta = 1 - \frac{T_1\left(\frac{T_4}{T_2} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)} = 1 - \frac{T_1\left(\frac{T_3}{T_2} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)} \] (\[ \because \text{equation (7.41d)} \])
\[ \therefore \eta = 1 - \frac{T_1}{T_2} \]
\[ \therefore \eta = 1 - \left( \frac{P_1}{P_2} \right)^{\frac{1}{\gamma}} \]
\[ \therefore \eta = 1 - \left( \frac{1}{r_p} \right)^{\frac{1}{\gamma}} \] (7.42)
7. Gas Power Cycles

- Thermal efficiency of Brayton cycle is function of pressure ratio. Efficiency increases with pressure ratio as shown in Fig. 7.12.

![Graph showing the effect of pressure ratio on the efficiency of Brayton cycle](image)

*Fig. 7.12 Effect of pressure ratio on the efficiency of Brayton cycle*

- The curve tends to become flat at higher pressure ratios, which implies that though the efficiency is increasing, the rate of increase starts diminishing at higher pressures.

7.9 The Open Cycle Gas Turbine OR Actual Brayton Cycle:

- The fundamental gas turbine unit is one operating on the open cycle. In Open cycle gas turbine, the products of combustion coming out from the turbine are exhausted to the atmosphere as they cannot be used any more. The working fluids (air and fuel) must be replaced continuously as they are exhausted into the atmosphere.

- In practice, it is not possible to achieve either isentropic compression or isentropic expansion because of internal friction, turbulence and leakage.

- If pressure drop is neglected in combustion chamber, the actual Brayton cycle on T-S diagram is shown by process 1-2’-3-4’ in Fig. 7.13.

![Diagram of an actual Brayton cycle](image)

*Fig. 7.13 Schematic and T-S diagram of an actual Brayton cycle*
In actual cycle, the temperatures at the end of compression and at the end of expansion are higher than in an ideal case for the same pressure ratio.

**Efficiency of compressor,**
\[
\eta_c = \frac{\text{Isentropic temperature rise}}{\text{Actual temperature rise}}
\]
\[
\therefore \eta_c = \frac{T_2 - T_1}{T_2' - T_1\gamma} = \frac{T_2 - T_1}{T_2' - T_1}\quad (7.43)
\]

**Efficiency of turbine,**
\[
\eta_t = \frac{\text{Actual decrease in temperature}}{\text{Isentropic decrease in temperature}}
\]
\[
\therefore \eta_t = \frac{T_3 - T_4'}{T_3 - T_4} = \frac{T_3 - T_4'}{T_3 - T_4}\quad (7.44)
\]

The performance of Brayton cycle can be improved by using multi stage compression with inter-cooling, multi stage expansion with reheating and regeneration.

**Pressure Ratio for Maximum Net Work**

- From equation 7.41a to 7.41c (Refer Page No. 7.25),

  **Pressure ratio,**
  \[
  r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4} = \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_2'}{T_1} = \frac{T_3'}{T_4'} = \frac{r_p^{\gamma - 1}}{\gamma - 1} = (r_p)^{\gamma - 1} - \quad (7.41a)
  \]

  For isentropic compression process \((1 - 2),\)
  \[
  \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma - 1} = \left(\frac{r_p}{P_1}\right)^{\gamma - 1} - \quad (7.41b)
  \]

  For isentropic expansion process \((3 - 4),\)
  \[
  \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\gamma - 1} = \left(\frac{r_p}{P_4}\right)^{\gamma - 1} - \quad (7.41c)
  \]

  Thus from equation \((7.41b)\) and \((7.41c),\)
  \[
  \frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{r_p}{P_1}\right)^{\gamma - 1} = \left(\frac{r_p}{P_4}\right)^{\gamma - 1} = x
  \]

- **Actual compression work,**
  \[
  W_{c_a} = h_2' - h_1 = \frac{h_2 - h_1}{\eta_c} = \frac{C_p(T_2 - T_1)}{\eta_c}
  \]

- **Actual turbine work,**
  \[
  W_{t_a} = h_3 - h_4 = \eta_t (h_3 - h_4) = \frac{C_p(T_3 - T_4)\eta_t}{\eta_t}
  \]
Actual net work,

\[ W_{\text{net}_a} = W_t - W_c \]

\[ \therefore W_{\text{net}_a} = C_p(T_3 - T_4)\eta_t - \frac{C_p(T_2 - T_1)}{\eta_c} \]

\[ \therefore W_{\text{net}_a} = C_p\eta_tT_3\left(1 - \frac{T_4}{T_3}\right) - \frac{C_pT_1}{\eta_c}\left(\frac{T_2}{T_1} - 1\right) \]

\[ \therefore W_{\text{net}_a} = C_p\eta_tT_3\left(1 - \frac{1}{x}\right) - \frac{C_pT_1}{\eta_c}(x - 1) \]

For maximum work, differentiate above equation w.r.t. \( x \) while keeping \( T_1 \) & \( T_3 \) constants.

\[ \therefore \frac{dW_{\text{net}_a}}{dx} = 0 \]

\[ \therefore C_p\eta_tT_3\left(\frac{1}{x^2}\right) - \frac{C_pT_1}{\eta_c} = 0 \]

\[ \therefore x = \sqrt{\frac{T_3}{\eta_t\eta_cT_1}} \]

\[ \therefore (r_p)^{\gamma - 1} = \left(\frac{\eta_t\eta_c}{T_3}\right)^{1/2} \]

\[ \therefore r_{\text{opt}} = \left(\frac{\eta_t\eta_c}{T_1}\right)^{\gamma/2(\gamma - 1)} \]

\[ \therefore r_{\text{opt}} = \left(\frac{\eta_t\eta_cT_{\text{max}}}{T_{\text{min}}}\right)^{\gamma/2(\gamma - 1)} \]

For ideal cycle,

\[ r_{\text{opt}} = \left(\frac{T_{\text{max}}}{T_{\text{min}}}\right)^{\gamma/2(\gamma - 1)} \]

For maximum work, the temperature \( T_1 \) at compressor entry should be as low as possible and the temperature \( T_3 \) at entry to the turbine should be as high as possible.

The compressor inlet temperature is normally at atmospheric temperature (say 288K at sea level), while the turbine inlet temperature is decided by metallurgical considerations (the maximum value of about 1000K that the metal can withstand).

The performance of an actual gas turbine plant depends upon both the pressure ratio and the temperature ratio.
7.10 Methods for Improvement of Thermal Efficiency of Open Cycle Gas Turbine Plant

- The following methods are employed to increase the specific output and thermal efficiency of the plant: 1) Intercooling 2) Reheating and 3) Regeneration.

1. Intercooling

- A compressor in a gas turbine cycle utilizes the major percentage of power developed by the gas turbine. The work required by the compressor can be reduced by compressing the air in two stages and incorporating an intercooler between the two as shown in Fig. 7.14.

- The ideal cycle for this arrangement is 1-2-3-4-5-6. The compression process without intercooling is shown as 1-L’ in the actual case and 1-L in the ideal isentropic case.

- The corresponding T-s diagram for the unit is shown in Fig. 7.15. The actual processes take place as follows:
  1-2’ ---- Low Pressure Compression
  2’-3 ---- Intercooling
  3-4’ ---- High Pressure Compression
  4’-5 ---- Heat addition in combustion chamber
  5-6’ ---- Expansion in turbine

![Fig. 7.14 Open cycle gas turbine plant with intercooler](image-url)
- It is evident from the Fig. 7.15, that the turbine work is the same as for the simple cycle.

- Work input (with intercooling),
  \[ W_{\text{input}} = C_p(T_2' - T_1) + C_p(T_4' - T_3) \]  
  \[ = C_p(T_L' - T_1) \]  
  \[ = C_p(T_2' - T_1) + C_p(T_L' - T_2') \]  
  \[ = C_p(T_L' - T_2') \]  
  \[ (7.45) \]  

- Work input (without intercooling),
  \[ W_{\text{input}} = C_p(T_L' - T_1) \]  
  \[ = C_p(T_2' - T_1) + C_p(T_L' - T_2') \]  
  \[ = C_p(T_L' - T_2') \]  
  \[ (7.46) \]  

- By comparing equation 7.45 & 7.46, it can be observed that the work input with intercooling is less than the work input without intercooling, when \( C_p(T_4' - T_3) < C_p(T_L' - T_2') \).

- Heat supplied in the combustion chamber (with intercooling),
  \[ q = C_p(T_5 - T_4') \]  

- Heat supplied in the combustion chamber (without intercooling),
  \[ q = C_p(T_5 - T_4') \]  

- Thus, the heat supplied when intercooling is used is greater than without intercooling and hence thermal efficiency of the cycle is reduced.

- It is expected because the heat supplied in the intercooler is a complete loss, no part of it is converted into useful work.
Thermal Efficiency for Open Brayton Cycle with Intercooling:

- **Turbine work**, 
  \[ W_t = h_5 - h'_6 = C_p(T_5 - T'_6) \]

- **Compressor work**, 
  \[ W_c = (h'_2 - h_1) + (h'_4 - h_3) = C_p(T'_2 - T_1) + C_p(T'_4 - T_3) \]

- **Heat Supplied**, 
  \[ Q_s = h_5 - h'_4 = C_p(T_5 - T'_4) \]

- **Thermal efficiency**, 
  \[
  \eta_{th} = \frac{Net \ Work}{Heat \ Supplied} = \frac{W_t - W_c}{Q_s} \]

\[ \therefore \eta_{th} = \frac{C_p(T_5 - T'_6) - C_p(T'_2 - T_1) - C_p(T'_4 - T_3)}{C_p(T_5 - T'_4)} \]

\[ \therefore \eta_{th} = \frac{(T_5 - T'_6) - (T'_2 - T_1) - (T'_4 - T_3)}{(T_5 - T'_4)} \quad (7.47) \]

2. Reheating

- The output of a gas turbine can be improved by expanding the gases in two stages with a reheater between the two as shown in Fig. 7.16. The corresponding T-s diagram is shown in Fig. 7.17.

- By reheating or adding heat to the gases after they have passed through the turbine stage, a further increase in work done is obtained.

- In reheating, the gas temperature, which has dropped due to expansion, is brought back to approximately the initial temperature for the expansion in the next stage.

![Fig. 7.16 Open cycle gas turbine plant with reheating](image)
The actual processes take place as follows:

1-2' ---- Compression in compressor

2'-3 ---- Heat addition in combustion chamber

3-4' ---- Expansion in high pressure turbine

4'-5 ---- Heat addition in reheater

5-6' ---- Expansion in low pressure turbine

- **Work output** (with reheating),
  \[ W = C_p(T_3 - T_4') + C_p(T_5 - T_6') \]

- Work output (without reheating),
  \[ W = C_p(T_3 - T_4') + C_p(T_4' - T_6') \]

Since the pressure lines diverge to the right on T-s diagram, it can be seen that the temperature difference \((T_5 - T_6')\) is always greater than \((T_4' - T_6')\). So the reheating increases the net work.

Although the net work is increased by reheating, the heat to be supplied is also increased and hence thermal efficiency can be reduced.
Thermal Efficiency for Open Brayton Cycle with Reheating:

- **Turbine work**, 
  \[ W_t = (h_3 - h'_4) + (h_5 - h'_6) = C_p(T_3 - T'_4) + C_p(T_5 - T'_6) \]

- **Compressor work**, 
  \[ W_c = (h'_2 - h_1) = C_p(T'_2 - T_1) \]

- **Heat Supplied**, 
  \[ Q_s = (h_3 - h'_2) + (h_5 - h'_4) = C_p(T_3 - T'_2) + C_p(T_5 - T'_4) \]

- **Thermal efficiency**, 
  \[ \eta_{th} = \frac{\text{Net Work}}{\text{Heat Supplied}} = \frac{W_t - W_c}{Q_s} \]

  \[ \therefore \eta_{th} = \frac{C_p(T_3 - T'_4) + C_p(T_5 - T'_6) - C_p(T'_2 - T_1)}{C_p(T_3 - T'_2) + C_p(T_5 - T'_4)} \]

  \[ \therefore \eta_{th} = \frac{(T_3 - T'_4) + (T_5 - T'_6) - (T'_2 - T_1)}{(T_3 - T'_2) + (T_5 - T'_4)} \]

3. **Regeneration**

- The temperature of exhaust gases leaving the turbine of a gas turbine engine is considerably higher than the temperature of air delivered by the compressor.

- Therefore, high pressure air leaving the compressor can be heated by hot exhaust gases, thereby reducing the mass of fuel supplied in the combustion chamber. Hence the thermal efficiency can be increased.

- The heat exchanger used to transfer the heat from exhaust gases to compressed air is known as regenerator.

- The net work is not affected by the addition of the recuperator or regeneration to the cycle, only the heat required to be supplied in the combustion chamber is decreased which gives the gain in thermal efficiency.

- The corresponding schematic and T-s diagram is shown in Fig. 7.18 & 7.19 respectively. The actual processes take place as follows:

  1-2' ---- Compression in compressor

  2'-3 ---- Heat addition into the compressed air during its passage through the heat exchanger

  3-4 ---- Heat addition in the combustion chamber

  4-5' ---- Expansion in turbine

  5'-6 ---- Heat rejection in heat exchanger to the compressed air
Fig. 7.18 Open cycle gas turbine plant with regeneration

Fig. 7.19 T-s diagram of open cycle gas turbine with regeneration
- The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice.
- The effectiveness of the heat exchanger is given by,

\[ \varepsilon = \frac{\text{Increase in enthalpy per kg of air}}{\text{Available increase in enthalpy per kg of air}} \]

\[ \therefore \varepsilon = \frac{T_3 - T_2'}{T_5 - T_2'} \] (7.49)

**Thermal Efficiency for Open Brayton Cycle with Regeneration:**

- Turbine work,

\[ W_t = (h_4 - h_5') = C_p(T_4 - T_5') \]

- Compressor work,

\[ W_c = (h_2' - h_1) = C_p(T_2' - T_1) \]

- Heat Supplied,

\[ Q_s = (h_4 - h_3) = C_p(T_4 - T_3) \]

- Thermal efficiency,

\[ \eta_{th} = \frac{\text{Net Work}}{\text{Heat Supplied}} = \frac{W_t - W_c}{Q_s} \]

\[ \therefore \eta_{th} = \frac{C_p(T_4 - T_5') - C_p(T_2' - T_1)}{C_p(T_4 - T_3)} \] (7.50)

- Under ideal conditions, \( T_3 = T_5' \) for \( \varepsilon = 1 \), then

\[ \therefore \eta_{th} = 1 - \frac{(T_2' - T_1)}{(T_4 - T_3)} \]

### 7.11 Solved Numerical

**Ex 9.1.** [GTU; Jun-2010; 3 Marks]

An engine uses 6.5 Kg of oil per hour of calorific value of 30,000 kJ/Kg. If the Brake power of engine is 22 kW and mechanical efficiency is 85% calculate (a) indicate thermal efficiency (b) Brake thermal efficiency (c) Specific fuel consumption in Kg/B.P/hr.

**Solution:**

- **Given Data:**
  \[ \dot{m}_f = 6.5 \text{ kg/hr} \]
  \[ C.V. = 30000 \text{ kJ/kg} \]
  \[ B.P. = 22 \text{ kW} \]
  \[ \eta_m = 85\% \]

- **To be Calculated:**
  a) \( \eta_{ith} =? \)
  b) \( \eta_{bth} =? \)
  c) \( BSFC =? \)

\[ \Rightarrow \text{Indicated Power,} \]

\[ \eta_m = \frac{B.P.}{I.P.} \]

\[ \therefore I.P. = \frac{22}{0.85} = 25.8824 \text{ kW} \]

\[ \Rightarrow \text{Indicated Thermal Efficiency:} \]
\[ \eta_{ith} = \frac{1. P.}{m_f \times C.V.} \times 25.8824 \]
\[ \therefore \eta_{ith} = \left( \frac{6.5}{3600} \right) \times 30000 \]
\[ \therefore \eta_{ith} = 0.4778 = 47.78\% \]

\[ \Rightarrow \text{Break Thermal Efficiency:} \]
\[ \eta_{bth} = \frac{B.P.}{m_f \times C.V.} \]
\[ \therefore \eta_{bth} = \left( \frac{6.5}{3600} \right) \times 30000 \]
\[ \therefore \eta_{bth} = 0.4062 = 40.62\% \]

\[ \Rightarrow \text{Break Specific Fuel Consumption:} \]
\[ BSFC = \frac{\dot{m}_f (kg/hr)}{B.P. (kW)} \]
\[ \therefore BSFC = \frac{6.5}{22} \]
\[ \therefore BSFC = 0.2955 \text{ kg/kWh} \]

**Ex 9.2.** [Ex 13.3; P. K. Nag]

In a Carnot cycle, the maximum pressure and temperature are limited to 18 bar and 410°C. The ratio of isentropic compression is 6 and isothermal expansion is 1.5. Assuming the volume of the air at the beginning of isothermal expansion as 0.18 m³, determine:

(a) The temperature and pressures at main points in the cycle.
(b) Change in entropy during isothermal expansion.
(c) Mean thermal efficiency of the cycle.
(d) Mean effective pressure of the cycle.
(e) The theoretical power if there are 210 working cycles per minute.

**Solution:**

**Given Data:**

\[ p_1 = 18 \text{ bar} \]
\[ T_H = T_1 = T_2 = 410\°C \]
\[ V_4 = 6 \]
\[ V_2 = 1.5 \]
\[ V_1 = 0.18 \text{ m}^3 \]
\[ \text{No. of cycles} = 210 / \text{min} \]

\[ N = 1.4 \text{ for air} \]

\[ \Rightarrow \text{Temperatures & Pressures at the main point of the cycle:} \]

For Process 4-1 (Isentropic Compression),
\[
\frac{T_L}{T_H} = \left(\frac{V_1}{V_4}\right)^{y-1}
\]
\[
\therefore T_L = 683 \times \left(\frac{1}{6}\right)^{14-1}
\]
\[
\therefore T_L = T_3 = T_4 = 333.5494 \text{ K}
\]

Also,

\[
p_1V_1^y = p_4V_4^y
\]
\[
\therefore p_4 = p_1 \times \left(\frac{V_1}{V_4}\right)^y
\]
\[
\therefore p_4 = 18 \times \left(\frac{1}{6}\right)^{1.4}
\]
\[
\therefore p_4 = 1.465 \text{ bar}
\]

For Process 1-2 (Isothermal Expansion),

\[
p_1V_1 = p_2V_2
\]
\[
\therefore p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)
\]
\[
\therefore p_2 = 18 \times \left(\frac{1}{1.5}\right)
\]
\[
\therefore p_2 = 12 \text{ bar}
\]

For Process 2-3 (Isentropic Expansion),

\[
p_2V_2^y = p_3V_3^y
\]
\[
\therefore p_3 = p_2 \times \left(\frac{V_2}{V_3}\right)^y
\]
\[
\therefore p_3 = p_2 \times \left(\frac{V_1}{V_4}\right)^y \left(\therefore \frac{V_4}{V_1} = \frac{V_3}{V_2}\right)
\]
\[
\therefore p_3 = 12 \times \left(\frac{1}{6}\right)^{1.4}
\]
\[
\therefore p_3 = 0.9767 \text{ bar}
\]

\(\Rightarrow\) **Change in Entropy:**

From T-S diagram,

\[
Q_s = T_H \times (S_2 - S_1)
\]
\[
\therefore \Delta S = S_2 - S_1 = \frac{Q_s}{T_H} = \frac{p_1V_1 \ln \left(\frac{V_2}{V_1}\right)}{T_H}
\]
\[
\therefore \Delta S = 18 \times 10^5 \times 0.18 \times \ln 1.5
\]
\[
\therefore \Delta S = 192.3436 \text{ J/K}
\]

\(\Rightarrow\) **Thermal Efficiency of the Cycle:**

Heat Supplied,

\[
Q_s = p_1V_1 \ln \left(\frac{V_2}{V_1}\right)
\]
\[
\therefore Q_s = T_H \times (S_2 - S_1)
\]
\[
\therefore Q_s = 683 \times 192.3436
\]
\[ Q_s = 131370.6788 \, J = 131.370 \, kJ \]

Heat Rejected,
\[ Q_r = p_4 V_4 \ln \frac{V_3}{V_4} \]
\[ \therefore Q_r = 333.5494 \times 192.3436 \]
\[ \therefore Q_r = 64156.0923 \, J = 64.1561 \, kJ \]

Efficiency,
\[ \eta = \frac{Q_s - Q_r}{Q_s} = \frac{131.370 - 64.1561}{131.370} \]
\[ \therefore \eta = 0.5116 = 51.16\% \]

\( \Rightarrow \) **Mean Effective Pressure of the Cycle:**
\[ \frac{V_4}{V_1} = 6 \, \& \, \frac{V_2}{V_1} = 1.5 \]

Also,
\[ \frac{V_4}{V_1} = \frac{V_3}{V_2} \]
\[ \therefore \frac{V_3}{V_2} \times \frac{V_2}{V_1} = 6 \times 1.5 \]
\[ \therefore \frac{V_3}{V_1} = 9 \]

Swept Volume,
\[ V_s = V_3 - V_1 = 9V_1 - V_1 = 8V_1 \]
\[ \therefore V_s = 8 \times 0.18 \]
\[ V_s = 1.44 \, m^3 \]

Mean Effective Pressure,
\[ p_m = \frac{Net \, Work}{Swept \, Volume} = \frac{Q_s - Q_r}{V_s} \]
\[ \therefore p_m = \frac{131370.6788 - 64156.0923}{1.44} \]
\[ \therefore p_m = 46676.7961 \, Pa \]

\( \Rightarrow \) **Power of the Engine:**
\[ P = \frac{Work \, Done}{Cycle} \times \frac{No. \, of \, Cycle}{Sec} \]
\[ \therefore P = (131370.6788 - 64156.0923) \times \frac{210}{60} \]
\[ \therefore P = 235251.0528 \, W = 235.251 \, kW \]

**Ex 9.3. [GTU; Jan-2015; 7 Marks]**
In an I C Engine working with the Otto cycle, the cylinder diameter is 250mm and a stroke is 375mm. If the clearance volume is 0.00263m³, and the initial pressure and temperature are 1bar and 50°C, calculate (a) The air standard efficiency and (b) Mean effective pressure of the cycle. The maximum cycle pressure is limited to 25bar.
Solution:

Given Data:

\[
\begin{align*}
D &= 0.250 \text{ m} \\
L &= 0.375 \text{ m} \\
V_c &= 0.00263 \text{ m}^3 \\
p_1 &= 1 \text{ bar} \\
T_1 &= 323 \text{ K} \\
p_3 &= 25 \text{ bar}
\end{align*}
\]

To be Calculated:

a) \( \eta = ? \)

b) \( p_m = ? \)

\( \Rightarrow \) Swept Volume,

\[
V_s = \frac{\pi}{4} D^2 L
\]

\[
\therefore V_s = \frac{\pi}{4} 0.250^2 \times 0.375
\]

\[
\therefore V_s = 0.0184 \text{ m}^3
\]

\( \Rightarrow \) Total Volume,

\[
V_1 = V_s + V_c
\]

\[
\therefore V_1 = 0.0184 + 0.00263
\]

\[
\therefore V_1 = 0.02103 \text{ m}^3
\]

\( \Rightarrow \) Compression Ratio,

\[
r = \frac{V_1}{V_2} = \frac{0.02103}{0.00263}
\]

\[
\therefore r = 7.9961
\]

\( \Rightarrow \) Air Standard Efficiency:

\[
\eta = 1 - \frac{1}{r^{\gamma-1}}
\]

\[
\therefore \eta = 1 - \frac{1}{(7.9961)^{1.4-1}}
\]

\[
\therefore \eta = 0.5646 = 56.46\%
\]

\( \Rightarrow \) For Process 1-2 (Isentropic Compression),

\[
\frac{T_2}{T_1} = (r)^{\gamma-1}
\]

\[
\therefore T_2 = 323 \times (7.9961)^{1.4-1}
\]

\[
\therefore T_2 = 741.9144 \text{ K}
\]

And,

\[
p_1 V_1^\gamma = p_2 V_2^\gamma
\]

\[
\therefore p_2 = p_1 \times \left(\frac{V_1}{V_2}\right)^\gamma
\]

\[
\therefore p_2 = 1 \times (7.9961)^{1.4}
\]

\[
\therefore p_2 = 18.3666 \text{ bar}
\]
For Process 2-3 (Constant Volume Heat Addition)

\[
\frac{T_3}{T_2} = \frac{P_3}{P_2} \quad (\because V_2 = V_3)
\]

\[
\therefore T_3 = 741.9144 \times \frac{25}{18.3666}
\]

\[
\therefore T_3 = 1009.869 \text{ K}
\]

⇒ Mass of Air,

\[
p_1V_1 = mRT_1
\]

\[
\therefore m = \frac{1 \times 10^5 \times 0.02103}{0.287 \times 10^3 \times 323}
\]

\[
\therefore m = 0.02268 \text{ kg}
\]

⇒ Heat Supplied,

\[
Q_s = mc_v(T_3 - T_2)
\]

\[
\therefore Q_s = 0.02268 \times 0.718 \times 10^3 \times (1009.869 - 741.9144)
\]

\[
\therefore Q_s = 4363.437 \text{ J} = 4.3634 \text{ kJ}
\]

⇒ Net Work,

\[
\eta = \frac{W_{net}}{Q_s}
\]

\[
\therefore W_{net} = 0.5646 \times 4363.437
\]

\[
\therefore W_{net} = 2463.5965 \text{ J}
\]

⇒ Mean Effective Pressure:

\[
p_m = \frac{W_{net}}{V_s}
\]

\[
\therefore p_m = \frac{2463.5965}{0.0184}
\]

\[
\therefore p_m = 133891.1141 \text{ Pa}
\]

\[
\therefore p_m = 1.3389 \text{ bar}
\]

Ex 9.4. [GTU; Nov-2011; 7 Marks]

In an air standard diesel cycle the compression ratio is 16. At the beginning of isentropic compression the temperature is 15 °C and pressure is 0.1 MPa. Heat is added until the temperature at the end of constant pressure process is 1480°C. Calculate: (a) cut off ratio, (b) cycle efficiency and (c) M. E. P.

Solution:

<table>
<thead>
<tr>
<th>Given Data:</th>
<th>To be Calculated:</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 16</td>
<td>a) ( \rho = ? )</td>
</tr>
<tr>
<td>( p_1 = 0.1 \text{ MPa} = 1\text{bar} )</td>
<td>b) ( \eta = ? )</td>
</tr>
<tr>
<td>( T_1 = 288 \text{ K} )</td>
<td>c) ( p_m = ? )</td>
</tr>
<tr>
<td>( T_3 = 1753 \text{ K} )</td>
<td></td>
</tr>
</tbody>
</table>

Ex. 9.4.
\( \bullet \textbf{Cut off Ratio:} \)

For Process 1-2 (Isentropic Compression),
\[
\frac{T_2}{T_1} = (r)^{\gamma - 1}
\]
\[
\therefore T_2 = 288 \times (16)^{1.4 - 1}
\]
\[
\therefore T_2 = 873.0527 \text{ K}
\]

For Process 2-3 (Constant Pressure Heat Addition)
\[
\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad (\because p_2 = p_3)
\]
\[
\therefore \frac{V_3}{V_2} = 1753
\]
\[
\therefore \rho = \frac{V_3}{V_2} = 2.007
\]

\( \bullet \textbf{Cycle Efficiency:} \)
\[
\eta = 1 - \frac{1}{r^{\gamma - 1}} \left( \frac{\rho^\gamma - 1}{\gamma (\rho - 1)} \right)
\]
\[
\therefore \eta = 1 - \frac{1}{16^{0.4}} \left( \frac{2.007^{1.4} - 1}{1.4(2.007 - 1)} \right)
\]
\[
\therefore \eta = 0.6134 = 61.34\%
\]

\( \bullet \textbf{Mean Effective Pressure:} \)

Heat Supplied (per unit mass),
\[
q_s = C_p(T_3 - T_2)
\]
\[
\therefore q_s = 1.005(1753 - 873.0527)
\]
\[
\therefore q_s = 884.347 \text{ kJ kg}^{-1}
\]

Net Work,
\[
\eta = \frac{w_{net}}{q_s}
\]
\[
\therefore w_{net} = 0.6134 \times 884.347
\]
\[
\therefore w_{net} = 542.4584 \text{ kJ kg}^{-1}
\]

Swept Volume,
\[
V_s = V_1 - V_2 = V_1 \left( 1 - \frac{V_2}{V_1} \right)
\]
\[
\therefore v_s = \frac{RT_1}{p_1} \left( 1 - \frac{1}{r} \right)
\]
\[
\therefore v_s = \frac{287 \times 288}{1 \times 10^5} \left( 1 - \frac{1}{16} \right)
\]
\[
\therefore v_s = 0.7749 \text{ m}^3 \text{ kg}^{-1}
\]

Mean Effective Pressure,
\[
p_m = \frac{w_{net}}{V_s}
\]
\[
\therefore p_m = \frac{542.4584 \times 10^3}{0.7749} \\
\therefore p_m = 700036.6499 \text{ Pa} \\
\therefore p_m = 7.0003 \text{ bar}
\]

### 7.12 References


PROPERTIES OF GASES AND GAS MIXTURE

Course Contents

8.1 Introduction
8.2 Equation of State
8.3 Ideal Gas and Ideal Gas equation of State
8.4 Vander Waals’ Equation of State for Real Gas
8.5 Reduced Properties
8.6 Law of Corresponding State
8.7 Compressibility Factor
8.8 Dalton’s Law and Gibbs-Dalton Law
8.9 Avogadro’s Law
8.1 Introduction
- A pure substance is defined as a substance having a constant and uniform chemical composition. A homogeneous mixture of gases which do not react with one another may, therefore, be considered a pure substance. For example, air is a homogeneous mixture of nitrogen, oxygen and traces of other substances like argon, helium, carbon dioxide, etc., and as they do not react with one another, air is regarded a pure substance. The properties of such a mixture can be determined and tabulated just like those of any other pure substance.
- The properties of air and some combustion products have been determined and tabulated in gas tables. But it is not possible to determine the properties of the unlimited number of mixtures possible, the properties of the mixtures are determined from the properties of the constituent gases.

8.2 Equation of State
- “The relation between the independent properties, such as pressure, specific volume and temperature for a pure substance is known as the equation of state.”
- The equation of state can be expressed as,
  \[ f(p, v, T) = 0 \]
- If two of these properties are known, the third can be evaluated from the equation of state. Two independent properties are sufficient to describe the state of the system. Thus equation of state for this type of system may be written as,
  \[ p = f(v, T) \]
  \[ v = \phi(p, T) \]
  \[ T = \psi(p, v) \]
- For the perfect gas, equation of state relating pressure, specific volume and temperature is expressed by, \( pv = RT \)

8.3 Ideal Gas and Ideal Equation of State
- Following are the characteristics of an ideal gas:
  1. An ideal gas having no forces of intermolecular attraction and repulsion and does not change its phase during a thermodynamic process.
  2. It obeys the gas laws at all ranges of pressures and temperatures are. However, ‘real gases’ follow these laws at low pressures or high temperatures or both. For examples real gases like hydrogen, oxygen, nitrogen, helium etc. behave nearly the same way as perfect gases.
  3. A perfect gas obeys the law \( pv = RT \) and has constant specific heats which are the function of temperature only.
- Consider unit mass of an ideal gas that passes from state \( p_1, v_1, T_1 \) to another state identified by \( p_2, v_2, T_2 \) as shown in Fig. 8.1. This change be at first at constant pressure \( p_1 \) to some intermediate volume \( v_i \) and then at constant temperature \( T_i \) to final volume \( v_2 \).
Process 1-2' is at constant pressure and therefore, state of the gas changes by following Charle’s law. Thus,
\[
\frac{v_1}{T_1} = \frac{v'_2}{T'_2}; \quad v_2 = v'_2 = \frac{T_2}{T'_1}
\]

Process 2'-2 is at constant temperature and, therefore change in the state of gas by following the Boyle’s law. Thus
\[
p_1v_1 = p_2v_2; \quad v_2 = v'_2 = \frac{p_2}{p_1}
\]

Compare value of \(v_2\),
\[
v_1 \frac{T'_2}{T_1} = \frac{p_2}{p_1}
\]

But \(T'_2 = T_2\) and \(p_1 = p'_2\), we get,
\[
v_1 \frac{T_2}{T_1} = p_2 \frac{v_2}{p_1} \quad \text{or} \quad p_1v_1 = p_2v_2 T_2
\]
\[
\therefore \frac{pv}{T} = R
\]

But \(v = \frac{V}{m}\) where \(V\) is the volume of gas and \(m\) is the mass of gas then above equation becomes,
\[
[pv = mRT]
\]

Above equation is called the equation of state or characteristic gas equation.

8.4 Vander Waals’ Equation of State for Real Gas

The ideal gas equation \(pv = RT\) can be derived analytically using the kinetic theory of gases by making the following assumptions:

1. A finite volume of gas contains large number of molecules. The volume occupied by the gas molecules is negligible as compared to the volume of the gas.
2. The collisions of molecules with one another and with the walls of the container are perfectly elastic.
3. The molecules are separated by large distances compared to their own dimensions.
4. The molecules do not exert forces on one another except when they collide. Real gases differ from ideal ones due to presence of the intermolecular forces and also to the finite molecular volumes.
   - Thus, the Van der Waal’s is equation of state has been set up by incorporating the following two corrections into the equation of state, \( pv = RT \). These modifications take into account the departure of a real gas from an ideal one.

**Modification-I (Correction for the size of molecules)**
- This correction is due to the size of the molecule. Consider some quantity of a gas contained in a vessel as shown in Fig. It is known that the molecules of a gas have a finite size; therefore the space available for the molecules to move about is less than the volume of the gas.

**Modification-II (Correction for the intermolecular attractions)**
- Consider a gas molecule A well within the body of the gas inside the vessel as shown in Fig. It is attracted equally by other molecules in all directions with the same force and the resultant force on it is zero. But when molecule (B) strikes the wall of the vessel, it is pulled back by other molecules. Its velocity, and hence the momentum, with which it strikes the wall would be less than the momentum with which it will strike in the absence of the force of attraction. It is obvious that when the pressure of the gas drops, the momentum of the molecules also decreases. It is due to this fact that the decrease in pressure is proportional to: (i) The number of attracting molecules per unit volume, and (ii) the number of attracting molecules striking a unit area of the walls of the containing vessel per unit time.

\[
\Delta p = a \rho^2 = \frac{a}{v^2}
\]
\[
\left(p + \frac{a}{v^2}\right)(v-b) = RT
\]

**Determination of Constants ‘\(a\)’ and ‘\(b\)’**

- Van der Waal’s equation can be expanded and written in the form,
  \[
  pv^3 - (pb + RT)v^2 + av - ab = 0
  \]
- For given values of \(p\) and \(T\). This cubic equation in \(v\) will have three roots with the following characteristics:
  1. The three roots are real and +ve for a certain range of pressure at low temperatures.
     With reference to Fig. 8.2 these three values for volume have been indicated as \(v_1\), \(v_2\) and \(v_3\).
  2. With rise in temperature, the three real roots approach each other and become equal at the critical point denoted by \(C\).
  3. Above critical temperature, the there is only one real root for all values of pressure.

![Graph showing Van der Waal's equation](image)

- At the critical point, where roots coincide, the isotherm has zero slope. Further, the slope changes at the critical point; it exhibits a point of inflection. That is,
  \[
  \left(\frac{\partial p}{\partial v}\right)_c = 0 \quad \text{and} \quad \left(\frac{\partial^2 p}{\partial v^2}\right)_c = 0
  \]
  where the subscript \(c\) refers to value at critical point.
- The Van der Waal’s equation in terms of critical properties is,
  \[
  \left(p_c + \frac{a}{v_c^2}\right)(v_c-b) = RT_c
  \]
  \[
  p_c = \frac{RT_c}{v_c-b} - \frac{a}{v_c^3}
  \]
  \[
  \left(\frac{\partial p}{\partial v}\right)_c = \frac{-RT_c}{(v_c-b)^2} \cdot \frac{2a}{v_c^3}
  \]
  \[
  RT_c = \frac{2a}{v_c^3}(v_c-b)^2 \quad \text{..................(i)}
  \]
Further, by second derivative
\[
\left( \frac{\partial^2 p}{\partial v^2} \right)_c = \frac{2RT_c}{(v_c-b)^3} - \frac{6a}{v_c^3} = 0
\]
\[
RT_c = \frac{3a}{v_c} (v_c-b)^3 \quad \text{..........................(ii)}
\]
- From expression (i) and (ii),
\[
\frac{2a}{v_c^3} (v_c-b)^2 = \frac{3a}{v_c} (v_c-b)^3
\]
\[
v_c = 3b
\]
- Put \(v_c = 3b\) in expression (i),
\[
RT_c = \frac{3a}{27b^3} (3b-b)^3
\]
\[
T_c = \frac{8a}{27bR}
\]
- When the values of \(v_c\) and \(T_c\) are substituted in Van der Wall’s equation, we get
\[
p_c = \frac{R \left( \frac{8a}{27b^2} \right)}{3b-b} - \frac{a}{9b^2}
\]
\[
p_c = \frac{a}{27b^2}
\]
- As we know that,
\[
T_c = \frac{8a}{27bR} = \frac{8 \times 27b^2 p_c}{27Rb} = \frac{8bp_c}{R}
\]
\[
b = \frac{RT_c}{8p_c}
\]
- Further, from equation \(a = 27b^2 p_c = 27 \left( \frac{RT_c}{8p_c} \right)^2 p_c\)
\[
a = \frac{27 R^2 T_c^2}{64 p_c}
\]

### 8.5 Reduced properties
- It is defined as the ratio of property of substance at the given state to the value of the same property at critical state of the substance.
- Thus, Reduced temperature, \(T_r = \frac{T}{T_c}\)

\[
\text{Reduced pressure, } p_r = \frac{P}{p_c}
\]
\[
\text{Reduced volume, } v_r = \frac{v}{v_c}
\]
Reduced Property Equation of State
- It refers to the Van der Wall’s equation of state in terms of reduced property.
- The Van der Wall’s equation of state is given by,
\[
(p + \frac{a}{v^2})(v - b) = RT
\]
- In above equation, the property \( p, v \) and \( T \) in terms of their reduced property is,
\[
\begin{align*}
p &= p_r, v = v_r = 3b v_r, \\
T &= T_r = \frac{8a}{27 R b} T_r
\end{align*}
\]
- When these values are substitute in Van der Wall’s equation,
\[
\left( \frac{a}{27 b^2} p_r + \frac{a}{9 b^3 v_r^2} \right) (3b v_r - b) = \frac{8a}{27 R b} T_r
\]
\[
\frac{a}{27b^2} \left( p_r + \frac{3}{v_r^2} \right) b (3v_r - 1) = \frac{8a}{27b} T_r
\]
\[
\left( p_r + \frac{3}{v_r^2} \right) (3v_r - 1) = 8T_r
\]
- Above equation contains only reduced property and is called reduced property equation.

8.6 Law of Corresponding States
- Two gases said to be in corresponding states if the gases have same values for any two of the three reduced variables, they also have the same value for the third variable.
- Denoting reduced variables of the gases by subscript 1 and 2, we have,
\[
\left( p_{r1} + \frac{3}{v_{r1}^2} \right) (3v_{r1} - 1) = 8T_{r1}
\]
\[
\left( p_{r2} + \frac{3}{v_{r2}^2} \right) (3v_{r2} - 1) = 8T_{r2}
\]
- Thus the gases are considered to be in corresponding states if their pressure, volume and temperature are of the same fractions of the critical pressure, volume and temperature of the two gases. Mathematically,
\[
T_r = f \left( p_r, v_r \right)
\]
- Above relation is referred as generalized equation of state.

8.7 Compressibility Factor
- Compressibility Factor is defined as the ratio of the actual volume of the gas to the value of predicted by the ideal gas equation at the same temperature and pressure. Thus,
The factor $Z$ is dimensionless and becomes unity for an ideal gas at all temperatures and pressures. The magnitude of $Z$ for a certain gas at a particular temperature and pressure gives an indication of the extent of deviation of the gas from ideal gas behaviour. The equation $Z = \frac{pv}{RT/p}$ shows that the compressibility factor $Z$ is a function of $p$ and $T$. Thus the experimental $p$-$v$-$T$ data of a pure substance can be fitted into an equation of the form, $Z = f(p, T)$. Accordingly, a plot can be made of isotherms (the lines of constant temperature) on the coordinates of $p$ and $Z$ in fig. Such a plot is referred to as the compressibility chart.

Conclusions from the charts

1. A different compressibility chart is needed for each gas.
2. A general similarity exists in the compressibility chart of different gases; the difference lies in the scales (numerical values).
3. As the pressure is reduced, the compressibility factor for any gas approaches unity, as $p \to 0; Z \to 1$. This aspect stems from the fact that the gases behave more like a perfect gas as the pressure is reduced.
4. The value of $Z$ for any gas may be more or less than unity depending upon the pressure and temperature of the gas. For real gases $Z < 1$ or $Z > 1$. The value of $Z > 1$ means that the actual density is greater, and the $Z > 1$ means that the actual density is less than that predicted by the ideal equation of state.
5. From the law corresponding states that $f(p_r, v_r, T_c) = 0$ and the compressibility factor $Z$ can be expressed as,

$$Z = \frac{pv}{RT} = \frac{p_r v_r}{R} \times \frac{v_r v_c}{T_r T_c} = \frac{p_r v_r}{RT} \times \frac{v_r v_c}{T_c} = Z_c \times \frac{p_r v_r}{T_r}$$
The factor $Z_c$ is called critical compressibility factor, and for a gas obeying Van der Wall’s equation of state,

$$Z_c = \frac{p_c v_c}{RT_c} = \frac{3}{8}$$

$$Z = \frac{3}{8} \frac{p v}{T}$$

The Van der Wall’s equation of state, expressed in the reduced co-ordinates,

$$\left[ \frac{p}{p_r} + \frac{27 p_r^2}{64 T_r^2} Z^2 \right] \left[ \frac{8 T_r Z}{p_r} - 1 \right] = 8 T_r$$

$$Z^3 - \left[ \frac{p}{8T_r} + 1 \right] Z^2 + \left[ \frac{27 p_r}{64 T_r^2} \right] Z - \frac{27 p_r^2}{512 T_r^2} = 0$$

This is another form of Van der Waal’s equation and can be expressed as,

$$Z = f \left( p_r, T_r \right)$$

Above equation is referred as generalized equation of state.

**Observations from the generalised compressibility chart are:**

- At all temperatures $Z \rightarrow 1$ as $p_r \rightarrow 0$
- At temperature $T_r = 2$ and above, $Z = 1$ over a wide range of pressures up to $p_r = 5$.
- In the region $Z < 1$, the intermolecular attractive forces play a dominant role. Accordingly the pressure exerted by the gas is less than that exerted by an ideal gas.
- In the region $Z > 1$, the volume occupied by the gas molecules plays a dominant role. As such the volume occupied by the actual gas is greater than that occupied by the ideal gas.
8.8 Dalton’s Law and Gibbs-Dalton Law

Dalton’s law

– Let us consider a closed vessel of volume V at temperature T, which contains a mixture of perfect gases at a known pressure. If some of the mixture were removed, then the pressure would be less than the initial value. If the gas removed were the full amount of one of the constituents then the reduction in pressure would be equal to the contribution of that constituent to the initial total pressure. Each constituent contributes to the total pressure by an amount which is known as the partial pressure of the constituent.

– The relationship between the partial pressures of the constituents is expressed by Dalton’s law, as follows:
  – The pressure of a mixture of gases is equal to the sum of the partial pressures of the constituents.
  – The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixtures at the same temperature.

This is expressed diagrammatically in Fig. The gases A and B, originally occupying volume V at temperature T are mixed in the third vessel which is of the same volume and is at the same temperature.

By the consideration of mass,

\[ m = m_A + m_B \]

By Dalton’s law, \[ p = p_A + p_B \]

Dalton’s law is based on experiment and is found to be obeyed more accurately by gas mixtures at low pressures. As shown in Fig. each occupant occupies the whole vessel.

The example given in Fig. and relationship in equation refer to a mixture of two gases, but the law can be extended to any number of gases,

\[ m = m_A + m_B + m_C + \ldots \] or \[ m = \Sigma m_i \]

where \( m_i \) = Mass of a constituent.

Similarly \[ p = p_A + p_B + p_C + \ldots \] or \[ p = \Sigma p_i \]

where \( p_i \) = The partial pressure of a constituent.
Gibbs-Dalton law

- Dalton’s law was re-formulated by Gibbs to include a second statement on the properties of mixtures. The combined statement is known as the Gibbs-Dalton law, and is as follows:
  - The internal energy, enthalpy, and entropy of a gaseous mixture are respectively equal to the sums of the internal energies, enthalpies, and entropies, of the constituents.
  - Each constituent has that internal energy, enthalpy and entropy, which it could have if it occupied alone that volume occupied by the mixture at the temperature of the mixture.
  - This statement leads to the following equations
    \[ mu = m_A u_A + m_B u_B + \ldots \quad \text{or} \quad mu = \sum m_i u_i \]
    \[ mh = m_A h_A + m_B h_B + \ldots \quad \text{or} \quad mh = \sum m_i h_i \]
    \[ ms = m_A s_A + m_B s_B + \ldots \quad \text{or} \quad ms = \sum m_i s_i \]

8.9 Avogadro's Law

- Avogadro’s law states that "Under identical conditions of temperature and pressure, equal volumes of all gases have same number of molecules."
- Consider two different ideal gases having the same volume, same pressure and same temperature. Further, let their molecular masses be denoted by \( M_1 \) and \( M_2 \) respectively. In accordance with Avogadro’s law, each gas will contain the same number of molecules, say ‘n’ and so the masses of the gases would be,
  \[ m_1 \propto n M_1 = k n M_1 \]
  \[ m_2 \propto n M_2 = k n M_2 \]
  where \( k \) is the proportionality constant, \( \rho_1 \) and \( \rho_2 \) are the densities of the gases,
  \[ m_i = \rho_i V \quad \text{and} \quad m_2 = \rho_2 V \quad (\because V_1 = V_2 = V) \]
  is the volume of each gas
  When the values of masses are substituted in equation, we get
  \[ \frac{\rho_1 V}{\rho_2 V} = \frac{k n M_1}{k n M_2} \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{M_1}{M_2}; \quad \frac{v_2}{v_1} = \frac{M_1}{M_2} \]
  - Thus the densities of the gases are directly proportional to their molecular masses at the same temperature and pressure.
  - Substituting \( \rho = \frac{1}{V} \) in above equation, we obtain
    \[ \frac{v_2}{v_1} = \frac{M_1}{M_2} \]
  - Thus the specific volumes of gases are inversely proportional to their molecular masses at the same temperature and pressure.
  Equation may be recast as,
  \[ v_1 M_1 = v_2 M_2 \quad \text{and} \quad vM = \text{Constant} \]
Above equation indicates that the product of molecular mass and specific volume is constant for all ideal gases under identical conditions of pressure and temperature. The quantity $\nu M$ is called molar volume; it represents the volume of one kilo mole of an ideal gas. One kilo mole is the quantity of a substance whose mass in kilograms is numerically equal to its molecular mass.

**Properties of gases and gas mixtures summary**

The first part of the chapter deals with general ideal gas mixture considerations and begins by describing mixture composition in terms of the mass fractions or mole fractions. The Dalton model, which brings in the partial pressure concept, is then introduced for the $p$–$y$–$T$ relation of ideal gas mixtures.

**Reference Books:**

1. Thermal Science and Engineering by D. S. Kumar
2. Engineering Thermodynamics by R. K. Rajput
3. Fundamental of Engineering Thermodynamics by Michael J. Moran
4. Engineering Thermodynamics by P. K. Nag

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*Fill the brain with high thoughts, highest ideas, place them day and night before you, and out of that will come great work.*

– Swami Vivekananda