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Applied Fluid Mechanics
(2160602)

Module-1

Viscous Flow

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SYLLABUS

Sr. No.	Contents	Teaching hours	Weightage %
1	• Introduction • Continuity equation • Energy equation • Momentum equation • Major and minor energy losses, • Hydraulic gradient and total energy line • Pipes in series and parallel • Pipe networks • Hydraulic transmission of power • Navier-Stokes equation of motion • Initial conditions and boundary conditions • Viscous flow-Couette flow • Hagen-Poiseuille equation-flow between parallel plates • Turbulent flow in pipes • Prandtl's mixing length theory • Velocity distribution • Smooth and rough boundaries • Water hammer phenomenon	12	30

GTU MIMP QEUSTIONS LIST

Sr. No.	Questions	Exam Year	Marks
1	Explain the terms: Hydraulic gradient line and Total energy line. **OR** Define: (i) Critical depth (ii) Total energy line (iii) Hydraulic gradient line.	May-2019 Nov-2017	3 3
2	Derive an expression for the loss of head due to sudden enlargement of a pipe.	May-2019 Oct-2016 May-2016	4 7 7
3	Prove that the velocity distribution for viscous flow between two parallel plates when both plates are fixed across a section is parabolic in nature.	May-2019	7
4	Explain the terms: Pipes in parallel and Equivalent pipe.		3
5	Derive an expression for the loss of head due to friction in pipe. **OR** Derive Darcy Weisbach formula for the loss of head due to friction in pipe line.	May-2019 Nov-2018 Nov-2017 Oct-2016	4 7 7 7
6	Enlist the major and minor loses in pipes. Derive the expression for loss of head due to sudden contraction. **OR** What are the minor losses? Under what circumstances will they be negligible? Derive the expression for loss of head due to sudden contraction.	Nov-2018 April-2017	7 7

GTU MIMP QEUSTIONS LIST

Sr. No.	Questions	Exam Year	Marks
7	Explain hydraulically smooth and rough pipes	Nov-2018 Nov-2017	3 3
8	Calculate the head loss due to friction using Darcy Equation and power required to maintain 60 liters per second of liquid flow through a steel pipe 0.08 m radius and 900 m long. Take Sp. Gravity of the liquid = 0.85 and co-efficient of friction f=0.0025.	Nov-2018	7
9	Enlist the important applications of Navier-stoke equations	Nov-2018 April-2017	4 3
10	Define (i) Cavitation (ii) Prandtl Mixing length (iii) Water Hammer **OR** Describe water hammer phenomenon in pipes. **OR** Explain briefly Prandtl's mixing layer theory.**OR** (i) Shear Velocity	Nov-2018 Nov-2017 April-2017	3 3 3
11	Explain Hagen-Poiseuille theory. **OR** Derive the Hagen-Poiseuille equation and state the assumptions made. **OR** Derive an expression for the velocity distribution of viscous flow through a circular pipe and prove that the ratio of maximum velocity to average velocity is 2.	April-2018 Nov-2017 April-2017 Oct-2016 May-2016	3 7 7 7 7

GTU MIMP QEUSTIONS LIST

Sr. No.	Questions	Exam Year	Marks
12	Describe relation between shear stress and velocity gradient.	April-2018	4
13	What is couette flow? Derive an expression of velocity and shear stress for couette flow.		7
14	Explain different types of shear theories for turbulent flow.		3
15	Write advantages and disadvantages of shear theories.		4
16	A rough pipe of 30 cm diameter carries water. If the mean point velocity and the velocity gradient at a distance of 3 cm from pipe wall are 2 m/sec and 12.5 sec-1 respectively, determine the average height of roughness projection, wall shear stress, friction factor and mean velocity of flow. Take ρ =1000 kg/m3 and κ(kappa) = 0.4		7
17	What do you mean by pipes in series and pipes in parallel? How the loss of head is determined in both systems		3
18	Write a brief note on major and minor losses in pipes.	Nov-2017	4

<table>
<tr><th>Sr. No.</th><th>Questions</th><th>Exam Year</th><th>Marks</th></tr>
<tr><td>19</td><td>A pipe system consists of three pipes arranged in series.
<table>
<tr><th>Sr. No</th><th>Pipe</th><th>Length (m)</th><th>Diameter (cm)</th></tr>
<tr><td>1.</td><td>AB</td><td>1000</td><td>40</td></tr>
<tr><td>2.</td><td>BC</td><td>2500</td><td>30</td></tr>
<tr><td>3.</td><td>CD</td><td>3000</td><td>20</td></tr>
</table>
Transform the system to (i) an equivalent length of 30 cm diameter pipe, and (ii) an equivalent diameter for the pipe 6500 m long.</td><td>Nov-2017</td><td>7</td></tr>
<tr><td>20</td><td>Write the assumptions made in derivation of the Dynamic Equation of the Gradually varied flow.</td><td rowspan="4">April-2017</td><td>3</td></tr>
<tr><td>21</td><td>What are the differences between pipe flow and open channel flow? Also write the uses of pipes for hydraulic transmission of fluid.</td><td>4</td></tr>
<tr><td>22</td><td>Derive the continuity equation for one dimensional flow and discuss its application.</td><td>7</td></tr>
<tr><td>23</td><td>Enlist the forces acting on Fluid in motion.</td><td>4</td></tr>
</table>

GTU MIMP QEUSTIONS LIST

Sr. No.	Questions	Exam Year	Marks
24	Calculate the head loss due to friction using Darcy Equation and power required to maintain 50.3 liters per second of liquid flow through a steel pipe 0.1 m radius and 900 m long. Take Sp. Gravity of the liquid = 0.7 and co-efficient of friction f=0.0025.	April-2017	7
25	A horizontal pipe of 150 mm diameter is suddenly enlarged to 300 mm diameter. The rate of flow of water through a pipe is 0.2 m3/sec. The pressure intensity of smaller pipe 125 kPa. Determine (i) Loss of head due to sudden enlargement (ii) Pressure intensity in large pipe.	Oct-2016	7
26	A pipe of diameter of 20 cm conveying water. Calculate the discharge when centre line velocity is 3 m/sec and velocity at a point 4 cm from centre is 2.5 m/sec.		7
27	A smooth pipe of diameter 400 mm and length 800 m carries water at a rate of 0.40 m3/s. determine the head lost due to friction, wall shear stress, centre line velocity and thickness of laminar sub-layer. Take kinematic viscosity of water as 0.018 stokes and coefficient of friction, f = 0.0791/ (Re)1/4	May-2016	7

GTU MIMP QEUSTIONS LIST

Sr. No.	Questions	Exam Year	Marks
28	Two reservoirs are connected by two pipes in series of lengths 200 m and 300 m and of diameters 20 cm and 30 cm respectively. The difference of head between the two reservoir water surfaces is 10m. The friction factors for the two pipes are 0.02 and 0.015 respectively. Determine the flow rate.	May-2016	7

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DYNAMICS OF FLUID FLOW:

- It is the study of fluid motion with the forces causing flow.
- The dynamic bhaviour of the fluid flow is analyzed by the Newton's second law of motion.

$$F = m\,a$$

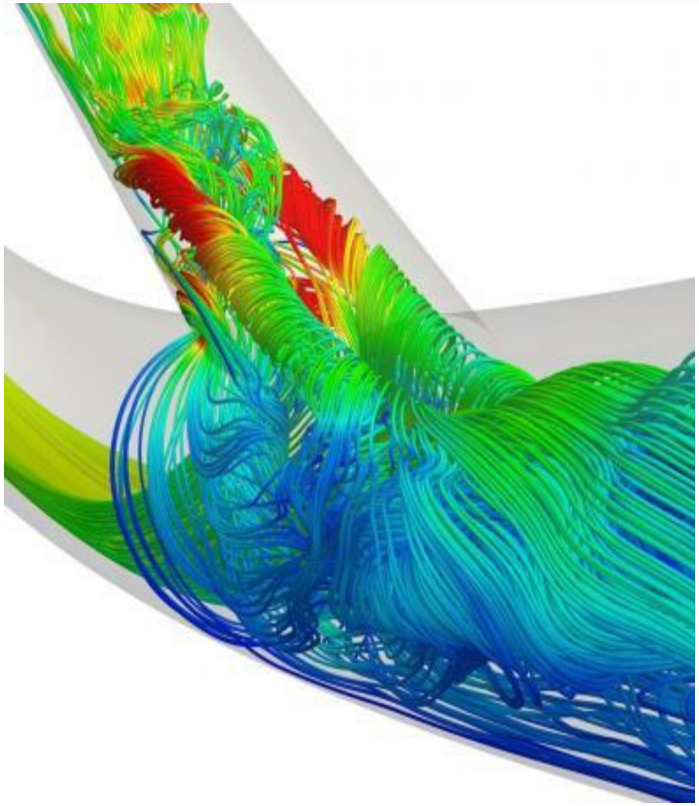
- In fluid flow, the following forces are present:
  - F_g, gravity force
  - F_p, pressure force
  - F_v, viscous force
  - F_t, turbulent force
  - F_c, force due to compressibility

$$F = F_g + F_p + F_v + F_t + F_c$$

- If F_c is neglected

$$F = F_g + F_p + F_v + F_t$$

Equation is known as Reynold's Equation of motion.



DYNAMICS OF FLUID FLOW:

- If F_t is neglected

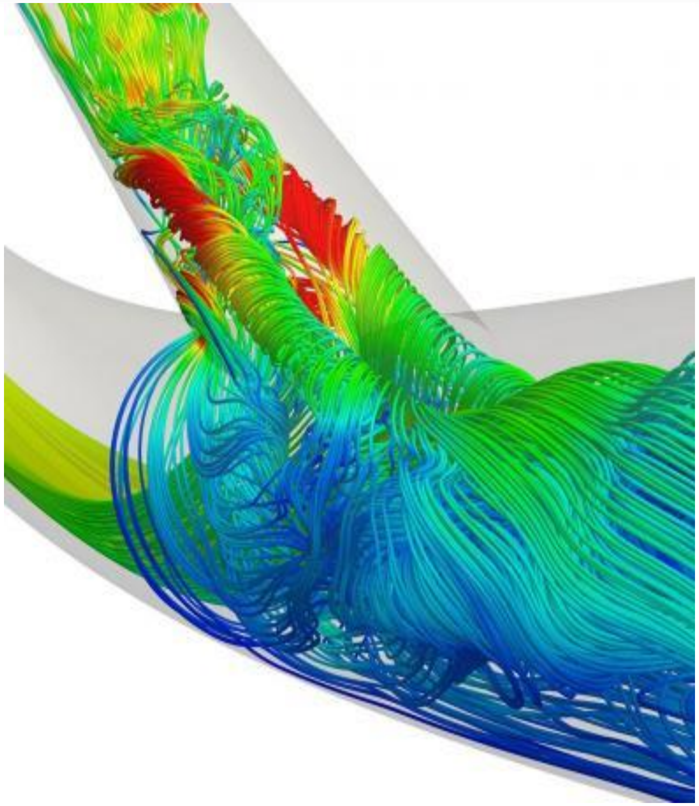
$$F = F_g + F_p + F_v$$

Equation is known as Navier-Stokes Equation.

- If is F_v neglected

$$F = F_g + F_p$$

Equation is known as Euler's equation of motion.

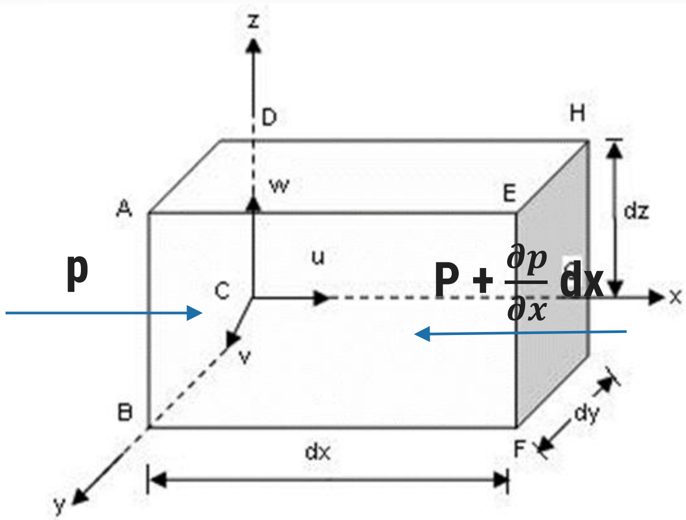


NAVIER-STOKES EQUATION OF MOTION:

- $F = F_g + F_p + F_v$

Equation is known as **Navier-Stokes Equation**.

- Consider small element of size dx, dy and dz in x, y and z direction.



- Let us consider pressure force acting on element is

On Face ABCD = p * dy dz

On Face EFGH = $[P + \frac{\partial p}{\partial x} dx] * dy\ dz$

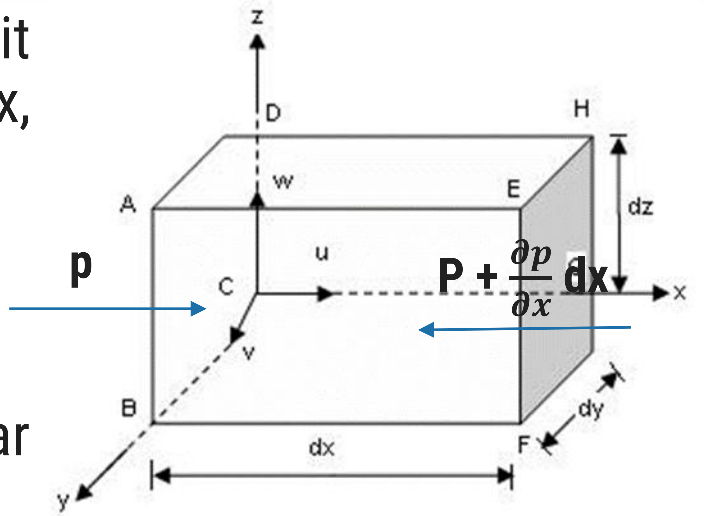
- Net pressure force in x-direction is

$= p * dy\ dz - [P + \frac{\partial p}{\partial x} dx] * dy\ dz$

$= -\frac{\partial p}{\partial x}$ **dx dy dz** ……………(1)

- Let us consider R is the body force per unit mass of fluid having component X, Y and Z in x, y and z dierection is

= **X * ρ * dx dy dz** …………… (2)



- Let T_x, T_y and T_z are the components of shear force in x, y and z direction. …………. (3)

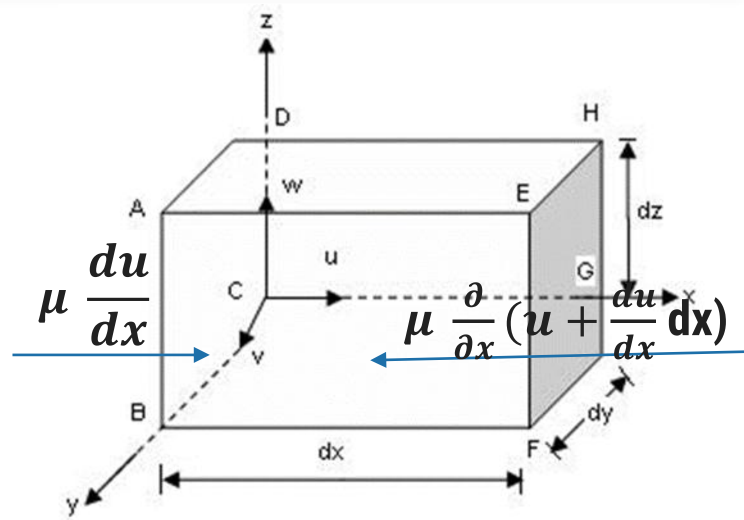
- Applying Newton's second law of motion in x – direction

Force = mass * Acceleration

$$X * \rho * dx\,dy\,dz - \frac{\partial p}{\partial x} dx\,dy\,dz - T_x = \rho * dx\,dy\,dz * \frac{du}{dt}$$

…………..(4)

NAVIER-STOKES EQUATION OF MOTION:



- Let us consider shear stress in x direction is

$$\tau = \mu \frac{du}{dx}$$

- The shear force acting on face ABCD is

$$= \mu \ (dy\ dz) \frac{du}{dx}$$

- The shear force acting on face EFGH is

$$= \mu \ (dy\ dz) \frac{\partial}{\partial x}(u + \frac{du}{dx} \text{ dx})$$

$$= \mu \ (dy\ dz) \ (\frac{\partial u}{\partial x} + \frac{d^2u}{dx^2} \text{ dx})$$

- The net shear force along x−direction on faces ABCD and EFGH is

$$= \mu \ (dy\ dz) \frac{du}{dx} - \mu \ (dy\ dz) \ (\frac{\partial u}{\partial x} + \frac{d^2u}{dx^2} \text{ dx})$$

$$= -\mu \frac{d^2u}{dx^2} \text{ dx } dy\ dz \qquad \text{.............(5)}$$

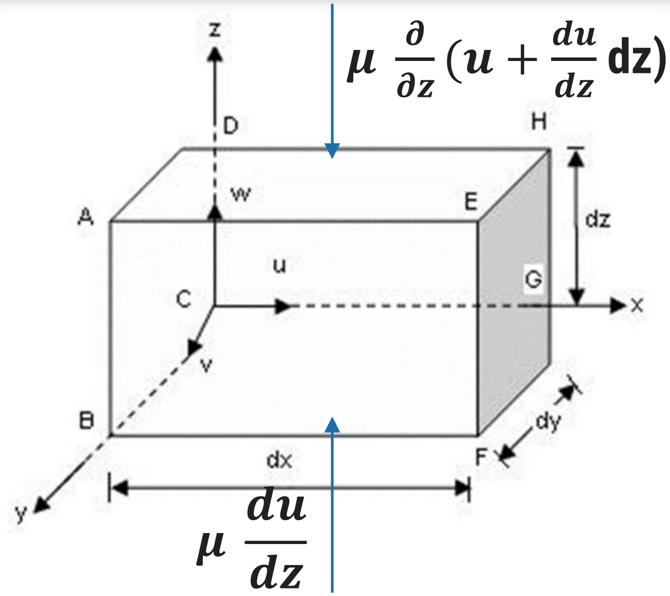
NAVIER-STOKES EQUATION OF MOTION:

- The net shear force along x−direction on faces BCGF and ADEH is

$$= -\mu \frac{d^2u}{dz^2}\,\mathrm{dx}\,dy\,dz \qquad \text{.......... (6)}$$

- The net shear force along x-direction on face CDGH and ABEF is

$$= -\mu \frac{d^2u}{dy^2}\,\mathrm{dx}\,dy\,dz \qquad \text{..........(7)}$$



- The net shear force along x−direction on all faces is

$$\mathrm{T_x} = -\mu \left[\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right] \mathrm{dx}\,dy\,dz \qquad \text{................... (8)}$$

- The net shear force along y−direction on all faces is

$$\mathrm{T_y} = -\mu \left[\frac{d^2v}{dx^2} + \frac{d^2v}{dy^2} + \frac{d^2v}{dz^2}\right] \mathrm{dx}\,dy\,dz \qquad \text{................... (9)}$$

- The net shear force along z−direction on all faces is

$$\mathrm{T_z} = -\mu \left[\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} + \frac{d^2w}{dz^2}\right] \mathrm{dx}\,dy\,dz \qquad \text{............... (10)}$$

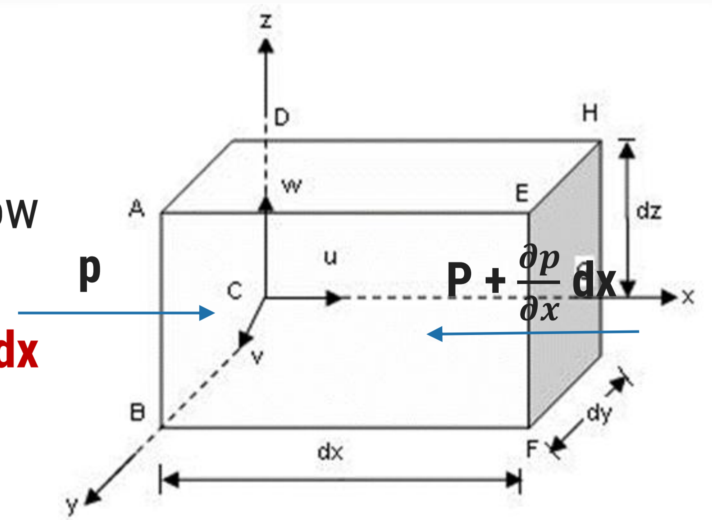
NAVIER-STOKES EQUATION OF MOTION:

- Remember equation (4)….

$$X * \rho * dx\,dy\,dz - \frac{\partial p}{\partial x} dx\,dy\,dz - T_x = \rho * dx\,dy\,dz * \frac{du}{dt}$$

- Let's put the value of T_x from equation (8) in below equation (4)

$$X * \rho * dx\,dy\,dz - \frac{\partial p}{\partial x} dx\,dy\,dz + \mu \left[\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right] dx\,dy\,dz = \rho * dx\,dy\,dz * \frac{du}{dt}$$



- Dividing on both the sides with ρ * dx dy dz

$$X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right] = \frac{du}{dt}$$

$$X - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt} - \frac{\mu}{\rho}\left[\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right]$$

$$X - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt} - \vartheta \left[\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right]$$

- Similarly in y and z direction,

$$Y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{dv}{dt} - \vartheta \left[\frac{d^2v}{dx^2} + \frac{d^2v}{dy^2} + \frac{d^2v}{dz^2}\right]$$

$$Z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{dw}{dt} - \vartheta \left[\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} + \frac{d^2w}{dz^2}\right]$$

The equations are known as Navier-Stokes equation.

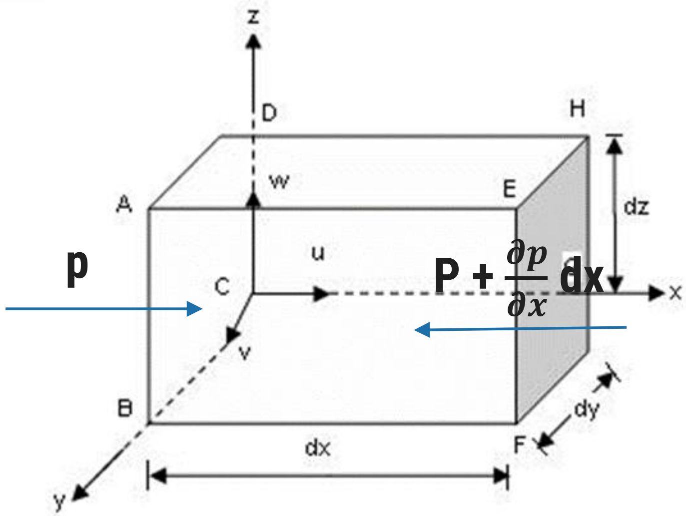
NAVIER-STOKES EQUATION OF MOTION:

- After rearranging the equation

$$\frac{du}{dt} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \vartheta\left[\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2}\right]$$

$$\frac{dv}{dt} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \vartheta\left[\frac{d^2v}{dx^2} + \frac{d^2v}{dy^2} + \frac{d^2v}{dz^2}\right]$$

$$\frac{dw}{dt} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \vartheta\left[\frac{d^2w}{dx^2} + \frac{d^2w}{dy^2} + \frac{d^2w}{dz^2}\right]$$



- In vector form

$$\frac{Dv}{Dt} = R - \frac{1}{\rho}\,\text{grad } p + \vartheta\,\nabla^2 v$$

NAVIER-STOKES EQUATION APPLICATION:

- The important applications of Navier-Stokes equations are
  - Laminar flow in circular pipe
  - Laminar flow between concentric rotating cylinders
  - Laminar flow between two fixed plates
  - Laminar flow between parallel plates having relative motion

  - The deign of aircraft and cars
  - Study of blood flow
  - The design of power station

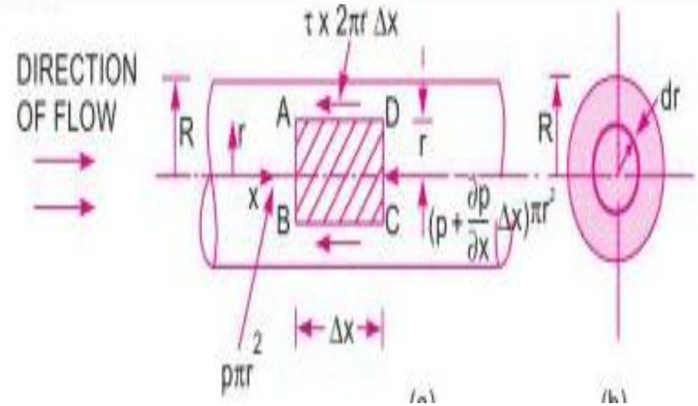
Hagen Poiseuille Formula:

- **FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE**:
- The flow through the circular pipe will be **viscous or laminar**, if the Reynolds number (**Re**) is **less than 2000**.

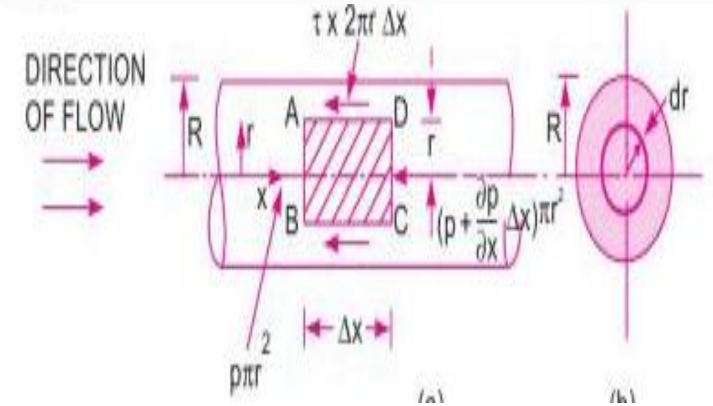
$$Re = \frac{\rho v d}{\mu}$$

- Let's consider pipe of radius R. Consider fluid element of radius r sliding in cylinder fluid element of radius r+dr.
- Consider length of fluid element is Δx.

1. p = intensity of pressure on the face AB
2. $(p+\frac{\partial p}{\partial x}\Delta x)$ = intensity of pressure on the face CD



Hagen Poiseuille Formula:



- The value of pressure force…

1. $P * \pi r^2$ on face AB
2. $(p+\frac{\partial p}{\partial x}\Delta x) * \pi r^2$ on face CD

- The value of shear force is

3. $\tau\, 2\pi r\Delta x$ on the surface of fluid element.

- Summation of all these forces must be equal to zero as no acceleration.

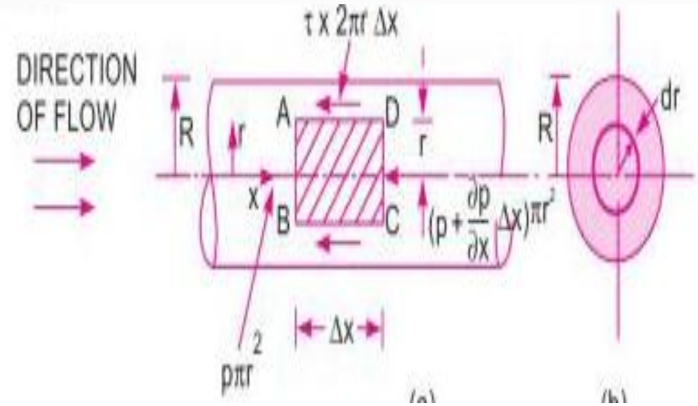
$$P * \pi r^2 - (p+\frac{\partial p}{\partial x}\Delta x) * \pi r^2 - \tau\, 2\pi r\Delta x = 0$$

$$\therefore \quad -\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau\, 2\pi r\Delta x = 0$$

$$\therefore \quad -\frac{\partial p}{\partial x} r - 2\tau = 0$$

So, $\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$ …………(1)

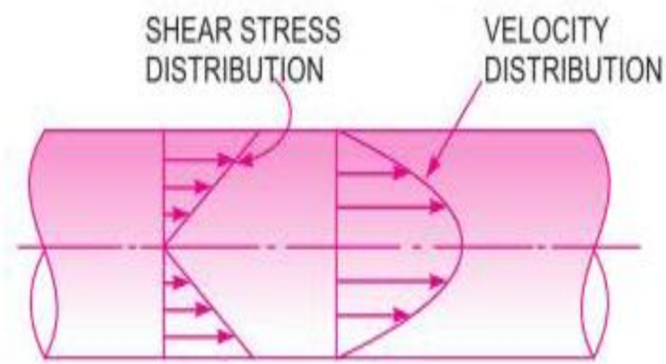
- The shear force and velocity distribution is shown in second figure.



1. **Velocity Distribution**:

- The value of shear stress…

$$\tau = \mu \frac{du}{dy}$$



y is measured from pipe wall so,

$y = R - r$

$dy = -dr$

$$\tau = -\mu \frac{du}{dr}$$

Put the above value in equation (1)

$$\therefore \qquad -\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$$\therefore \qquad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x}\, r$$

1. **Velocity Distribution**:

$$\frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x}\, r$$

Integrating above equation w.r.t. r

We get,

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\, r^2 + C$$

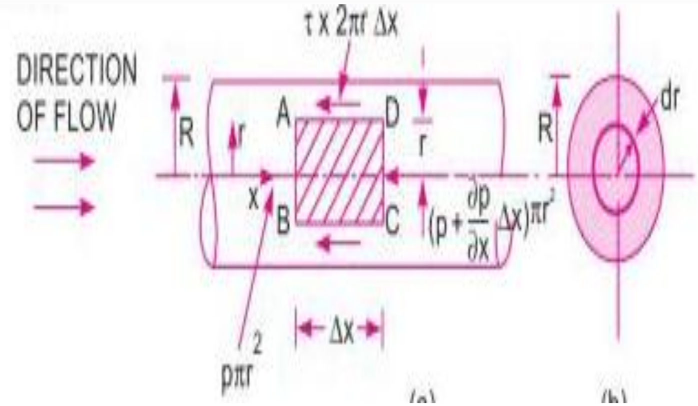
To find out the value of C let's apply boundary condition,

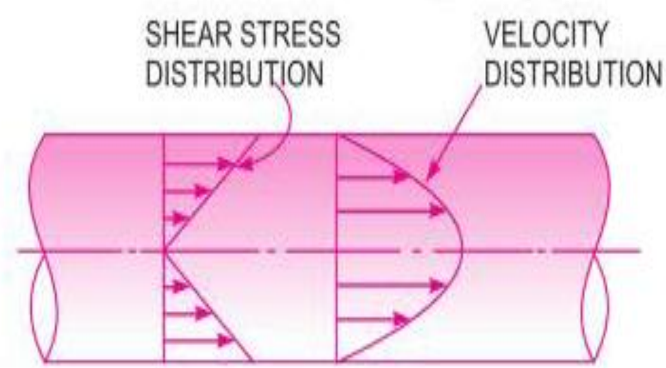
At r = R , u = 0

$$\therefore \quad 0 = \frac{1}{4\mu}\frac{\partial p}{\partial x}\, R^2 + C$$

$$\therefore \quad C = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\, R^2$$

So, $u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\, r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x}\, R^2$





Hagen Poiseuille Formula:

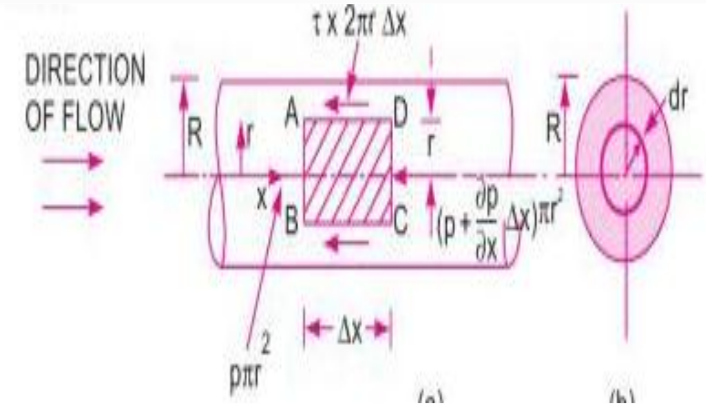
1. Velocity Distribution:

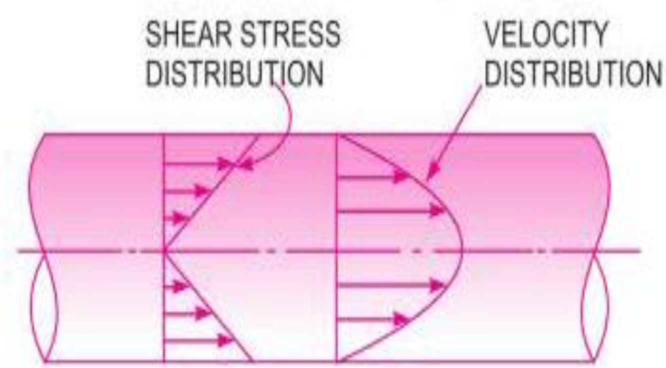
$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$$

The above equation of u is a equation of parabola.

- The velocity distribution in pipe is parabola as shown in figure.





2. Ratio of Maximum velocity to Average Velocity:

- The velocity is maximum when r=0,

so

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

Hagen Poiseuille Formula:

Average Velocity:

- The average velocity can be obtained by dividing discharge by area of pipe.
- The fluid flow through elementary ring is

dQ = velocity at radius r * area

$= u * 2\pi r\, dr$

$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] * 2\pi r\, dr$

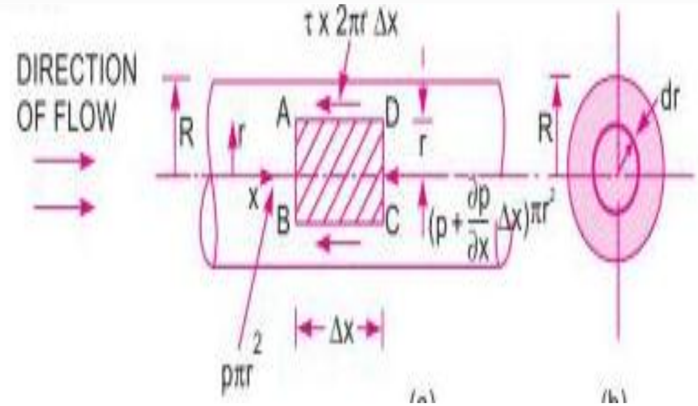
- The total discharge is….

$$Q = \int_0^R dQ$$

$$= \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] * 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x} 2\pi \int_0^R [R^2 - r^2]\, r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x} 2\pi \int_0^R [R^2 r - r^3]\, dr$$



$$Q = -\frac{1}{4\mu}\frac{\partial p}{\partial x} 2\pi \int_0^R [R^2 r - r^3]\, dr$$

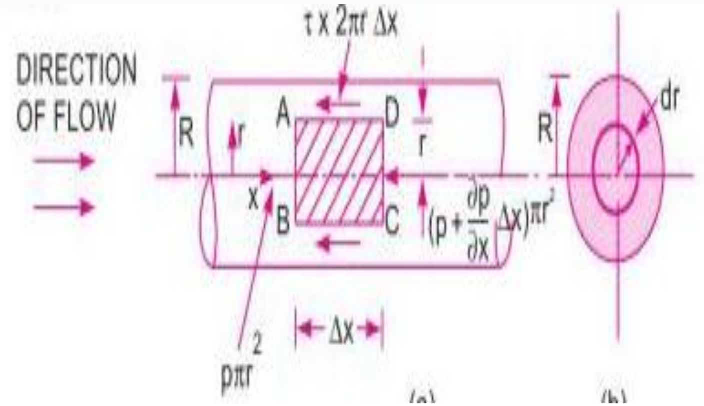
$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x} 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) 2\pi \frac{R^4}{4}$$

$$\mathbf{Q = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)\pi R^4}$$

Hagen Poiseuille Formula:

Average Velocity:



$$\bar{u} = \frac{Q}{A}$$

$$= \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^4}{\pi R^2}$$

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$$

Ratio of U_{max} to $\bar{u}$:

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2}$$

$$\frac{U_{max}}{\bar{u}} = 2$$

Hagen Poiseuille Formula:

3. Drop of pressure for a given length of pipe:

- The average velocity is…

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$$

Or

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

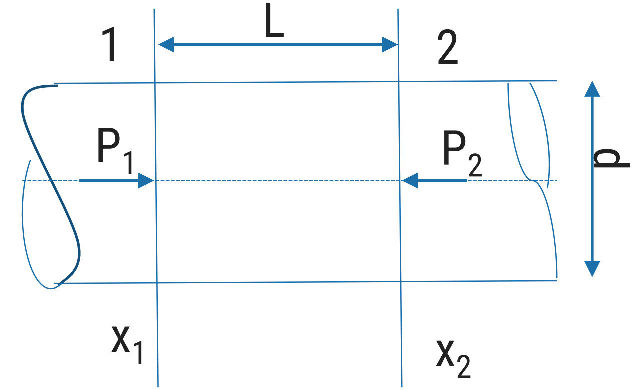
Integrating above equation w.r.t. x

$$\int_2^1 -dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\, dx$$

$$-[p_1-p_2] = \frac{8\mu\bar{u}}{R^2}[x_1-x_2]$$

$$[p_1-p_2] = \frac{8\mu\bar{u}}{R^2} L \quad \{L = x_2-x_1$$

$$[p_1-p_2] = \frac{8\mu\bar{u}L}{(D/2)2}$$



$$[p_1-p_2] = \frac{32\mu\bar{u}L}{D^2}$$

Now,

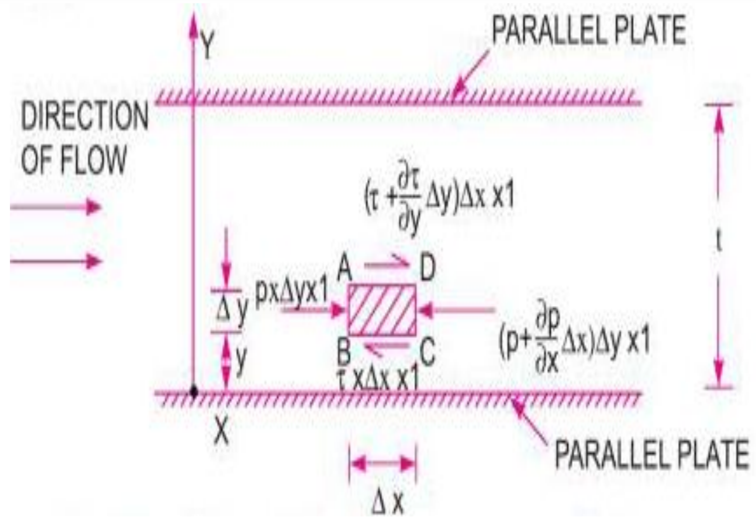
Loss of pressure head $= \frac{p_1-p_2}{\rho g}$

$$\frac{p_1-p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$$

The equation is Hagen Poiseuille Formula.

Example:

FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL PLATES:



- Consider two parallel fixed plates kept at a distance ‘t’ apart as shown in fig.
- If the width of the element in the direction perpendicular to the paper is unity then the forces acting on the fluid element are:
- The value of pressure force are

1. P * $\Delta y * 1$on face AB
2. (p+$\frac{\partial p}{\partial x}\Delta x$) * $\Delta y * 1$ on face CD

- The value of shear force are

3. $\tau \; * \Delta x * 1$ on face BC
4. (τ +$\frac{\partial \tau}{\partial y}\Delta y$) * $\Delta x * 1$ on face AD

- Summation of all these forces must be equal to zero as no acceleration.

- The value of pressure force are

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2. $(p+\frac{\partial p}{\partial x}\Delta x) * \Delta y * 1$ on face CD

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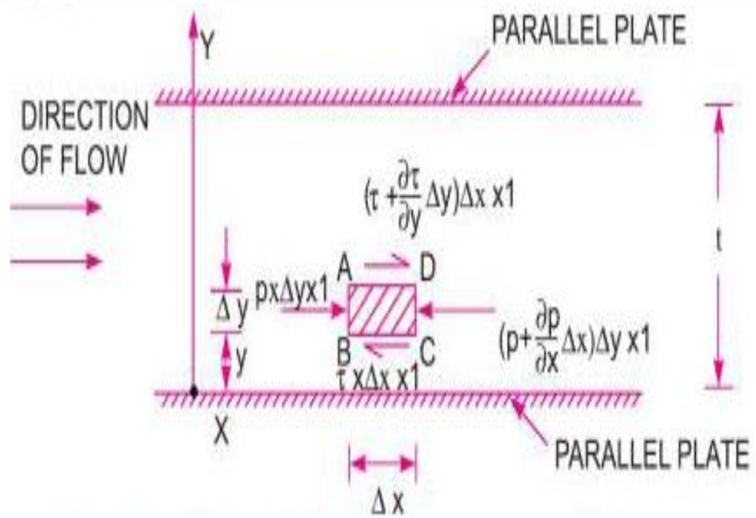


- Summation of all these forces must be equal to zero as no acceleration.

$$p\,\Delta y * 1 - (p+\frac{\partial p}{\partial x}\Delta x)\,\Delta y * 1 - \tau\,\Delta x * 1 + (\tau+\frac{\partial \tau}{\partial y}\Delta y)\,\Delta x * 1 = 0$$

$$\therefore \quad -\frac{\partial p}{\partial x}\Delta x\,\Delta y + \frac{\partial \tau}{\partial y}\Delta y\,\Delta x = 0$$

Dividing by $\Delta x\,\Delta y$

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = 0$$

$$\boldsymbol{\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}}$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$

1. Velocity Distribution:

The value of shear stress using Newton's law of viscosity is

$$\tau = \mu \frac{du}{dy}$$

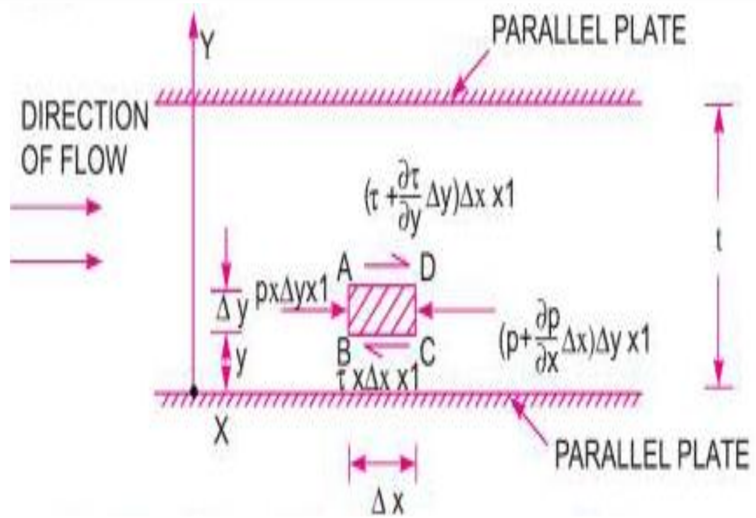
Putting the value in equation $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$

$$\frac{\partial p}{\partial x} = \frac{\partial(\mu \frac{du}{dy})}{\partial y}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating above equation w.r.t. y we get,



$$\frac{du}{dy} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + c_1$$

Integrating again

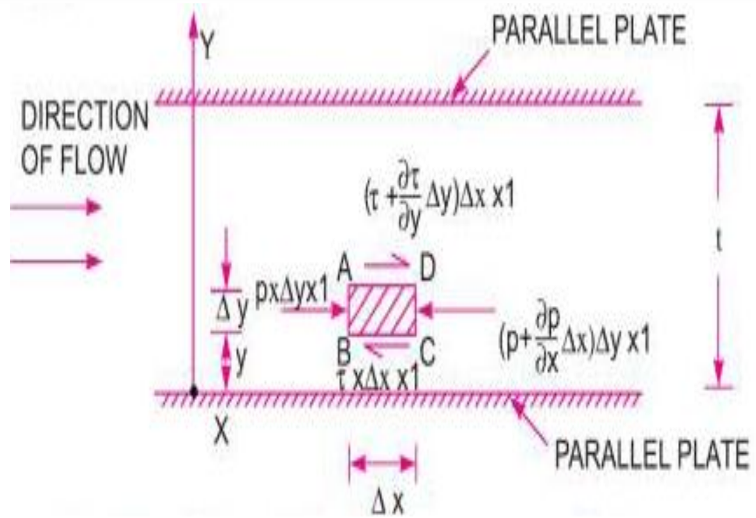
$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + c_1 y + c_2$$

c_1 and c_2 are constant of the equation.

FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL PLATES:

1. Velocity Distribution:

- $u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + c_1 y + c_2$
- c_1 and c_2 are find out by putting the boundary condition.
  - I. $y = 0$, $u = 0$
  - II. $y = t$, $u = 0$



- Applying first boundary condition

$$0 = 0 + 0 + c_2$$

$$\therefore c_2 = 0$$

- Applying second boundary condition

$$0 = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{t^2}{2} + c_1 t$$

$$c_1 = -\frac{1}{2\mu}\frac{\partial p}{\partial x} t$$

Substituting the value in equation

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + c_1 y + c_2$$

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + \left(-\frac{1}{2\mu}\frac{\partial p}{\partial x} t\right) y$$

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

1. **Velocity Distribution:**

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$

- The above value shows that velocity profile is parabola type.

2. **Ratio of Maximum velocity to Average velocity:**

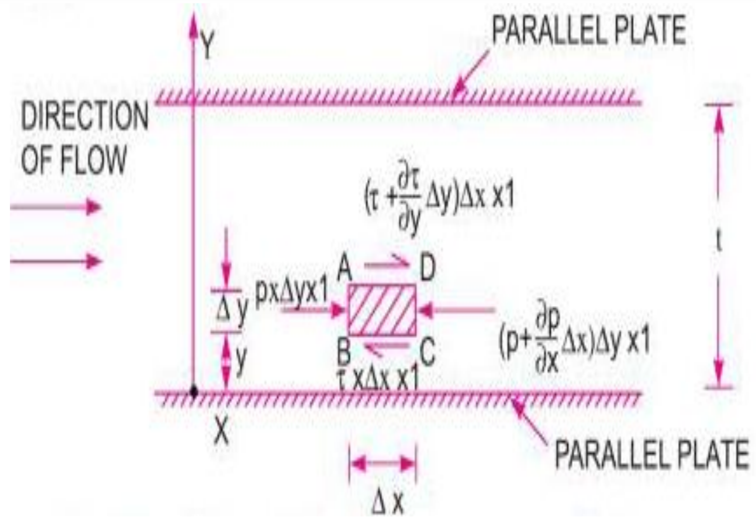
- The velocity is maximum when y = t/2, applying the value in above equation,

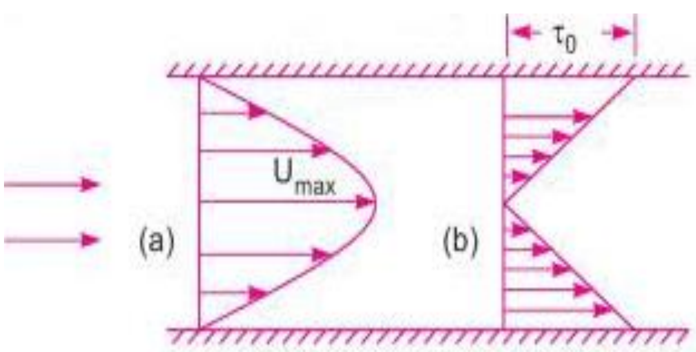
$$U_{max} = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[t\,(\tfrac{t}{2}) - (\tfrac{t}{2})^2]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}[\frac{t^2}{2} - \frac{t^2}{4}]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x} * \frac{t^2}{4}$$

$$U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2$$





2. Ratio of Maximum velocity to Average velocity:

Average Velocity:

- The average velocity can be obtained by dividing discharge by area of pipe.

dQ = velocity at distance y * area of strip

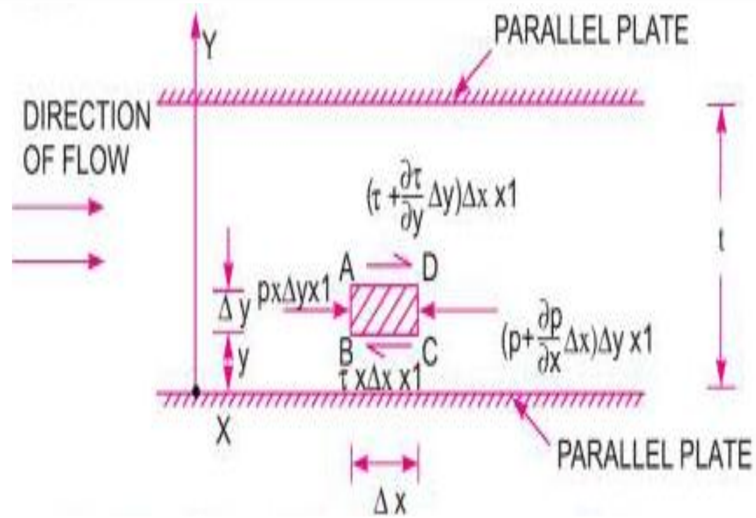
$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] * dy * 1$$

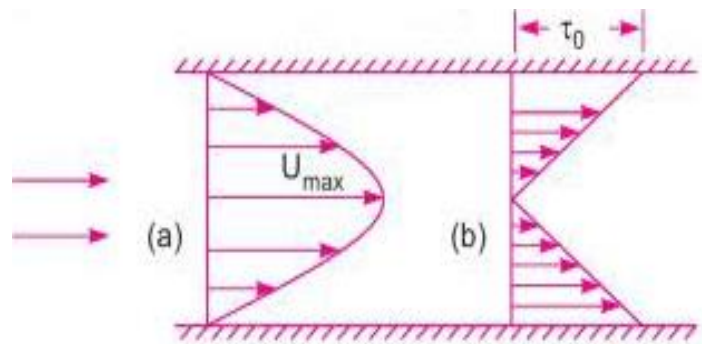
$$Q = \int_0^t dQ = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] * dy * 1$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{t^3}{2} - \frac{t^3}{3}\right]$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\frac{t^3}{6}$$

$$Q = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3$$





2. Ratio of Maximum velocity to Average velocity:

Average Velocity:

- The value of discharge,

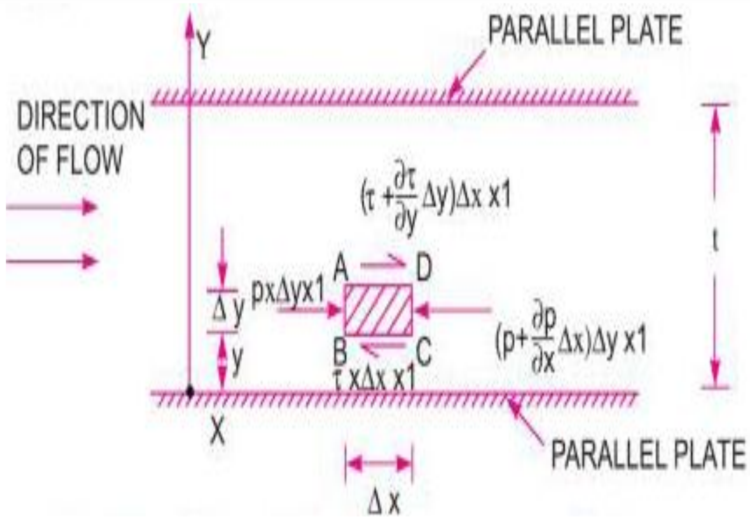
$$Q = -\frac{1}{12\mu}\frac{\partial p}{\partial x} t^3$$

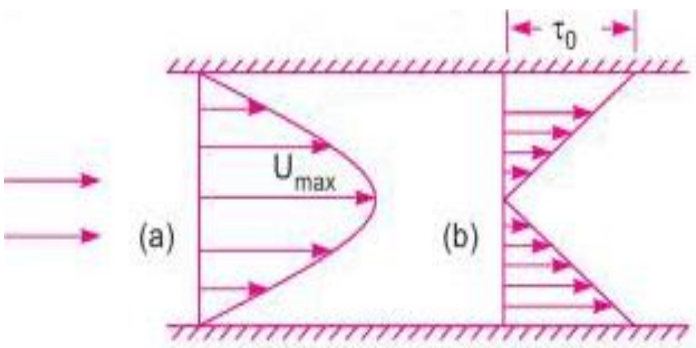
- The average velocity is given by,

$$\bar{u} = \frac{Q}{A}$$

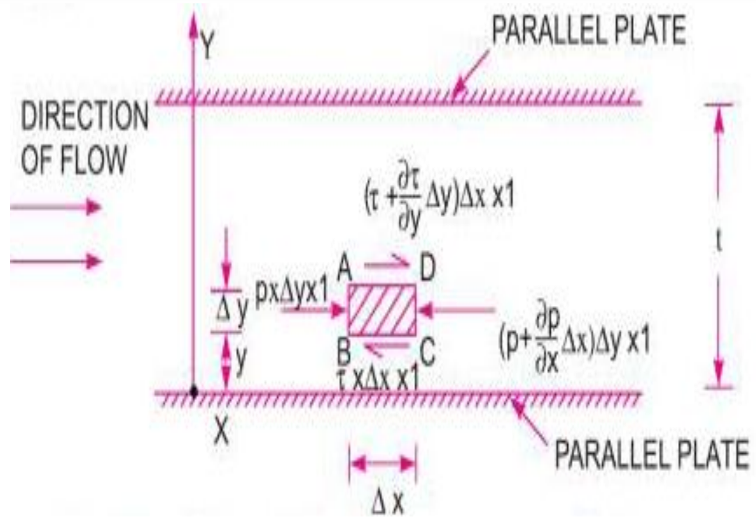
$$= \frac{-\frac{1}{12\mu}\frac{\partial p}{\partial x} t^3}{t*1}$$

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x} t^2$$





2. Ratio of Maximum velocity to Average velocity:

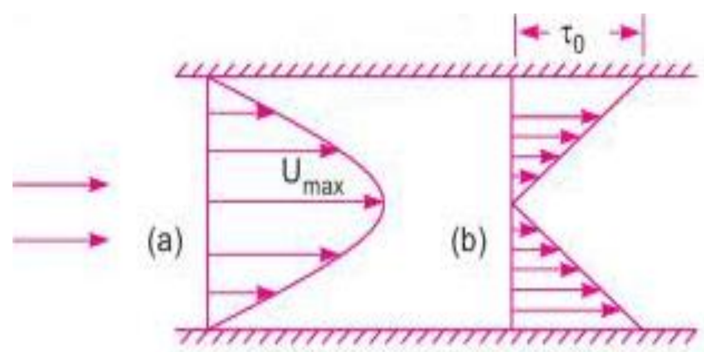


- Maximum velocity is

$$U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x} t^2$$

- Average velocity is

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x} t^2$$



- So ratio is

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x} t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x} t^2}$$

$$\frac{U_{max}}{\bar{u}} = \frac{3}{2}$$

3. Drop of pressure head for a given length:

- The value of average velocity is given by,

$$\bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2$$

Rewriting the above equation

$$\frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

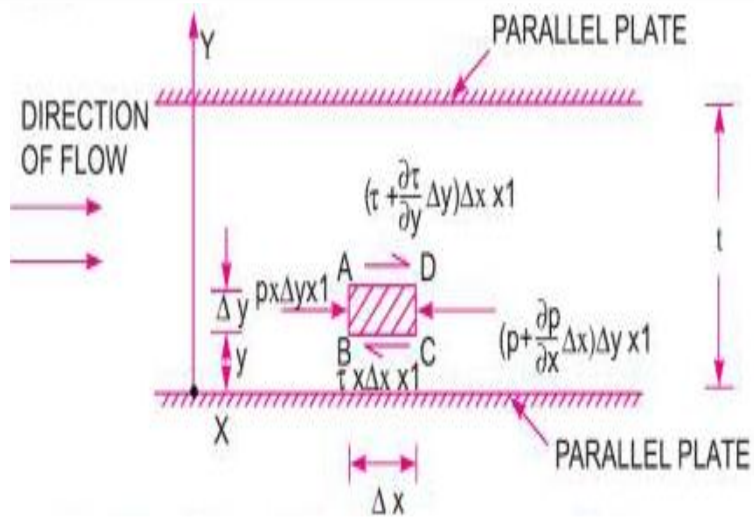
Integrating above equation w.r.t. x

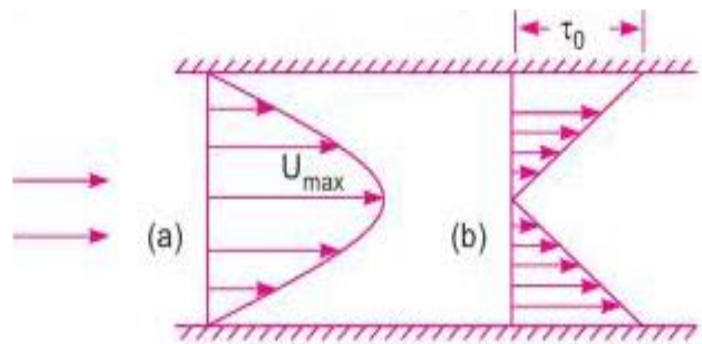
$$\int_2^1 dp = \int_2^1 -\frac{12\mu\bar{u}}{t^2}dx$$

$$[p_1-p_2] = -\frac{12\mu\bar{u}}{t^2}[x_1-x_2]$$

$$[p_1-p_2] = \frac{12\mu\bar{u}}{t^2}[x_2-x_1]$$

$$[p_1-p_2] = -\frac{12\mu\bar{u}L}{t^2}$$



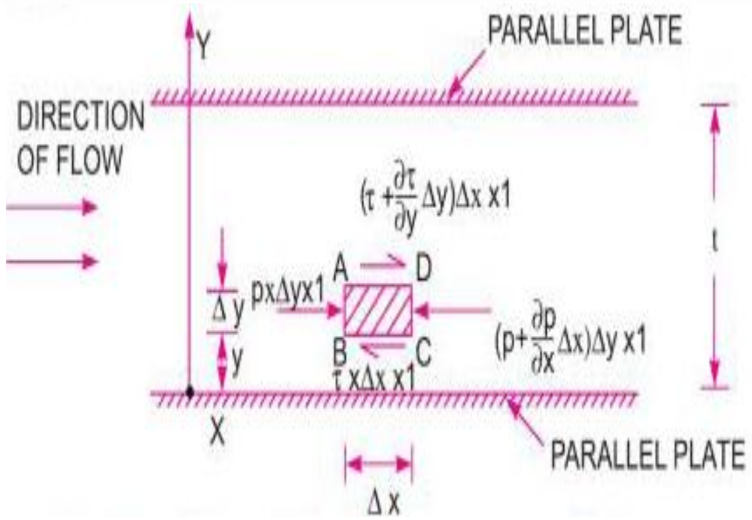


3. Drop of pressure head for a given length:

$$[p_1 - p_2] = -\frac{12\mu \bar{u} L}{t^2}$$

- If h_f is drop of pressure head then,

$$h_f = \frac{p_1 - p_2}{\rho g} = -\frac{12\mu \bar{u} L}{\rho g t^2}$$



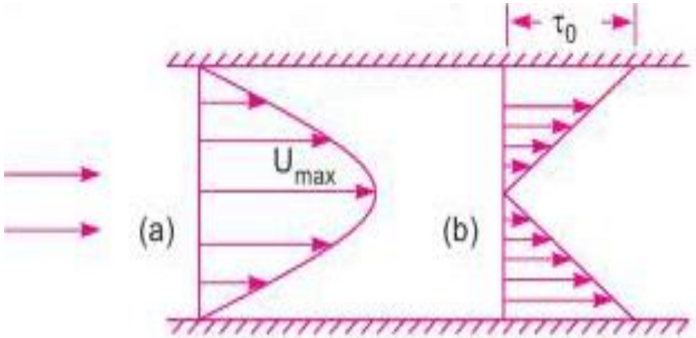
4. Shear stress Distribution:

The shear stress value is

$$\tau = \mu \frac{du}{dy}$$

And velocity is given by,

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$$



4. Shear stress Distribution:

- Putting the value of u in shear stress equation then,

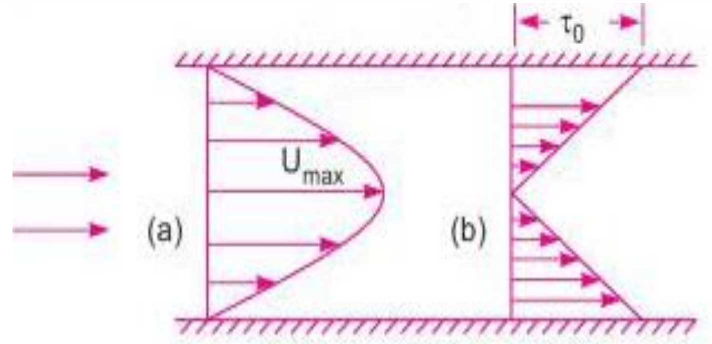
$$\tau = \mu \frac{d}{dy}\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(ty - y^2)\right]$$

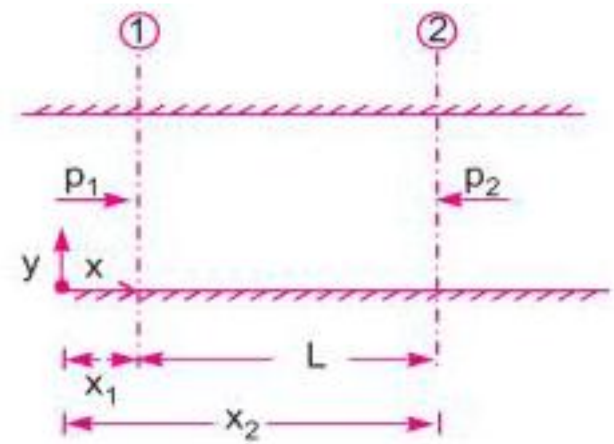
$$\tau = \mu\left[-\frac{1}{2\mu}\frac{\partial p}{\partial x}(t - 2y)\right]$$

$$\tau = -\frac{1}{2}\frac{\partial p}{\partial x}(t - 2y)$$

- The value of τ with y only, at y=0 or t shear stress is maximum,

$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x}\,t$$





Example:

Let me give you an

EXAMPLE

References:

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