UNIT-2 » SOME SPECIAL PROBABILITY DISTRIBUTIONS

❖ INTRODUCTION

- ✓ In this chapter we shall study some of the probability distribution that figure most prominently in statistical theory and application. We shall also study their parameters. We shall introduce number of discrete probability distribution that have been successfully applied in a wide variety of decision situations. The purpose of this chapter is to show the types of situations in which these distributions can be applied.
- ✓ Probability function of discrete random variable is called probability mass function (P.M.F.) and probability function of continuous random variable is called probability density function (P.D.F.).
- ✓ Some special probability distributions:
  - ➢ Binomial distribution (P.M.F.)
  - ➢ Poisson distribution (P.M.F.)
  - ➢ Normal distribution (P.D.F.)
  - ➢ Exponential distribution (P.D.F.)
  - ➢ Gamma distribution (P.D.F.)

❖ BERNOULLI TRIALS

- ✓ Suppose a random experiment has two possible outcomes, which are complementary, say success (S) and failure (F). If the probability $p(0 < p < 1)$ of getting success at each of the n trials of this experiment is constant, then the trials are called Bernoulli trials.

❖ BINOMIAL DISTRIBUTION

- ✓ A random experiment consists of n Bernoulli trials such that
  - ➢ The trials are independent.
  - ➢ Each trial results in only two possible outcomes, labeled as success and failure.
  - ➢ The probability of success in each trial remains constant.
- ✓ The random variable X that equals the number of trials that results in a success is a binomial random variable with parameters $0 < p < 1, q = 1 - p$ and $n = 1, 2, 3, \ldots$. The probability mass function of X is

$$P(X = x) = \binom{n}{x} p^x q^{n-x} \ ; x = 0, 1, 2, \dots \dots, n$$

- ✓ Examples of Binomial Distribution:
  - Number of defective bolts in a box containing n bolts.
  - Number of post-graduates in a group of n people.
  - Number of oil wells yielding natural gas in a group of n wells test drilled.
  - In the next 20 births at a hospital. Let X=the number of female births.
  - Flip a coin 10 times. Let X=number of heads obtained.
- ✓ NOTE:
  - The mean of binomial distribution is defined as $\mu = E(X) = np$.
  - The variance of the binomial distribution is defined as $V(X) = npq$ & $S.D.\ \sigma = \sqrt{npq}$.

METHOD – 1: BASIC EXAMPLES ON BINOMIAL DISTRIBUTION

C	1	12% of the tablets produced by a tablet machine are defective. What is the probability that out of a random sample of 20 tablets produced by the machine, 5 are defective? **Answer**: $\mathbf{0.0567}$	
C	2	20% Of the bulbs produced are defective. Find probability that at most 2 bulbs out of 4 bulbs are defective. **Answer**: $\mathbf{0.9728}$	
H	3	If 3 of 12 car drivers do not carry driving license, what is the probability that a traffic inspector who randomly checks 3 car drivers, will catch 1 for not carrying driving license. **Answer**: $\mathbf{\frac{27}{64}}$	

C	4	The probability that India wins a cricket test match against Australia is given to be $\frac{1}{3}$. If India and Australia play 3 tests matches, what is the probability that (a) India will lose all the three test matches? (b) India will win at least one test match? **Answer**: $\mathbf{0.2963, 0.7037}$	
H	5	What are the properties of Binomial Distribution? The average percentage of failure in a certain examination is 40. What is the probability that out of a group of 6 candidates, at least 4 passed in examination? **Answer**: $\mathbf{0.5443}$	
H	6	The probability that in a university, a student will be a post-graduate is 0.6. Determine probability that out of 8 students none, two and at least two will be post-graduate. **Answer**: $\mathbf{0.0007, 0.0413, 0.9915}$	
H	7	If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random, (a) 1, (b) 0, (c) less than 2, bolts will be defective. **Answer**: $\mathbf{0.4096, 0.4096, 0.8192}$	
H	8	Probability of man hitting a target is $\frac{1}{3}$. If he fires 6 times, what is the probability of hitting (a) at most 5 times? (b) at least 5 times? (c) exactly one? **Answer**: $\mathbf{0.9986, 0.0179, 0.2634}$	
C	9	The probability that an infection is cured by a particular antibiotic drug within 5 days is 0.75. Suppose 4 patients are treated by this antibiotic drug. What is the probability that (a) no patient, (b) exactly two patients, (c) at least two patients, are cured? **Answer**: $\mathbf{0.0039, 0.2109, 0.9492}$	

H	10	Assume that on average one telephone number out of fifteen called between 1 p.m. and 2 p.m. on weekdays is busy. What is the probability that, if 6 randomly selected telephone numbers were called, (a) not more than three, (b) at least three, of them would be busy? **Answer**: $\mathbf{0.9997, 0.0051}$	
C	11	Find the probability that in a family of 4 children there will be at least 1 boy. Assume that the probability of a male birth is 0.5. **Answer**: $\mathbf{0.9375}$	
H	12	Out of 2000 families with 4 children each, how many would you expect to have (a) at least 1 boy, (b) 2 boys, (c) 1 or 2 girls, (d) no girls? Assume equal probabilities for boys and girls. **Answer**: $\mathbf{1875, 750, 1250, 125}$	
C	13	Out of 800 families with 4 children each, how many would you expect to have (a) 2 boys and 2 girls? (b) at least 1 boy? (c) at most 2 girls? (d) no girls? Assume equal probabilities for boys and girls. **Answer**: $\mathbf{300, 750, 550, 50}$	W 2019 (7)
T	14	A multiple choice test consists of 8 questions with 3 answer to each question (of which only one is correct). A student answers each question by rolling a balanced dice & checking the first answer if he gets 1 or 2, the second answer if he gets 3 or 4 & the third answer if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answers. If there is no negative marking, what is the probability that the student secures a distinction? **Answer**: $\mathbf{P(X \geq 6) = 0.0197}$	
H	15	Ten coins are thrown simultaneously. Find the probability of getting at least seven heads. **Answer**: $\mathbf{0.1719}$	
H	16	A dice is thrown 6 times getting an odd number of success. Find probability of (a) five success, (b) at least five successes, (c) at most five successes. **Answer**: $\mathbf{\frac{3}{32}, \frac{7}{64}, \frac{63}{64}}$	

C	17	Find the probability that in five tosses of a fair die, 3 will appear (a) twice, (b) at most once, (c) at least two times. **Answer**: $\frac{625}{3888}, \frac{3125}{3888}, \frac{763}{3888}$	
C	18	Find probability of getting a sum of 7 at least once in 3 tosses of a pair of dice. **Answer**: $\frac{91}{216}$	
H	19	Find the binomial distribution for $n = 4$ and $p = 0.3$. **Answer**: $P(X = x) = \binom{4}{x}(0.3)^x(0.7)^{4-x}\,;\, x = 0, 1, 2, 3, 4.$	
C	20	Obtain the binomial distribution for which mean is 10 and variance is 5. **Answer**: $P(X = x) = \binom{20}{x}(0.5)^x(0.5)^{20-x}\,;\, x = 0, 1, 2, \dots, 20.$	
C	21	For the binomial distribution with $n = 20, p = 0.35$. Find mean, variance and standard deviation. **Answer**: $7, 4.55, 2.1331$	
H	22	If the probability of a defective bolt is 0.1, find mean and standard deviation of the distribution of defective bolts in a total of 400. **Answer**: $40, 6$	
H	23	A university warehouse has received a shipment of 25 printers, of which 10 are laser printers and 15 are inkjet models. If 6 of these 25 are selected at random to be checked by a particular technician, what is the probability that exactly 3 of those selected are laser printers (so that the other 3 are inkjets)? **Answer**: 0.2765	W 2019 (4)
H	24	Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, at least 4 samples contain the pollutant. **Answer**: 0.0982	W 2019 (3)

❖ POISSON DISTRIBUTION

- ✓ A discrete random variable X is said to follow Poisson distribution if it assume only non-negative values. Its probability mass function is given by

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!};\ x = 0,1,2,3, \dots.\ \ \&\ \lambda\ = \text{mean of the Poisson distribution}$$

- ✓ Examples of Poisson Distribution:
  - ➢ Number of telephone calls per minute at a switchboard.
  - ➢ Number of cars passing a certain point in one minute.
  - ➢ Number of printing mistakes per page in a large text.
  - ➢ Number of persons born blind per year in a large city.
- ✓ Properties of Poisson Distribution:
  - ➢ The Poisson distribution holds under the following conditions.
  - ➢ The random variable X should be discrete.
  - ➢ The number of trials n is very large.
  - ➢ The probability of success p is very small (very close to zero).
  - ➢ The occurrences are rare.
  - ➢ $\lambda = np$ is finite.
  - ➢ The mean and variance of the Poisson distribution with parameter λ are defined as follows.

$$\text{mean } \mu = E(X) = \lambda = np\ \ \&\ \ \text{variance } V(X) = \sigma^2 = \lambda$$

METHOD – 2: EXAMPLES ON POISSON DISTRIBUTION

C	1	In a company, there are 250 workers. The probability of a worker remain absent on any one day is 0.02. Find the probability that on a day seven workers are absent. **Answer**: $\mathbf{P(X = 7) = 0.1044}$	

C	2	A book contains 100 misprints distributed randomly throughout its 100 pages. What is the probability that a page observed at random contains at least two misprints? **Answer**: $\mathbf{0.2642}$	
H	3	Suppose a book of 585 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free from error? **Answer**: $\mathbf{0.4795}$	
H	4	100 Electric bulbs are found to be defective in a lot of 5000 bulbs. Find the probability that at the most 3 bulbs are defective in a box of 100 bulbs. **Answer**: $\mathbf{P(X \leq 3) = 0.8571}$	
C	5	Average number of accidents on any day on a national highway is 1.8. Determine the probability that the number of accidents are (a) at least 1, (b) at most 1. **Answer**: $\mathbf{0.8347, 0.4628}$	
C	6	If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001. Determine the probability that out of 2000 individuals, (a) exactly 3, (b) more than 2, individuals will suffer a bad reaction. **Answer**: $\mathbf{0.1804, 0.3233}$	
H	7	The probability that a person catch corona virus is 0.001. Find the probability that out of 3000 persons (a) exactly 3, (b) more than 2 persons will catch the virus. **Answer**: $\mathbf{0.2240, 0.5768}$	
H	8	Suppose 1% of the items made by machine are defective. In a sample of 100 items find the probability that the sample contains all good, 1 defective and at least 3 defectives. **Answer**: $\mathbf{P(X = 0) = 0.3679, P(X = 1) = 0.3679, P(X \geq 3) = 0.0803}$	

C	9	Potholes on a highway can be serious problems. The past experience suggests that there are, on an average, 2 potholes per mile after a certain amount of usage. It is assumed that Poisson process applies to random variable "no. of potholes". What is the probability that no more than four potholes will occur in a given section of 5 miles? **Answer**: $\mathbf{P(X \le 4) = 0.0293}$	
H	10	A car hire firm has two cars, which are hires out day by day. The number of demands for a car on each day is distributed on a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and proportion of days on which some demand is refused. ($e^{-1.5} = 0.2231$). **Answer**: $\mathbf{P(X = 0) = 0.2231, 1 - P(X \le 2) = 0.1912}$	
C	11	In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain exactly two defective parts? **Answer**: **271**	
C	12	In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packet of 10. Calculate the approximate number of packets containing no defective, one defective, two defective blades in a consignment of 10000 packets. **Answer**: **9802, 196, 2**	
H	13	In a bolt manufacturing company, it is found that there is a small chance of $\frac{1}{500}$ for any bolt to be defective. The bolts are supplied in a packed of 20 bolts. Use Poisson distribution to find approximate number of packets containing (a) no defective bolt, (b) containing two defective bolt, in the consignment of 10000 packets. **Answer**: **9608, 8**	
C	14	For Poisson variant X, if $P(X = 3) = P(X = 4)$, then find $P(X = 0)$. **Answer**: $\mathbf{P(X = 0) = e^{-4}}$	

H	15	For Poisson variant X, if $P(X = 1) = P(X = 2)$. Find mean and standard deviation of this distribution. Also, find $P(X = 3)$. **Answer**: $\mathbf{2, \sqrt{2}, 0.1804}$	
H	16	Assume that the probability that a wafer contains a large particle of contamination is 0.01 and that the wafers are independent; that is, the probability that a wafer contains a large particle is not dependent on the characteristics of any of the other wafers. If 15 wafers are analyzed, what is the probability that no large particles are found? **Answer**: $\mathbf{0.8607}$	W 2019 (3)
H	17	If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is .005 and errors are independent from page to page. What is the probability that one of its 400-page novels will contain (a) exactly one page with errors? (b) at most three pages with errors? **Answer**: **(a)** $\mathbf{0.2707}$, **(b)** $\mathbf{0.8571}$	W 2019 (7)

❖ EXPONENTIAL DISTRIBUTION

- ✓ A random variable X is said to have an Exponential distribution with parameter $\theta > 0$, if its probability density function is given by

$$f(X = x) = \begin{cases} \theta e^{-\theta x}; & x \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

- ✓ Here, $\theta = \frac{1}{\text{mean}}$ or $\text{mean} = \frac{1}{\theta}$ and variance $= \frac{1}{\theta^2}$.
- ✓ Exponential distribution is a special case of Gamma distribution.
- ✓ Exponential distribution is used to describe lifespan and waiting times.
- ✓ Exponential distribution can be used to describe (waiting) times between Poisson events.
- ✓ In exponential distribution we can find the probability as given below.
  - ➢ $P(X \leq x) = 1 - e^{-\theta x}$ & $P(X \geq x) = e^{-\theta x}$
  - ➢ $P(a \leq X \leq b) = e^{-a\theta} - e^{-b\theta}$

METHOD – 3: EXAMPLES ON EXPONENTIAL DISTRIBUTION

T	1	Define exponential distribution. Obtain its mean and variance.	
C	2	The lifetime T of an alkaline battery is exponentially distributed with $\theta =$ 0.05 per hour. What are mean and standard deviation of batteries lifetime? **Answer**: $\mathbf{20, 20}$	
C	3	The lifetime T of an alkaline battery is exponentially distributed with $\theta =$ 0.05 per hour. (a) What are the probabilities for battery to last between 10 and 15 hours? (b) What are the probabilities for the battery to last more than 20 hr? **Answer**: $\mathbf{0.1342, 0.3679}$	
C	4	The time between breakdowns of a particular machine follows an exponential distribution with a mean of 17 days. Calculate the probability that a machine breakdown in 15-day period. **Answer**: $\mathbf{0.5862}$	
C	5	The arrival rate of cars at a gas station is 40 customers per hour. (a) What is the probability of having no arrivals in 5 min. interval? (b) What is the probability of having 3 arrivals in 5 min.? **Answer**: $\mathbf{0.0356, 0.2202}$	
T	6	In a large corporate computer network, user log-on to the system can be modeled as a Poisson process with a mean of 25 log-on per hours. (a) what is the probability that there are no log-on in an interval of six min.? (b) what is the probability that time until next log-on is between 2 & 3 min.? **Answer**: $\mathbf{0.0821, 0.1481}$	
H	7	The time intervals between successive barges passing a certain point on a busy waterway have an exponential distribution with mean 8 minutes. Find the probability that the time interval between two successive barges is less than 5 minutes. **Answer**: $\mathbf{0.4647}$	

H	8	Accidents occur with Poisson distribution at an average of 4 per week. (a) Calculate the probability of more than 5 accidents in any one week. (b) What is probability that at least two weeks will elapse between accidents? **Answer**: $\mathbf{0.3895, 0.0003}$	
C	9	A random variable has an exponential distribution with probability density function given by $f(x) = 3e^{-3x}\,; x > 0\ \&\ f(x) = 0\,; x \le 0$. What is the probability that X is not less than 4? **Answer**: $\mathbf{e^{-12}}$	
T	10	The income tax of a man is exponentially distributed with $f(x) = \frac{1}{3}e^{-\left(\frac{x}{3}\right)}\,; x > 0$. What is the probability that his income will exceed Rs. 17000? Assume that the income tax is levied at the rate of 15% on the income above Rs. 15000. **Answer**: $\mathbf{e^{-100}}$	

❖ GAMMA DISTRIBUTION

✓ A random variable X is said to have a Gamma distribution with parameter $r, \theta > 0$, if its probability density function is given by

$$f(x) = \begin{cases} \dfrac{\theta^r\, x^{r-1}\, e^{-\theta x}}{\Gamma(r)} & ;\ x \ge 0 \\ 0 & ;\ \text{otherwise} \end{cases}$$

➢ Here, mean $= \frac{r}{\theta}$ and variance $= \frac{r}{\theta^2}$.

METHOD – 4: EXAMPLES ON GAMMA DISTRIBUTION

H	1	Define Gamma distribution. Obtain its mean and variance.	
C	2	Given a gamma random variable X with $r = 3$ and $\theta = 2$. Find E(X), V(X) and $P(X \le 1.5)$. **Answer**: $\mathbf{1.5, 0.75, 0.5768}$	

H	3	The time spent on a computer is a gamma distribution with mean 20 and variance 80. (a) What are the value of r & θ? (b) What is $P(X < 24)$? (c) What is $P(20 < X < 40)$? **Answer**: $\mathbf{r = 5, \theta = 4, P(X < 24) = 0.715, P(20 < X < 40) = 0.411}$	
C	4	The daily consumption of milk in a city, in excess of 20000 liters, is approximately distributed as a gamma variate with $r = 2$ and $\theta = \frac{1}{10000}$. The city has daily stock of 30000 liters. What is the probability that the stock is insufficient on a particular day? **Answer**: $\mathbf{0.736}$	
T	5	Suppose that the time it takes to get service in a restaurant follows a gamma distribution with mean 8 min and standard deviation 4 minutes. (a) Find the parameters r and θ of the gamma distribution. (b) You went to this restaurant at 6:30. What is the probability that you will receive service before 6:36? **Answer**: $\mathbf{r = 4, \theta = \frac{1}{2}, 0.3528}$	
H	6	Suppose you are fishing and you expect to get a fish once every $\frac{1}{2}$ hour. Compute the probability that you will have to wait between 2 to 4 hours before you catch 4 fish. **Answer**: $\mathbf{0.1239}$	
C	7	The daily consumption of electric power in a certain city is a random variable X having probability density function $f(x) = \begin{cases} \frac{1}{9} x e^{-\frac{x}{3}}; & x > 0 \\ 0 \quad ; & x \le 0 \end{cases}$. Find the probability that the power supply is inadequate on any given day if the daily capacity of the power plant is 12 million KW per hour. **Answer**: $\mathbf{0.0916}$	

❖ NORMAL DISTRIBUTION

✓ A continuous random variable X is said to follow a normal distribution if its probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]; -\infty < x < \infty \ \& \ \sigma > 0$$

- ✓ Where, μ = mean of the distribution and σ = standard deviation of the distribution
- ✓ μ (mean) & σ^2 (variance) are called parameters of the distribution.
- ✓ If X is a normal random variable with mean μ and standard deviation σ, and if we find the random variable $Z = \frac{X-\mu}{\sigma}$ with mean 0 and standard deviation 1, then Z in called the standard (standardized) normal variable.
- ✓ The probability destiny function for the normal distribution in standard form is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \ ; \ -\infty < z < \infty$$

- ✓ The distribution of any normal variate X can always be transformed into the distribution of the standard normal variate Z.

$$P(x_1 \le X \le x_2) = P\left(\frac{x_1-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{x_2-\mu}{\sigma}\right) = P(z_1 \le Z \le z_2)$$

- ✓ For normal distribution,
  - ➢ $P(-\infty \le z \le \infty) = 1$ (Total area)
  - ➢ $P(-\infty \le z \le 0) = P\ (0 \le z \le \infty) = 0.5$
  - ➢ $P(-z_1 \le z \le 0) = P(0 \le z \le z_1) \ ; \ z_1 > 0$

METHOD – 5: EXAMPLES OF NORMAL DISTRIBUTION

C	1	For a random variable having the normal distribution with $\mu = 18.2$ and $\sigma = 1.25$, find the probabilities that it will take on a value (a) less than 16.5, (b) between 16.5 and 18.8.[$P(z = 1.36) = 0.4131, P(z = 0.48) = 0.1843$] **Answer**: **0.0869, 0.5974**	

H	2	The compressive strength of the sample of cement can be modelled by normal distribution with mean $6000\ kg/cm^2$ and standard deviation $100\ kg/cm^2$. (a) What is the probability that a sample strength is less than $6250\ kg/cm^2$? (b) What is probability if sample strength is between 5800 and $5900\ kg/cm^2$? (c) What strength is exceeded by 95% of the samples? $[P(z = 2.5) = 0.4798, P(z = 1) = 0.3413]$ $[P(z = 2) = 0.4773, P(z = 1.65) = 0.45]$ **Answer**: $\mathbf{0.9798, 0.136, 6165}$	
C	3	In a photographic process, the developing time of prints may be looked upon as a random variable having normal distribution with mean of 16.28 seconds and standard deviation of 0.12 seconds. Find the probability that it will take (a) anywhere from 16.00 to 16.50 sec to develop one of the prints, (b) at least 16.20 sec to develop one of the prints, (c) at most 16.35 sec to develop one of the prints. $[P(z = 1.83) = 0.4664, P(z = 2.33) = 0.4901]$ $[P(z = 0.67) = 0.2486, P(z = 0.58) = 0.2190]$ **Answer**: $\mathbf{0.9565, 0.7486, 0.7190}$	
H	4	A sample of 100 dry battery cell tested & found that average life is 12 hours & standard deviation 3 hours. Assuming data to be normally distributed what % of battery cells are expected to have life (a) more than 15 hrs.? (b) less than 6 hrs.? (c) between 10 & 14 hrs.? $[P(z = 1) = 0.3413, P(z = 2) = 0.4773, P(z = 0.67) = 0.2486]$ **Answer**: $\mathbf{15.87\%, 2.27\%, 49.72\%}$	
H	5	The breaking strength of cotton fabric is normally distributed with $E(x) = 16$ and $\sigma(x) = 1$. The fabric is said to be good if $x \geq 14$. What is the probability that a fabric chosen at random is good? $[P(z = 2) = 0.4773]$ **Answer**: $\mathbf{0.9773}$	

H	6	The customer accounts of certain department store have an average balance of 120 Rs. & standard deviation of 40 Rs. Assume that account balances are normally distributed. (a) What proportion of the account is over 150 Rs.? (b) What proportion of account is between 100 & 150 Rs.? (c) What proportion of account is between 60 & 90 Rs.? $[P(z = 0.75) = 0.2734, P(z = 0.5) = 0.1915, P(z = 1.5) = 0.4332]$ **Answer**: $\mathbf{0.2266, 0.4649, 0.1598}$	
C	7	Weights of 500 students of college is normally distributed with μ=95 lbs. & σ =7.5 lbs. Find how many students will have the weight between 100 and 110 lbs.$[P(z = 2) = 0.4773, P(z = 0.67) = 0.2486]$ **Answer**: $\mathbf{114}$	
H	8	Distribution of height of 1000 soldiers is normal with mean 165 cm & standard deviation 15 cm. How many soldiers are of height (a) less than 138 cm? (b) more than 198 cm? (c) between 138 & 198 cm? $[P(z = 1.8) = 0.4641, P(z = 2.2) = 0.4861]$ **Answer**: $\mathbf{36, 14, 950}$	W 2019 (7)
T	9	Assuming that the diameters of 1000 brass plugs taken consecutively from a machine form a normal distribution with mean 0.7515 cm and standard deviation 0.002 cm. Find the number of plugs likely to be rejected if the approved diameter is 0.752 ± 0.004 cm. $[P(z = 1.75) = 0.4599, P(z = 2.25) = 0.4878]$ **Answer**: $\mathbf{52}$	
T	10	In a company, amount of light bills follows normal distribution with $\sigma =$ 60. 11.31% of customers pay bill less than 260. Find average amount of light bill.$[P(z = 1.21) = 0.3869]$ **Answer**: $\mathbf{332.60}$	
C	11	In a normal distribution, 31% of items are below 45 & 8% are above 64. Determine the mean and standard deviation of this distribution. $[P(z = 0.22) = 0.19, P(z = 1.41) = 0.42]$ **Answer**: $\boldsymbol{\mu = 47.6634, \sigma = 11.58}$	

H	12	In an examination, minimum 40 marks for passing and 75 marks for distinction are required. In this examination 45% students passed and 9% obtained distinction. Find average marks and standard deviation of this distribution of marks.[$P(z = 0.125) = 0.05$ and $P(z = 1.34) = 0.41$] **Answer**: $\mathbf{36.40, 28.81}$	W 2019 (7)

❖ BOUNDS ON PROBABILITIES

- ✓ If the probability distribution of a random variable is known, $E(X)$ & $V(x)$ can be computed. Conversely, if $E(X)$ & $V(X)$ are known, probability distribution of X cannot be constructed and quantities such as $P\{|X - E(X)| \leq K\}$ cannot be evaluate.
- ✓ Several approximation techniques have been developed to yield upper and/or lower bounds to such probabilities. The most important of such technique is Chebyshev's inequality.

❖ CHEBYSHEV'S INEQUALITY

- ✓ If X is a random variable with mean μ and variance σ^2, then for any positive number k,

$$P\{|X - \mu| \geq k\sigma\} \leq \frac{1}{k^2} \quad \text{or} \quad P\{|X - \mu| < k\sigma\} \geq 1 - \frac{1}{k^2}$$

METHOD – 6: EXAMPLES ON CHEBYSHEV'S INEQUALITY

C	1	A random variable X has a mean 12, variance 9 and unknown probability distribution. Find $P(6 < X < 18)$. **Answer**: $\mathbf{P(6 < X < 18) \geq \frac{3}{4}}$	
H	2	If X is a variate such that $E(X) = 3$, $E(X^2) = 13$, show that $P(-2 < X < 8) \geq \frac{21}{25}$.	

H	3	The number of customers who visit a car dealer's showroom on Sunday morning is a random variable with mean 18 and standard deviation 2.5. What is the bound of probability that on Sunday morning the customers will be 8 to 28? **Answer**: $\mathbf{P(8 < X < 28) \geq \frac{15}{16}}$	
C	4	Variate X takes values $-1, 1, 3, 5$ with associate probability $\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{2}$. Compute $p = P\{\lvert x - 3\rvert \geq 1\}$ directly and find an upper bound to 'p' by Chebyshev's inequality. **Answer**: $\mathbf{0.83, 5.33}$	
T	5	Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that $P\{\lvert X - 7\rvert \geq 3\} < \frac{35}{54}$. Compare this with actual probability. **Answer**: $\mathbf{\frac{1}{3}}$	
H	6	A random variable X has mean 10, variance 4 and unknown probability distribution. Find 'c' such that $P\{\lvert X - 10\rvert \geq c\} < 0.04$. **Answer**: $\mathbf{10}$	

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