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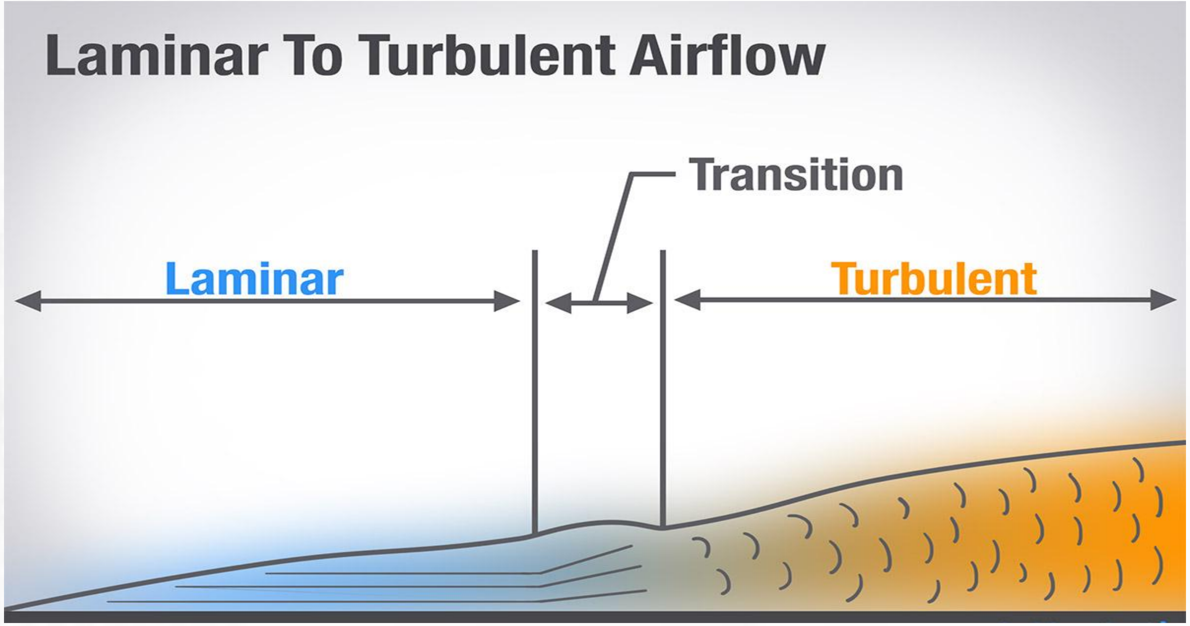

Applied Fluid Mechanics
(2160602)

Module-2

Boundary Layer

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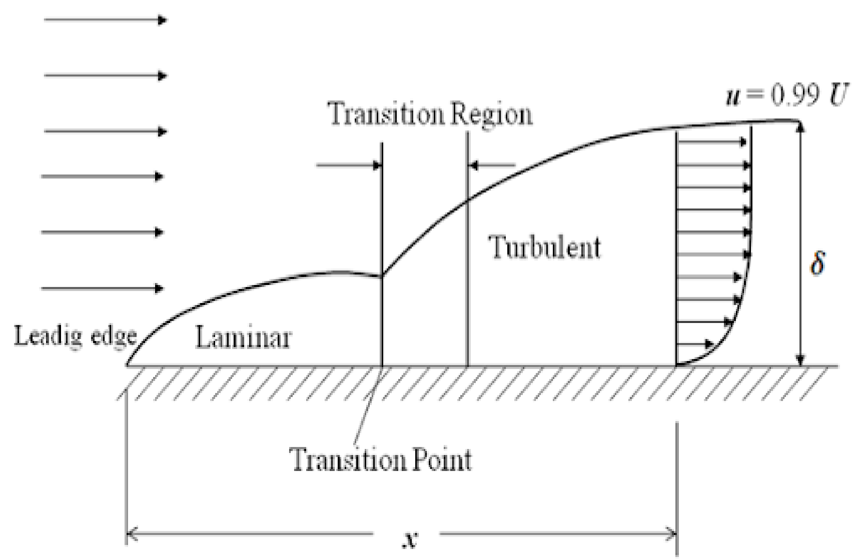
INTRODUCTION:

- According to boundary layer theory, the flow of fluid in the neighborhood of the solid boundary may be divided into two regions.

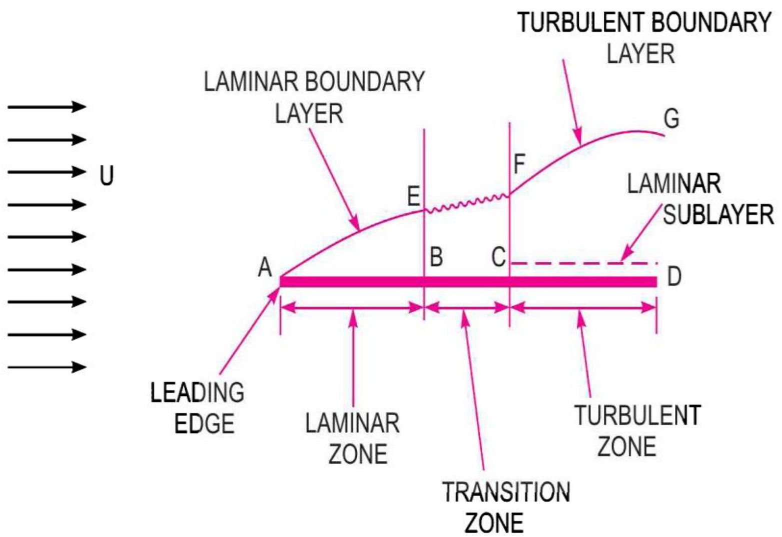
1. A very thin layer of the fluid, called the boundary layer, in the immediate neighborhood of the solid boundary, where the variation of velocity from zero at the solid boundary to free-stream velocity in the direction normal to the boundary takes place.
2. The remaining fluid, which is outside the boundary layer. The velocity outside the boundary layer is constant and equal to free-stream velocity.



DEFINITIONS:

1. Laminar Boundary Layer:

- Consider the flow of a fluid, having free-stream velocity (U), over a smooth thin plate which is flat and placed parallel to the direction for free stream of fluid as shown in Fig.
- Plate is stationary and hence velocity of fluid on the surface of the plate is zero.
- A velocity gradient is set up in the fluid near the surface of the plate. This velocity gradient develops shear resistance, which retards the fluid.
- The boundary layer region begins at the sharp leading edge.



- At subsequent points downstream the leading edge, the boundary layer region increases because the retarded fluid is further retarded. This is also referred as the **growth of boundary layer**.

DEFINITIONS:

1. Laminar Boundary Layer:

- Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar though the main flow is turbulent. This layer of the fluid is said to be **laminar boundary layer**. This is shown by AE in Fig.
- The Reynold number is given by

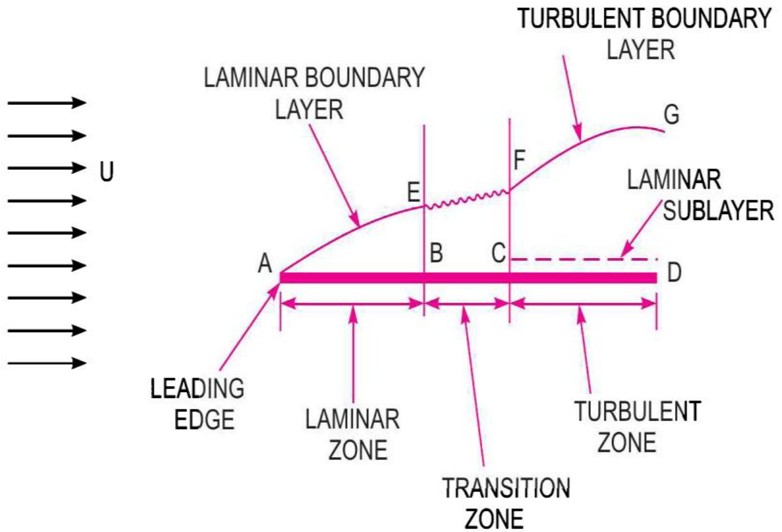
$$(R_e)_x = \frac{U \times x}{\nu}$$

x = Distance from leading edge,

U = Free-stream velocity of fluid,

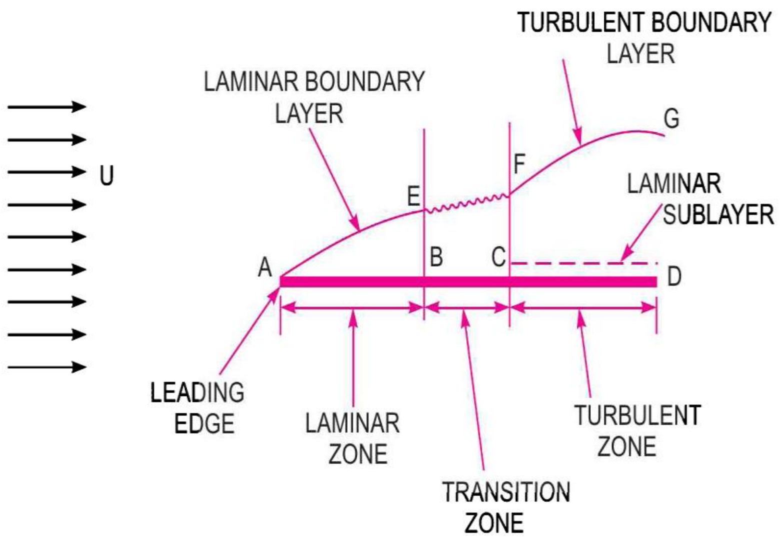
v = Kinematic viscosity of fluid,

- Reynold number equal to 5×10^5 for a plate.



2. Turbulent Boundary Layer:

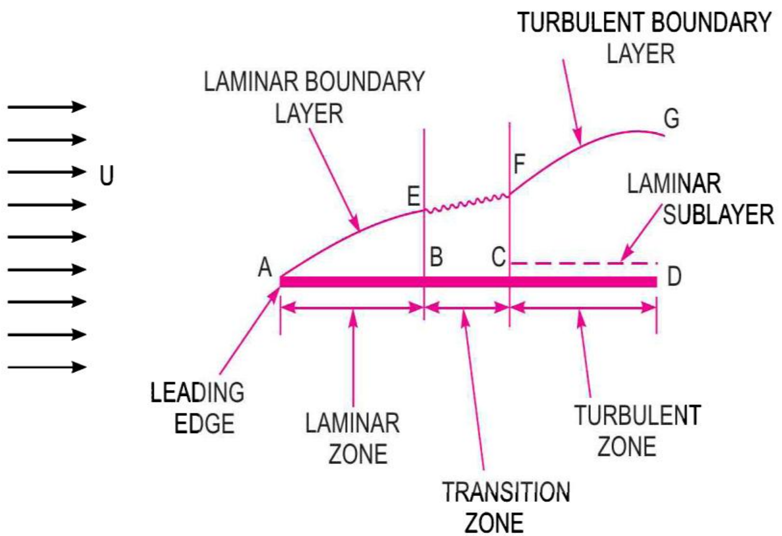
- The thickness of boundary layer will go on increasing in the downstream direction.
- Then the laminar boundary layer becomes unstable and motion of fluid within it, is disturbed and irregular which leads to a transition from laminar to **turbulent boundary layer**.
- This short length over which the boundary layer flow changes from laminar to turbulent is called **transition zone**.



DEFINITIONS:

3. Laminar Sub-layer:

- This is the region in the turbulent boundary layer zone, adjacent to the solid surface of the plate as shown in Fig.
- In this zone, the velocity variation is influenced only by viscous effects.
- The velocity gradient can be considered constant. Therefore, the shear stress in the laminar sub-layer would be constant and equal to the boundary shear stress τ_0.



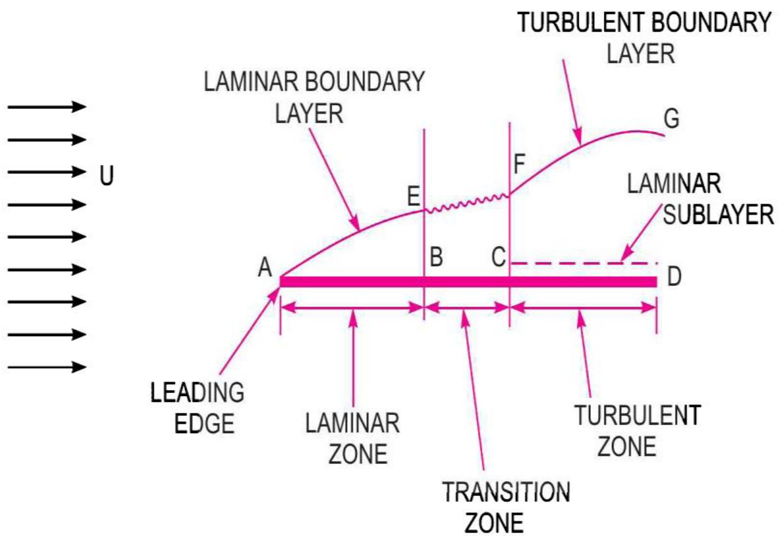
$$\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu \frac{u}{y}$$

DEFINITIONS:

4. Boundary Layer Thickness (δ):

- It is defined as the distance from the boundary of the solid body measured in the y-direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity (U) of the fluid.
- For laminar and turbulent zone it is denoted as :

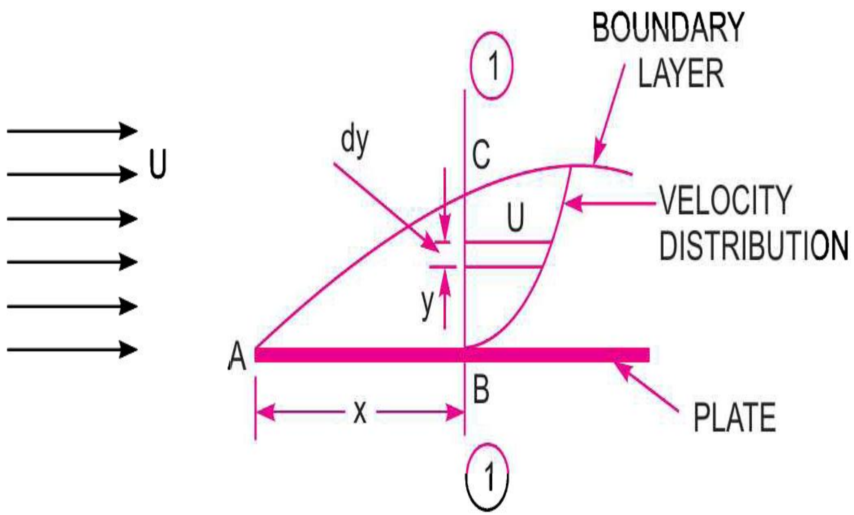
1. δ_{lam} = Thickness of laminar boundary layer,
2. δ_{tur} = Thickness of turbulent boundary layer,
3. δ’ = Thickness of laminar sub-layer,



DEFINITIONS:

5. Displacement Thickness (δ *):

- It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.
- At a distance x from the leading edge consider a section 1-1.
- The velocity of fluid at B is zero and at C, which lies on the boundary layer, is U.
- Thus velocity varies from zero at B to U at C, where BC is equal to the thickness of boundary layer Distance BC = δ



- Let

y = distance of elemental strip from the plate,

dy=thickness of the elemental strip,

u=velocity of fluid at the elemental strip,

b = width of plate.

DEFINITIONS:

5. Displacement Thickness (δ *):

- Then area of elemental strip,

  dA = b x dy

- Mass of fluid per second flowing through elemental strip

= p x Velocity x Area of elemental strip

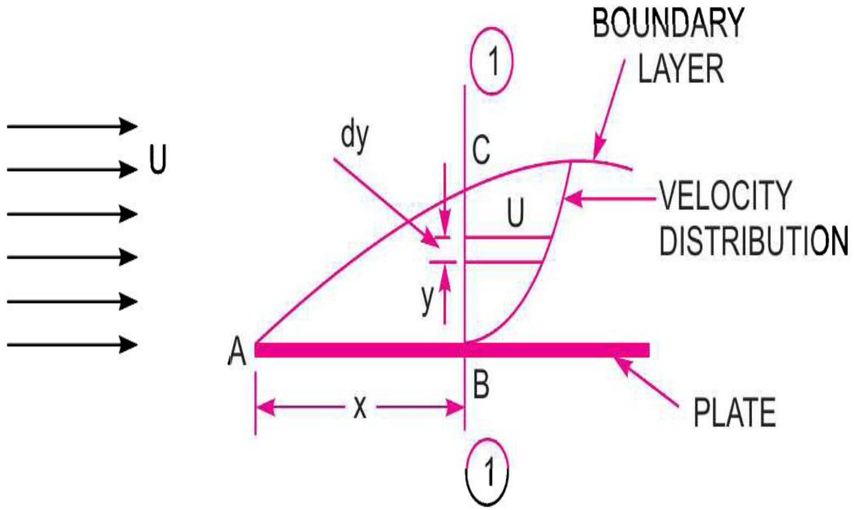
= p x u x dA

=px u x b x dy ...(i)

- Then mass of fluid per second flowing through elemental strip

= p x Velocity x Area

= p x U x b x dy ...(ii)



- This reduction in mass/sec flowing through elemental strip

= mass/sec given by equation (ii) - mass/sec given by equation (i)

= pUbdy – pubdy

= pb(U - u)dy.

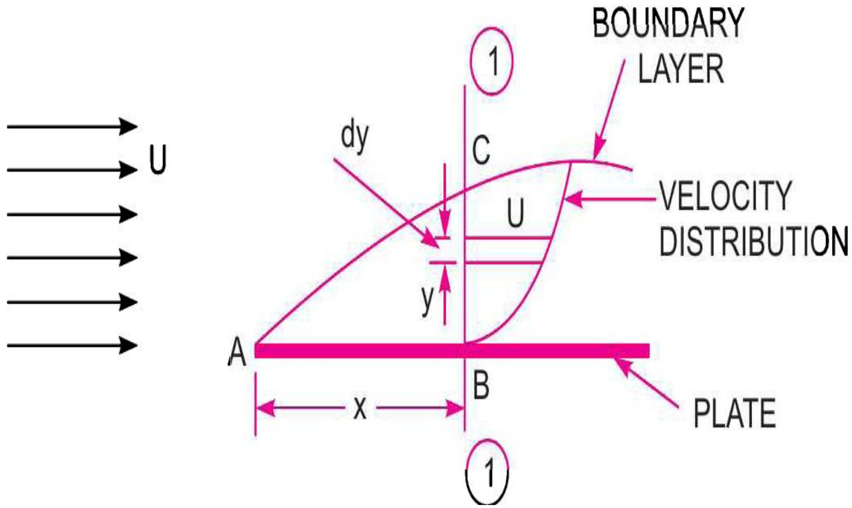
DEFINITIONS:

5. Displacement Thickness (δ *):

- Total reduction in mass of fluid/s flowing through BC due to plate

$$= \int_0^\delta \rho b(U-u)dy$$

$$= \rho b \int_0^\delta (U-u)dy \qquad \text{...(iii)}$$



- Loss of the mass of the fluid/sec flowing through the distance δ *

= p x Velocity x Area

= p x U x δ * x b ...(iv)

- Equating equation (iii) and (iv)

$$\rho b \int_0^\delta (U-u)dy = \rho \times U \times \delta^* b$$

- $$\int_0^\delta (U-u)dy = U \times \delta^*$$

$$\delta^* = \frac{1}{U}\int_0^\delta (U-u)dy$$

$$= \int_0^\delta \frac{(U-u)dy}{U}$$

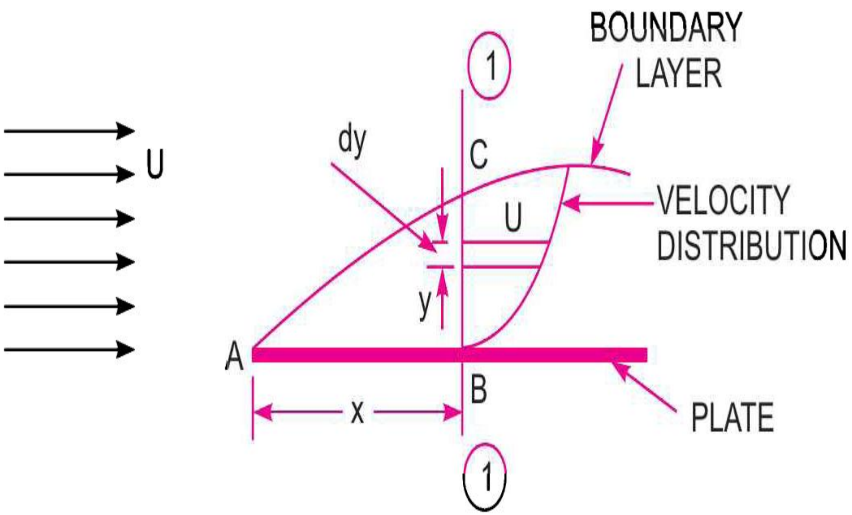
$$\boxed{\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right) dy}$$

6. Momentum Thickness (θ):

- Momentum thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation.
- Consider the section 1-1 at a distance x from leading edge. Take an elemental strip at a distance y from the plate having thickness (dy).
- Momentum of this fluid

= Mass x Velocity

= (pubdy)u



- Momentum of this fluid in the absence of boundary layer

= (pubdy)U

- Loss of momentum through elemental strip

= (pubdy)U - (pubdy) x u

= pbu(U - u)dy

DEFINITIONS:

6. Momentum Thickness (θ):

- Total loss of momentum/sec through BC

$$= \int_0^{\delta} \rho b u(U-u)dy \quad \text{...(i)}$$

- Let θ = distance by which plate is displaced when the fluid is flowing with a constant velocity U.

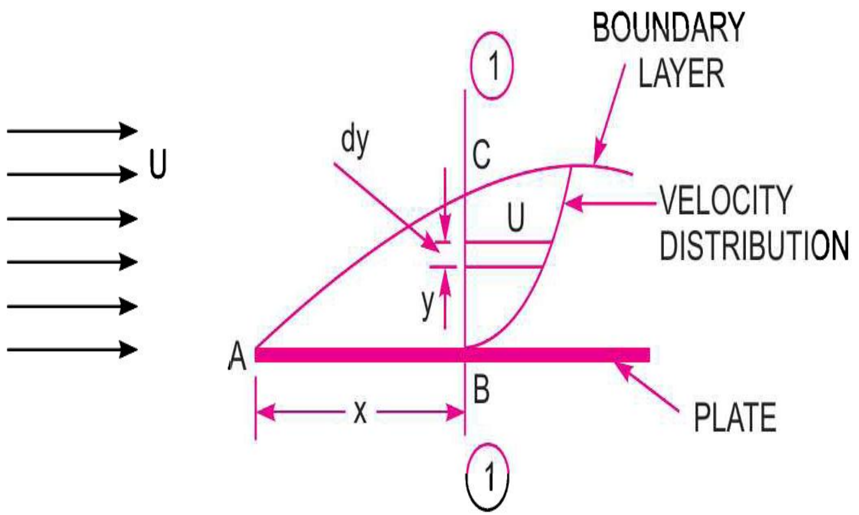
- Loss of momentum/sec of fluid flowing through distance θ with a velocity U

= Mass of fluid through θ x velocity

= (p x area x velocity) x velocity

= [p x θ x b x U] x U

= p θ bU^2 ...(ii)



- Equating equ. (i) and (ii)

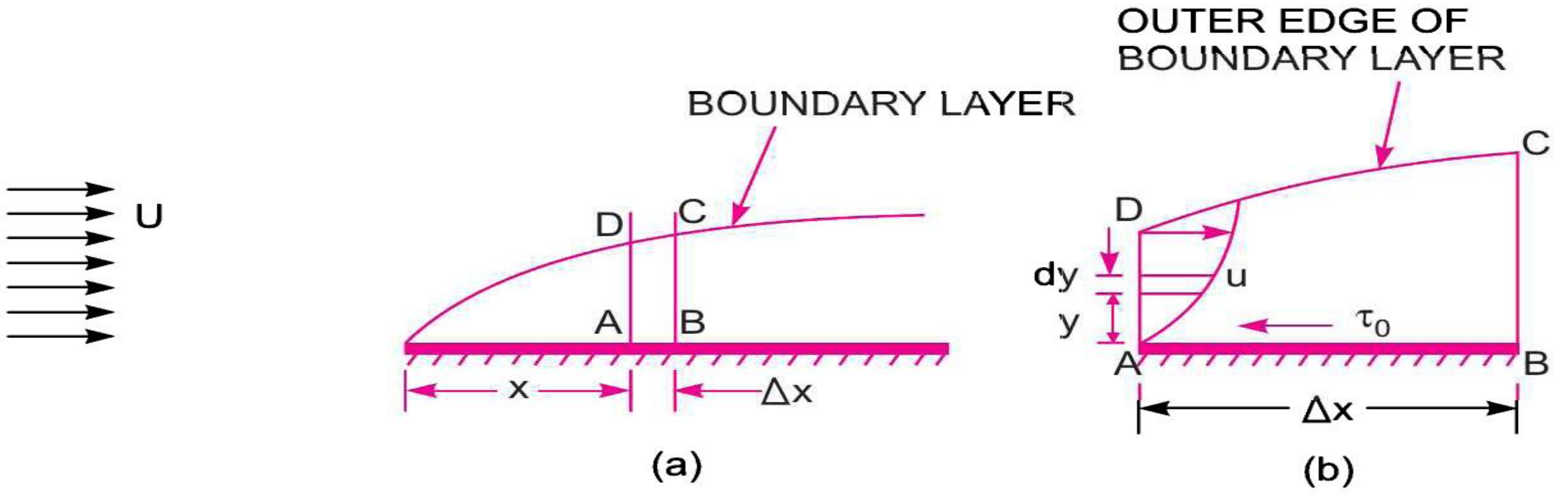
$$\rho\theta b U^2 = \int_0^{\delta} \rho b u(U-u)dy$$

$$= \rho b \int_0^{\delta} u(U-u)dy$$

$$\theta U^2 = \int_0^{\delta} u(U-u)dy$$

$$\boxed{\theta = \int_0^{\delta} \frac{u}{U}\left[1-\frac{u}{U}\right]dy}$$

Von Karman Momentum Integral Equation:



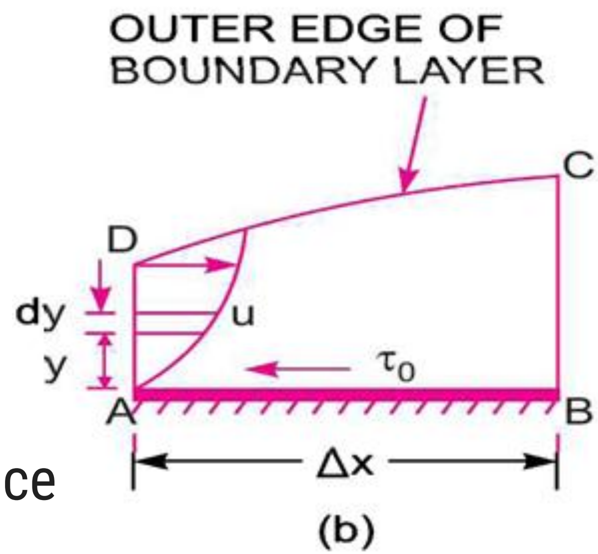
- Consider the flow of a fluid having free-stream velocity equal to U, over a thin plate as shown in Fig.
- The drag force on the plate can be determined if the velocity profile near the plate is known.
- Consider a small length Δx of the plate at a distance of x from the leading edge as shown in Fig. (a). The enlarged view of the small length of the plate is shown in Fig. (b).

Von Karman Momentum Integral Equation:

- The shear stress τ_0 is given by,

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0}$$

Where $\left(\frac{du}{dy}\right)_{y=0}$ is the velocity distribution near the plate at y = 0.



- Then drag force or shear force on a small distance Δx is given by,

ΔF_d = shear stress x area

$= \tau_0 \times \Delta x \times b$

where ΔF_d = drag force on distance Δx

- Let u = velocity at any point within the boundary layer

  b = width of plate

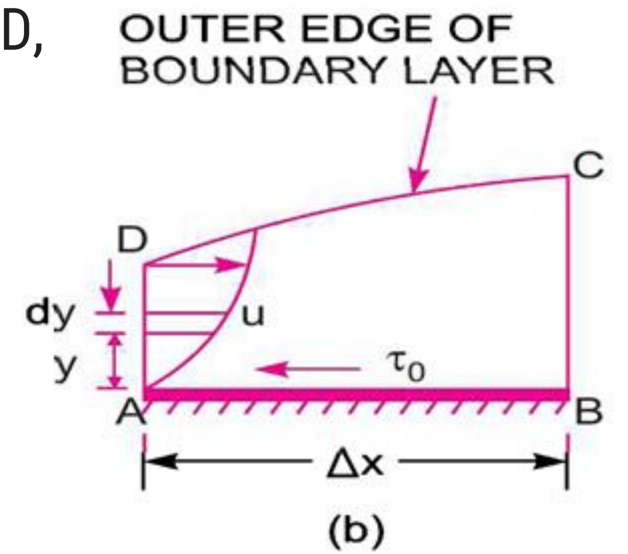
Von Karman Momentum Integral Equation:

- Then mass rate of flow entering through the side AD,

$$= \int_0^\delta \rho \times \text{velocity} \times \text{area of strip of thickness } dy$$

$$= \int_0^\delta \rho \times u \times b \times dy$$

$$= \int_0^\delta \rho u b dy$$



- Mass rate of flow leaving the side BC,

$$= \text{mass through } AD + \frac{\partial}{\partial x}(\text{mass through } AD) \times \Delta x$$

$$= \int_0^\delta \rho u b dy + \frac{\partial}{\partial x}\left[\int_0^\delta (\rho u b dy)\right] \times \Delta x$$

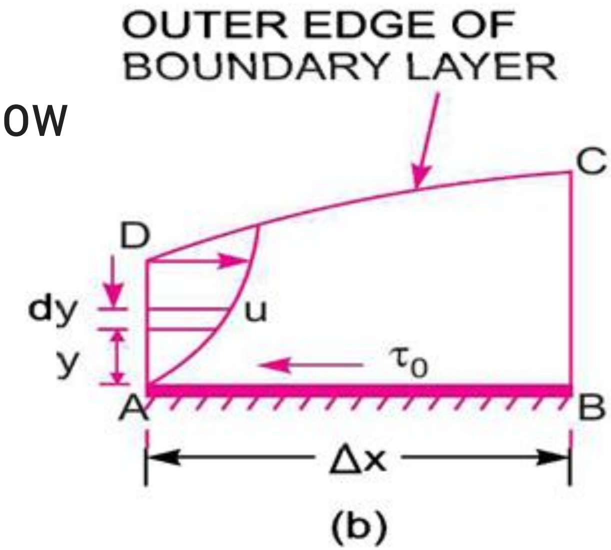
Von Karman Momentum Integral Equation:

- Mass rate of flow entering DC

= mass rate of flow through BC - mass rate of flow through AD,

$$= \int_0^\delta \rho u b dy + \frac{\partial}{\partial x}\left[\int_0^\delta \rho u b dy\right] \times \Delta x - \int_0^\delta \rho u b dy$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u b dy\right] \times \Delta x$$



- Now let us calculate momentum flux through control volume.
- Momentum flux entering through AD,

$$= \int_0^\delta \text{momentum flux through strip of thickness } dy$$

$$= \int_0^\delta \text{mass through strip} \times \text{velocity}$$

Von Karman Momentum Integral Equation:

- Momentum flux entering through AD,

$$= \int_0^\delta (\rho u b dy) \times u$$

$$= \int_0^\delta \rho u^2 b dy$$

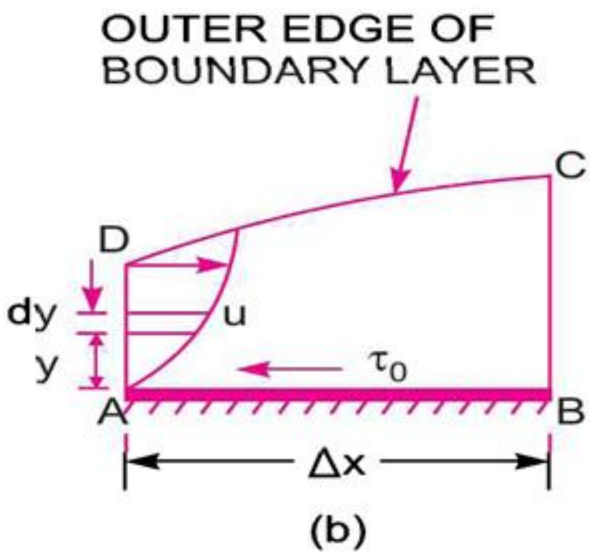
- Momentum flux leaving the side BC,

$$= \int_0^\delta \rho u^2 b dy + \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 b dy\right] \times \Delta x$$

- Momentum flux entering the side DC ,

= mass rate through DC x velocity

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u b dy\right] \times \Delta x \times U \qquad = \frac{\partial}{\partial x}\left[\int_0^\delta \rho u U b dy\right] \times \Delta x$$



Von Karman Momentum Integral Equation:

- Rate of change of momentum of the control volume,

= Momentum flux through BC - Momentum flux through AD - momentum flux through DC

$$= \int_0^\delta \rho u^2 b dy + \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 b dy\right] \times \Delta x - \int_0^\delta \rho u^2 b dy - \frac{\partial}{\partial x}\left[\int_0^\delta \rho u U b dy\right] \times \Delta x$$

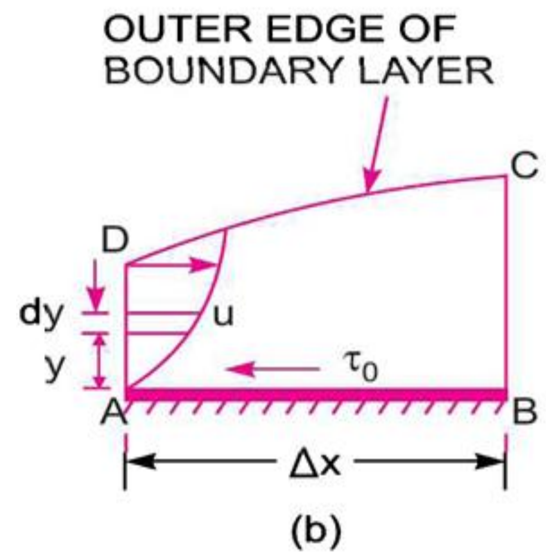
$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 b dy\right] \times \Delta x - \frac{\partial}{\partial x}\left[\int_0^\delta \rho u U b dy\right] \times \Delta x$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 b dy - \int_0^\delta \rho u U b dy\right] \times \Delta x$$

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \left(\rho u^2 b - \rho u U b\right) dy\right] \times \Delta x$$

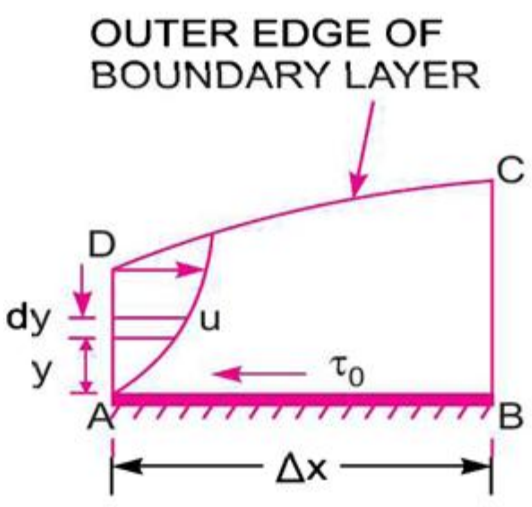
$$= \frac{\partial}{\partial x}\left[\rho b \int_0^\delta \left(u^2 - uU\right) dy\right] \times \Delta x$$

$$= \rho b \frac{\partial}{\partial x}\left|\int_0^\delta \left(u^2 - uU\right) dy\right| \times \Delta x \quad \text{...(i)}$$



Von Karman Momentum Integral Equation:

- The only external force acting on the control volume is the shear force acting on the side AB in the direction from B to A as shown in Fig. (b).



(b)

- The value of this force is given by equation as,

$\Delta F_d = \tau_0 x\ \Delta x\ x\ b$

- Total external force in the direction of rate of change of momentum,

$= -\tau_0 x\ \Delta x\ x\ b$ ...(ii)

- Equating equation (i) and (ii)

$$-\tau_0 \times \Delta x \times b = \rho b \frac{\partial}{\partial x}\left[\int_0^\delta \left(u^2 - uU\right)dy\right] \times \Delta x$$

$$-\tau_0 = \rho \frac{\partial}{\partial x}\left[\int_0^\delta \left(u^2 - uU\right)dy\right]$$

- Equating equation (i) and (ii)

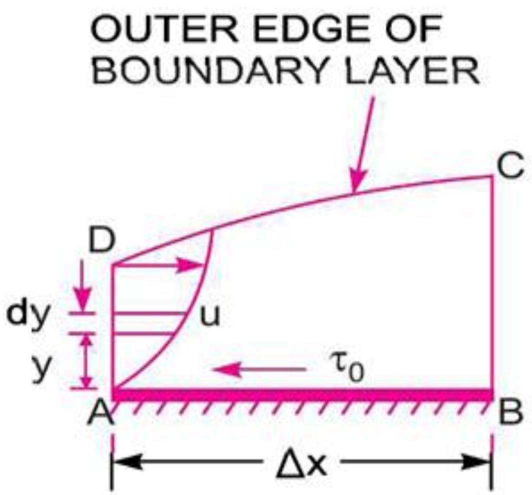
$$\tau_0 = -\rho \frac{\partial}{\partial x}\left[\int_0^\delta \left(u^2 - uU\right)dy\right]$$

$$= \rho \frac{\partial}{\partial x}\left[\int_0^\delta \left(uU - u^2\right)dy\right]$$

$$= \rho \frac{\partial}{\partial x}\left[\int_0^\delta U^2\left(\frac{u}{U} - \frac{u^2}{U^2}\right)dy\right]$$

$$= \rho U^2 \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right]dy\right]$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left[1 - \frac{u}{U}\right]dy\right]$$



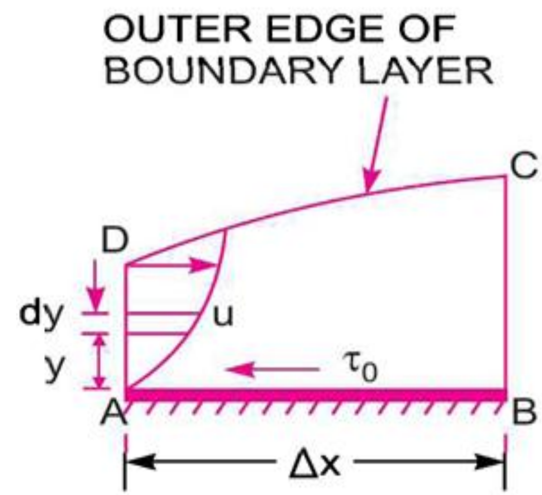
(b)

Von Karman Momentum Integral Equation:

- The equation $\int_0^\delta \frac{u}{U}\left[1-\frac{u}{U}\right]dy$ is equal to momentum thickness θ.

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x}\left[\int_0^\delta \frac{u}{U}\left[1-\frac{u}{U}\right]dy\right]$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}$$



(b)

The above Equation is known as **Von Karman momentum integral equation for boundary layer flows**.

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