5

DIMENSIONAL ANALYSIS AND SIMILARITIES

Course Contents

5.1 Introduction

- Dimensional analysis is a method of dimensions. It is a mathematical technique used in research work for design and for conducting model tests.

• Application of Dimensional analysis

  - To derive formula which representing the relationship between the physical quantities that affect a given physical phenomenon.
  - To check dimensional homogeneity of any equation of fluid flow.
  - To provide scaling laws that can convert data from small model to large model.
  - To develop dimensionless number which is useful to compare the different problems.

• Dimensions

- A dimension is measure by which physical quantity is expressed. A unit is a method of attaching a number to the quantitative dimension.
- For example: The length is a dimension of variables as distance, displacement, height etc.

• Fundamental Dimensions/Quantity

- All the physical quantities are measured by comparison, which is made with respect to arbitrary fixed value.
- Length L, mass m, and time T are three fixed dimensions. If in any problem heat is involved then temperature is also taken as fixed dimension.
- These fixed dimensions are called fundamental dimensions or fundamental quantity.

• Secondary/Derived quantity

- These are those quantities which possess more than one fundamental dimensions.
- For example: Velocity is denoted by distance per unit time (L/T), density is denoted by mass per unit volume (M/L^3).
- So velocity, density become as secondary quantities.

• Dimensional Variable

- It is quantities which vary during the given case.

• Dimensional Constant

- It is quantities which held constant during a given case. But it may vary from case to case.

Table 5.1 Quantities used in fluid mechanics

Sr. No.	Quantity	Symbol	Units (SI)	Dimensions (MLT System)
A	**Fundamental**			
1	Mass	M	Kg	$M^1L^0T^0$
2	Length	L	m	$M^0L^1T^0$
3	Time	T	Sec	$M^0L^0T^1$
B	**Geometric**			
1	Area	A	m^2	L^2
2	Volume	V	m^3	L^3
3	Moment of inertia	I	m^4	L^4
4	Roughness	K	m	L
C	**Kinematic**			
1	Linear Velocity	u, v	m/s	L^1T^{-1}
2	Angular Velocity	ω	rad/s	T^{-1}
3	Rotational speed	N	rev/min	T^{-1}
3	Acceleration	a	m/s^2	L^1T^{-2}
4	Angular Acceleration	α	rad/s^2	T^{-2}
5	Discharge	Q	m^3/sec	L^3T^{-1}
6	Kinematic Viscosity	ν	m^2/sec	L^2T^{-1}
D	**Dynamic**			
1	Force / Resistance/Weight	F/R/W	N ($kg\text{-}m/s^2$)	$M^1L^1T^{-2}$
2	Specific Weight	w	N/ m^3	$M^1L^{-2}T^{-2}$
3	Density	ρ	Kg/ m^3	M^1L^{-3}
4	Pressure	p	N/m^2	$M^1L^{-1}T^{-2}$
5	Shear stress	τ	N/m^2	$M^1L^{-1}T^{-2}$
6	Dynamic Viscosity	μ	Kg/m-sec	$M^1L^0T^{-2}$
7	Modulus of elasticity	E,K	N/m^2	$M^1L^{-1}T^{-2}$
8	Surface tension	σ	N/m	$M^1L^{-1}T^{-2}$
9	Work, Energy	W, E	N-m (Joule)	$M^1L^2T^{-2}$
10	Power	P	Watt (J/sec)	$M^1L^2T^{-3}$
11	Torque	T	Nm	$M^1L^2T^{-2}$

12	Momentum	M	Kgm/s	$M^1L^1T^{-1}$

5.2 Dimensional Homogeneity

- Dimensional homogeneity means the dimensions of each terms in an equation on both the sides equal.
- Thus if the dimension of the each term on both sides of equation are the same equation is known as dimensionally homogeneous equation.
- The power of fundamental dimension (L,M,T) on both side of equation will be identical for a dimensionally homogeneous equation.

Example:

let us consider the equation $V = \sqrt{2gH}$

$$\text{Dimension of L.H.S.} = V = \frac{L}{T} = LT^{-1}$$

$$\text{Dimension of R.H.S.} = \sqrt{2gH} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} = LT^{-1}$$

$$\therefore \text{Dimension of L.H.S.} = \text{Dimension of R.H.S.} = LT^{-1}$$

$$\therefore \text{Equation } V = \sqrt{2gH} \text{ is dimensionally homogeneous.}$$

5.3 Method of Dimensional Analysis

- If the number of variables involved in a physical phenomenon are known, then the relation among the variable can be determined by following method.
  1. Rayleigh's method
  2. Buckingham's π theorem

Rayleigh's method

- This method is used for determining the expression for a variable which depends upon maximum three or four variable only.
- If the number of variables becomes more than four, then it is very difficult to find the expression for the dependent variable.
- Let X is a variable, which depends on X_1, X_2, and X_3 variables. Then according to the Rayleigh's method, X is a function of X_1, X_2, and X_3 mathematically it is written as

$$X = KX_1^a \cdot X_2^b \cdot X_3^c$$

Where K is constant and a, b, and c are arbitrary powers.

- The value of a, b and c are obtain by comparing the power of fundamental dimension on both sides. Thus expression is obtained for dependent variables.

Buckingham's π Theorem

- The Rayleigh's method of dimensional analysis becomes more laborious if the variables are more than number of fundamental dimensions (M, L, T). These difficulties are overcome by using Buckingham's π - theorem.

  Theorem
- **"If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T) then variables are arranged in to (n-m) dimensionless number. Each term is called π term."**
- Let X_1, X_2, X_3 ……….. X_n are the variables involved in physical problems. Let X_1 be the dependent variable and X_2, X_3 ……….. X_n are the independent variable on which X_1 depends. Then X_1 is a function of X_2, X_3 ……….. X_n and mathematically it is expressed as

$$X_1 = f(X_2, X_3 \ldots\ldots\ldots\ldots X_n) \qquad \ldots\ldots\ldots\ldots(1)$$

- Equation (1) can also be written as

$$f_1(X_2, X_3 \ldots\ldots\ldots\ldots X_n) = 0 \qquad \ldots\ldots\ldots\ldots(2)$$

- Equation (2) is a dimensionally homogeneous equation.
- It contains n variables. If there are m fundamental dimensions then according to the Buckingham's π Theorem, equation (2) can be written in term of number of dimensionless group or π terms in which number of π-term is equal to (n - m).Hence equation (2) becomes as

$$f(\pi_1, \pi_2, \ldots, \pi_{n-m}) = 0 \qquad \cdots\cdots\cdots(3)$$

- Each of π-term is dimensionless and is independent of the system.
- Division or multiplication by a constant does not change the character of the π-term. Each π-term contains m+1 variables, where m is the number of fundamental dimensions and is also called repeating variables.
- Let in the above case X_2, X_3 and X_4 are repeating variables if the fundamental dimension m(M, L, T)=3. Then each π-term is written as

$$\left.\begin{aligned} \pi_1 &= X_2^{a_1} X_3^{b_1} X_4^{c_1} X_1 \\ \pi_2 &= X_2^{a_2} X_3^{b_2} X_4^{c_2} X_5 \\ &\vdots \qquad \vdots \\ \pi_{n-m} &= X_2^{a_{n-m}} X_3^{b_{n-m}} X_4^{c_{n-m}} X_n \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots(4)$$

- Each equation is solved by the principle of dimensional homogeneity and value of a_1, b_1, c_1 etc. are obtained. These values are substituted in equation (4) and values of π_1, π_2, π_3, …….π_{n-m} are obtained. These values of π's are substituted in equation (3).

The final equation for the phenomenon is obtained by expressing any one of the π-terms as a function of others as

$$\pi_1 = \phi\left[\pi_2, \pi_3, \cdots \pi_{n-m}\right]$$
$$\pi_2 = \phi_1\left[\pi_1, \pi_3, \cdots \pi_{n-m}\right]$$

Method of Selecting Repeating Variables

- The number of repeating variables are equal to the number of fundamental dimensions of the problem. The choice of repeating variable is governed by the following considerations:
  1. As far as possible, the dependent variable should not be selected ass repeating variable.
  2. The repeating variables should be choosen in such a way that one variable contains geometric property, other variable contains flow property and third variable contain fluid property.
  - Variables with Geometric property are
    - I. Length, l
    - II. Diameter d
    - III. Height, h etc
  - Variables with flow property are
    - I. Velocity, V
    - II. Acceleration etc.
  - Variables with fluid property
    - I. μ
    - II. ρ etc
  3. Repeating variables selected should not from a dimensionless group
  4. The repeating variable together must have the same number of fundamental dimensions
  5. No two repeating variables should have the same dimensions
- In most of fluid mechanics problems, the choice of repeating variable may be
  - I. d, μ ,ρ
  - II. l, ν, ρ
  - III. l, ν, μ
  - IV. d, ν, μ

5.4 Similitude-Types of similarities

- Similitude is define as the similarity between the model and its prototype in every aspect, which means that model and prototype have similar properties or model and prototype are completely similar.

– Three type of similarity must exist between the model and prototype. They are…
    1. Geometric Similarity
    2. Kinematic Similarity
    3. Dynamic Similarity

Geometric Similarity

– Geometric similarity is said to be exist between the model and prototype if the ratio of all corresponding linear dimension in the model and prototype are equal.

Let $L_m = \text{Length of model}$ $b_m = \text{Breath of the model}$
$D_m = \text{Diameter of model}$ $A_m = \text{Area of model}$
$V_m = \text{Volume of model}$
and $L_p, b_p, D_p, A_p, V_p = \text{corresponding values of prototype.}$

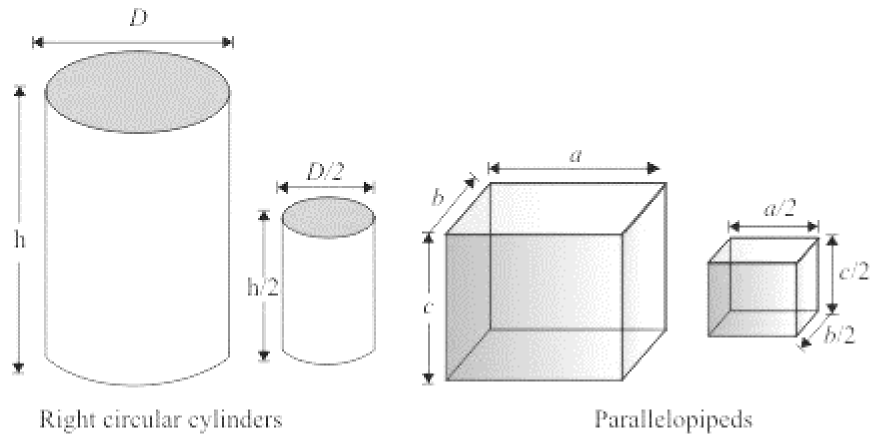

Fig. 5.1 Geometric similarity

– For geometric similarity between the model and prototype, we must have the relation,

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r \qquad \cdots\cdots\cdots\cdots()$$

Where L_r is called scale ratio.

– For area's ratio and volume's ratio relation should be as below:

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^{\,2}$$

$$\frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3$$

Kinematic similarity

- Kinematic similarity means the similarity of motion between model and prototype.
- Thus kinematic similarity is said to be exist between the model and prototype if the ratio of the velocity and acceleration at the corresponding points in the model at the corresponding points in the prototype are same.
- Since velocity and acceleration are vector quantities, hence not only the ratio of magnitude of velocity and acceleration at the corresponding points in the model and prototype should be same; but the direction of velocity and acceleration at the corresponding points in the model and prototype also should be parallel.

Let V_{p1} = Velocity of fluid at point 1 in prototype

V_{p2} = Velocity of fluid at point 2 in prototype

a_{p1} = Acceleration of fluid at point 1 in prototype

a_{p2} = Acceleration of fluid at point 2 in prototype

V_{m1}, V_{m2}, a_{m1}, a_{m2} = Corresponding values at the corresponding points of fluid velocity and acceleration in the model.

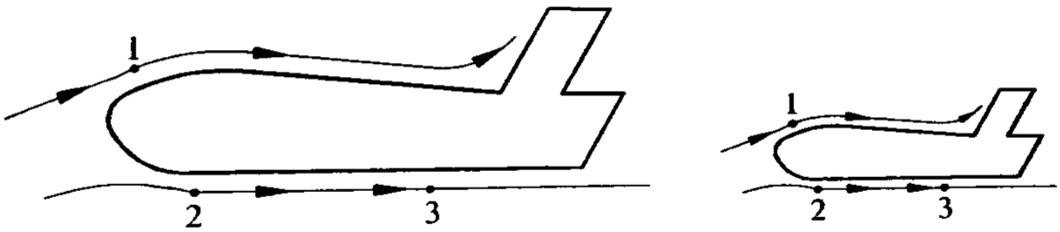

Fig. 7.2 Kinematic similarity

- For kinematic similarity we must have

$$\frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = V_r$$

Where V_r = velocity ratio

- For acceleration we must have

$$\frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_r$$

Where a_r = acceleration ratio

- Also the direction of the velocity in the model and prototype should be same.

Dynamic similarity

- Dynamic similarity means the similarity of forces between the model and prototype.
- Thus dynamic similarity is said to be exits between model and prototype if the ratios of corresponding forces acting at the corresponding points are equal.
- Also the direction of corresponding forces at the corresponding points should be same.

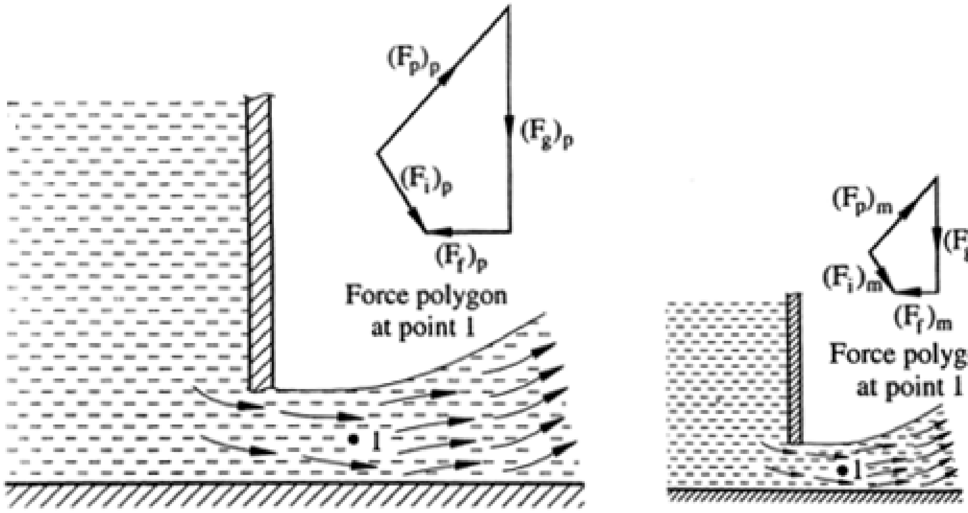

Fig. 7.3 Dynamic similarity

$(F_i)_p$ = Inertia force at a point in prototype

$(F_v)_p$ = Viscous force at a point in prototype

$(F_g)_p$ = Gravity force at a point in prototype

$(F_i)_m, (F_i)_m, (F_i)_m$ arecorresponding values of force at the corresponding point in model.

Then for dynamic simillarity, we have

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_i)_m} = \frac{(F_g)_p}{(F_g)_m} \qquad \text{Where } F_r = \text{force ratio}$$

- Also the direction of corresponding forces at the corresponding forces at corresponding points in the model and prototype should be same.

5.5 Dimensionless Numbers

- Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravitational force or pressure force or surface tension force or elastic force.

- As this is a ratio of one force to another force, it will be a dimensionless numbers. These dimensionless numbers are also called non-dimensional parameters.
- The followings are the important dimensionless numbers:

1. Reynold's number
2. Froude's number
3. Euler's number
4. Weber's number
5. Mach's number

1. Reynold's number

- It is define as the ratio of inertia force of flowing fluid and viscous force of the fluid.
- It is denoted by R_e.

$$R_e = \frac{\text{Inertia force}(F_i)}{\text{Viscous force}(F_v)}$$

$$= \frac{\rho A V^2}{\mu \frac{V}{L} A}$$

$$= \frac{\rho V L}{\mu}$$

$$= \frac{VL}{\upsilon} \quad (\because \upsilon = \text{Kinematic viscosity})$$

$$\boxed{R_e = \frac{VD}{\upsilon} = \frac{\rho V D}{\mu}} \quad (\text{In case of pipe flow})$$

Significance

- Reynold's number (Re) measures the relative magnitude of the inertia force to viscous force occurring in the flow.
- Higher the inertia Re, greater the inertia effect. Smaller the Re, greater the viscous stresses.
- The Reynold number is the criteria of dynamic similarity in the flow situations where the viscous force predominates.
- In this case dynamic similarity is said to be exist between the model and prototype when Re of model and prototype is same.
- Examples of such situation:
  I. Flow of incompressible fluid in a pipe.
  II. Motion of submarine completely in closed pipe.

2. Froude's number

- It is defined as the square root of the ratio of the inertia force and gravitational force.
- It is denoted by F_r .

$$F_r = \sqrt{\frac{\text{Inertia force}(F_i)}{\text{Gravity force}(F_g)}}$$

$$= \sqrt{\frac{\rho A V^2}{\rho A L g}}$$

$$= \sqrt{\frac{V^2}{Lg}}$$

$$\boxed{\therefore F_r = \frac{V}{\sqrt{Lg}}}$$

Significance:

- If the gravitational force is of prime importance, dynamic similarity is said to be exists between the model and prototype when the Froud number for model and prototype is same.
- Example of such situation
  I. Flow of liquid jet from the orifice
  II. Flow over notches, weirs of a dam.

3. Mach number

- It is define as the square root of the ratio of the inertia force to elastic force.It is denoted by M.

$$M = \sqrt{\frac{\text{Inertia force}(F_i)}{\text{Elastic force}(F_e)}}$$

$$M = \sqrt{\frac{\rho A V^2}{K L^2}}$$

$$M = \sqrt{\frac{\rho L^2 V^2}{K L^2}}$$

$$M = \frac{V}{\sqrt{K/\rho}}$$

$$\boxed{\therefore M = \frac{V}{C}}$$

where $C = \sqrt{K/\rho}$ = Speed of sound wave in flowing medium

Significance:

- The Mach number signifies predominance of effect of compressibility of fluid.
- Higher Mach number signifies the predominance of effect of compressibility of fluid.
- The mach number is important in compressible flow problems at higher velocities such as
  - I. Aerodynamic testing
  - II. Water hammer problem

4. Weber number

- It is define as the square root of ratio of the inertia force to surface tension force. It is denoted by W_e.

$$W_e = \sqrt{\frac{\text{Inertia force}(F_i)}{\text{Surface tension force}(F_s)}}$$

$$= \sqrt{\frac{\rho A V^2}{\sigma L}}$$

$$= \sqrt{\frac{\rho L^2 V^2}{\sigma L}}$$

$$\boxed{\therefore W_e = \frac{V}{\sqrt{\sigma / \rho l}}}$$

Significance:

- The Weber number is important in case where the surface tension force is predominating force, dynamic similarity is said to exist when weber number of model and prototype is equal.
- Practical application of weber number is as under:
  - I. Capillarity tube action
  - II. Flow of blood in veins and arteries

5. Euler's number

- It is define as the square root of ratio of inertia force to pressure force. It is denoted by E_u.

$$E_u = \sqrt{\frac{\text{Inertia force}(F_i)}{\text{Pressure force}(F_p)}}$$

$$= \sqrt{\frac{\rho A V^2}{p A}}$$

$$\boxed{\therefore E_u = \frac{V}{\sqrt{p / \rho}}}$$

Significance:

- The Euler number is signifies when pressure force is predominates.
- Examples of such flow situations are as
  - I. Flow through pipe
  - II. Discharge through orifice and mouthpieces.

5.6 Model Laws or Law of Similarity

- Law on which the models are designed for dynamic similarity are known as Model laws or similarity laws.
- Model laws are as
  1. Reynold's model law
  2. Froud model law
  3. Euler model law
  4. Weber model law
  5. Mach model law

1. Reynold's model law

- For the flow where in addition to inertia force, the various forces are predominant, the models are designed for dynamic similarity on Reynold number. This is called Reynold law.
- Mathematically,

$$(R_e)_{model} = (R_e)_{prototype}$$

$$\therefore \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

- **Application of Reynold's model law**
  - Complete enclosed flow: Flow through pipe and plates
  - Viscous flow
  - Flow in flowmeter in pipe: Venturimeter, orificemeter etc.
  - Incompressible fluid flow In pipe
  - Completely submerge flow: Aeroplanes, submarines

2. Froude model law

- For the flow where in addition to inertia force, the gravity force are predominant, the model are designed for dynamic similarity on Froude number. This is called Froude law.
- Mathematically,

$$(F_r)_{model} = (F_r)_{prototype}$$

$$\therefore \frac{V_m}{\sqrt{L_m g}} = \frac{V_p}{\sqrt{L_p g}}$$

$$\boxed{\therefore \frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}}$$

- **Application od Froude model law**
  - Free surface flow: open channel flow, Spillways, weirs, notches.
  - Flow of jet from an orifice or nozzle.
  - Where fluids of different densities flow over another.

3. Euler's model law

- For the flow where in addition to inertia force, the pressure force are predominant, the model are designed for dynamic similarity on Euler number. This is called Euler's model law.
- Mathematically,

$$(E_u)_{model} = (E_u)_{prototype}$$

$$\therefore \frac{V_m}{\sqrt{p_m / \rho_m}} = \frac{V_p}{\sqrt{p_p / \rho_p}}$$

- **Application of Euler's model law**
  - Enclosed fluid flow where the turbulent is fully developed(viscous force are negligible, also gravity and surface tension force absent).
  - Where phenomenon of cavitation take places.

4. Weber model law

- For the flow where in addition to inertia force, the surface tension effect predominant, the model are designed for dynamic similarity on weber number. This is called Weber's model law.
- Mathematically,

$$(W_e)_{model} = (W_e)_{prototype}$$

$$\therefore \frac{V_m}{\sqrt{\sigma_m / \rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p / \rho_p L_p}}$$

- **Application of weber model law**
  - Capillarity rise in narrow passage.
  - Capillarity movement of water in soil.
  - Thin sheet of liquid flows over a surface.

5. Mach model law

- For the flow where in addition to inertia force, the force due to elastic compression are predominant, the model are designed for dynamic similarity on Mach number. This is called Mach model law.
- Mathematically,

$$(M)_{model} = (M)_{prototype}$$

$$\therefore \frac{V_m}{\sqrt{K_m / \rho_m}} = \frac{V_p}{\sqrt{K_p / \rho_p}}$$

- **Application of Mach model law**
  - Aerodynamic testing.
  - Water hammer problem.
  - Supersonic flow – flow of aeroplane and projectiles through air.

5.7 Classification of models

- The hydraulic models are classified as :
  1. Undistorted model
  2. Distorted model

1. Undistorted model

- These are those models which are geometrically similar to their prototypes or in other words if the scale ratio for the linear dimensions of the model and its prototype is same, model is called undistorted model.
- The behavior of the prototype can be easily predicted from the result of undistorted model.

2. Distorted model

- A model is said to be distorted if it is not geometrically similar to its prototype.
- For a distorted model different scale ratios for the linear dimensions are adopted.
- For example, in case of rivers, reservoirs etc., two different scale ratios, one for horizontal and other for vertical dimensions are taken. Thus the model of river and reservoir will become a distorted model.
- If for the river, the horizontal and vertical scale ratios are taken to be same so that the model is undistorted model, then the depth of water in model of riverwill be very-very small which may not be measure accurately.
- Thus following are the advantages of distorted model:
  1. The vertical dimension of the model can be measured accurately.
  2. The cost of model can be reduced.
  3. Turbulent flow in the model can be maintained.

5.8 Solved Numerical

1. Find the expression for the power P, developed by pump when P depends upon the head H, the discharge Q and specific weight w of a fluid.

Solution: Power P is a function of……

I. Head H
II. Discharge Q
III. Specific weight w

$$P = KH^aQ^bw^c \qquad \cdots\cdots\cdots(1)$$

Where $K =$ Dimensionless constant

→ Substitute dimension on the both sides of equation

$$\therefore ML^2T^{-3} = K\left(M^0L^1T^0\right)^a\left(M^0L^3T^{-1}\right)^b\left(M^1L^{-2}T^{-2}\right)^c$$

→ Equating the power on the both sides of equation

Power of M	$1 = c$	$\therefore c = 1$
Power of L	$2 = a + 3b - 2c$	$\therefore a = 2 - 3b + 2c = 2 - 3 + 2 = 1$
Power of T	$-3 = -b - 2c$	$\therefore b = 3 - 2c = 3 - 2 = 1$

→ Substituting the value of a,b and c in equation (1)

$$\boxed{\therefore P = KHQw}$$

2. The efficiency η of a fan depends on density ρ, dynamic viscosity μ of a fluid, angular velocity ω, diameter D of the rotor and discharge Q. Express η in terms of dimensionless parameters using Buckingham's π -theorem.

Solution:

→ η is function of ρ, μ, ω, D and Q

$$\therefore \eta = f\left(\rho, \mu, \omega, D, Q\right)$$

$$\therefore f_1\left(\eta, \rho, \mu, \omega, D, Q\right) = 0 \qquad \cdots\cdots(1)$$

→ Here total no of variables $n = 6$

no of fundamental dimensions $m = 3$

$\therefore$ no of π terms $= n - m = 6 - 3 = 3$

→ Let these π terms are π_1, π_2, π_3

So equation (1) can be written as

$$f_1\left(\pi_1, \pi_2, \pi_3\right) = 0 \qquad \cdots\cdots(2)$$

→ no. of repeating variable $= m = 3$

Let these repeating variables are D, ω, ρ

$$\therefore \pi_1 = D^{a_1}\omega^{b_1}\rho^{c_1}\eta$$

$$\therefore \pi_2 = D^{a_2}\omega^{b_2}\rho^{c_2}\mu$$

$$\therefore \pi_3 = D^{a_3}\omega^{b_3}\rho^{c_3}Q$$

→ First π term (π_1):

$$\therefore \pi_1 = D^{a_3}\omega^{b_3}\rho^{c_3}\eta$$

$$\therefore \left(M^0L^0T^0\right) = \left(M^0L^1T^0\right)^{a_1}\left(M^0L^{-1}T^0\right)^{b_1}\left(M^1L^{-3}T^0\right)^{c_1}\left(M^0L^0T^0\right)$$

Equating power on both sides of equation

Power of M	$0 = c_1 + 0$	$\therefore c_1 = 0$
Power of L	$0 = a_1 + 0$	$\therefore a_1 = 0$
Power of T	$0 = -b_1 + 0$	$\therefore b_1 = 0$

Substituting the value of a_1, b_1 and c_1 in equation of π_1

$$\therefore \pi_1 = D^0\omega^0\rho^0\eta = \eta$$

$$\boxed{\therefore \pi_1 = \eta}$$

→ Second π term (π_2):

$$\therefore \pi_2 = D^{a_2}\omega^{b_2}\rho^{c_2}\mu$$

$$\therefore \left(M^0L^0T^0\right) = \left(M^0L^1T^0\right)^{a_2}\left(M^0L^{-1}T^0\right)^{b_2}\left(M^1L^{-3}T^0\right)^{c_2}\left(M^1L^{-1}T^{-1}\right)$$

Equating power on both sides of equation

Power of M	$0 = c_2 + 1$	$\therefore c_2 = -1$
Power of L	$0 = a_2 - 3c_2 - 1$	$\therefore a_2 = 3c_2 + 1 = -3 + 1 = -2$
Power of T	$0 = -b_2 - 1$	$\therefore b_2 = -1$

Substituting the value of a_2, b_2 and c_2 in equation of π_2

$$\therefore \pi_2 = D^{-2}\omega^{-1}\rho^{-1}\mu = \frac{\mu}{D^2\omega\rho}$$

$$\boxed{\therefore \pi_2 = \frac{\mu}{D^2\omega\rho}}$$

→ Third π term (π_3):

$$\therefore \pi_3 = D^{a_3}\omega^{b_3}\rho^{c_3}Q$$

$$\therefore \left(M^0L^0T^0\right) = \left(M^0L^1T^0\right)^{a_3}\left(M^0L^{-1}T^0\right)^{b_3}\left(M^1L^{-3}T^0\right)^{c_3}\left(M^0L^3T^{-1}\right)$$

Equating power on both sides of equation

Power of M	$0 = c_3$	$\therefore c_3 = 0$
Power of L	$0 = a_3 - 3c_3 + 3$	$\therefore a_3 = 3c_3 - 3 = -3$
Power of T	$0 = -b_3 - 1$	$\therefore b_3 = -1$

Substituting the value of a_3, b_3 and c_3 in equation of π_3

$$\therefore \pi_3 = D^{-3}\omega^{-1}\rho^0 Q = \frac{Q}{D^3\omega} \qquad \boxed{\therefore \pi_3 = \frac{Q}{D^3\omega}}$$

→ Substituting the value of π_1, π_2 and π_3 in equation (2)

$$\therefore f_1\left(\eta, \frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right) = 0 \quad \Rightarrow \quad \boxed{\therefore \eta = \phi\left(\frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right)}$$