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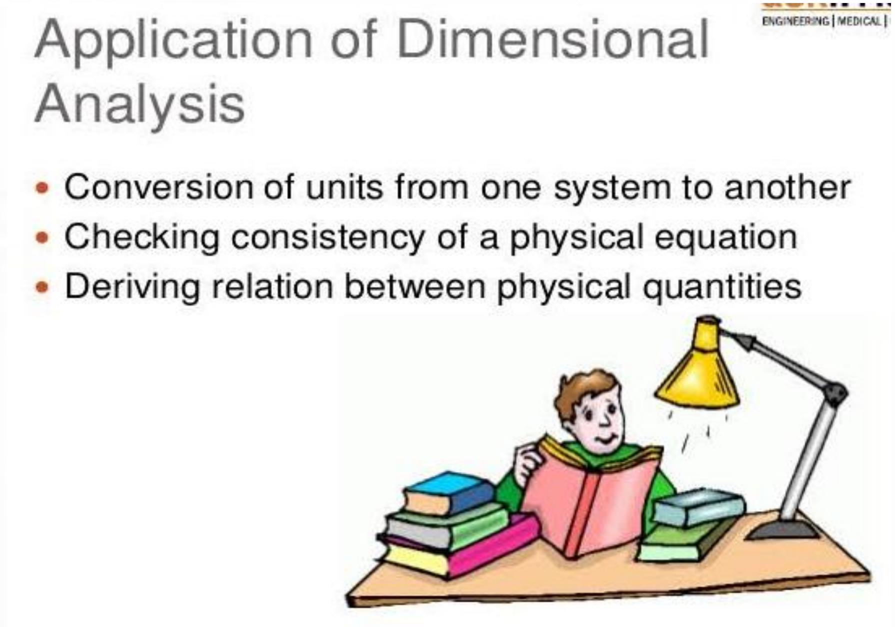

Applied Fluid Mechanics
(2160602)

Module-5

Dimensional Analysis and Similitude

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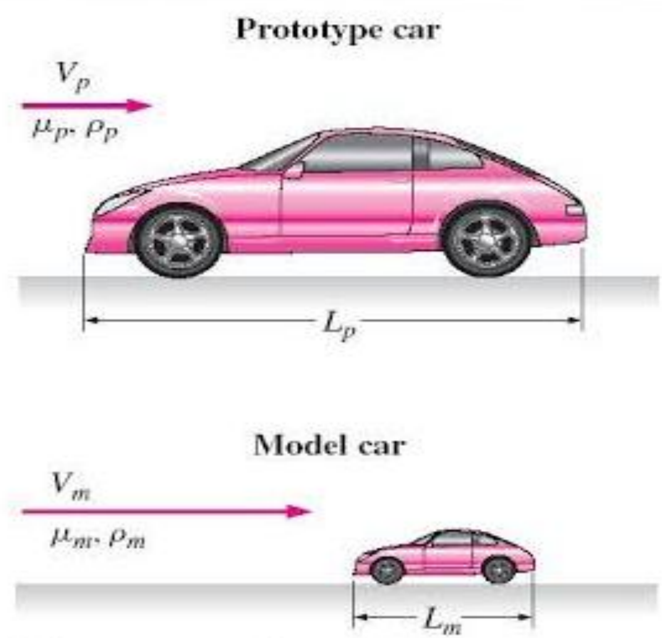
INTRODUCTION:

- Dimension Analysis:
  - It is a mathematical technique which make use of the study of dimensions as an aid to the solution of engineering problems.



- Dimensional analysis helps in determining a systematic arrangement of variables in the physical relationship and combining dimensional variables to form non dimensional parameters.
  - Ex. ρ,v, d, μ to form Reynolds number

- It is useful in both analytical and experimental investigation.

INTRODUCTION:

- Fluid characteristics
  - → Quantity: numbers or units
  - → Quality: w.r.t primary quantity

- Four basic **primary or fundamental** quantities:
  - **Length – L**
  - **Mass – M**
  - **Time – T**
  - **Temperature – θ**

- These are basic dimension to describe the secondary quantities.

INTRODUCTION:

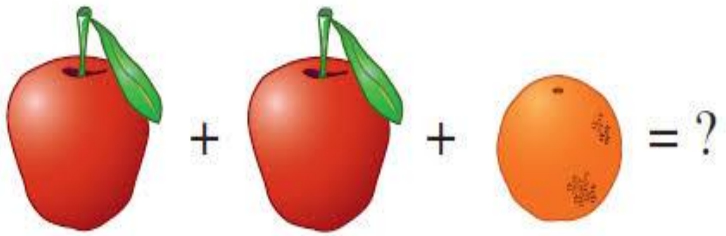
- Uses:

1. Testing the dimensional homogeneity of any equation of fluid motion.

   Ex. $v = v_0 + at$

   $LT^{-1} = LT^{-1} + (LT^{-2})T$

   $LT^{-1} = LT^{-1}$



2. Deriving equations expressed in terms of non dimensional parameters to show the relative significance of each parameter.

   $$Re = \frac{\rho\ v\ d}{\mu}$$

INTRODUCTION:

- Uses:

3. Planning model tests and presenting experimental results in a systematic manner in terms of non dimensional parameters; thus making it possible to analyze the complex fluid flow phenomenon.

PRIMARY or FUNDAMENTAL QUNTITIES:

Primary dimensions and their associated primary SI and English units

Dimension	Symbol*	SI Unit	English Unit
Mass	m	kg (kilogram)	lbm (pound-mass)
Length	L	m (meter)	ft (foot)
Time†	t	s (second)	s (second)
Temperature	T	K (kelvin)	R (rankine)
Electric current	I	A (ampere)	A (ampere)
Amount of light	C	cd (candela)	cd (candela)
Amount of matter	N	mol (mole)	mol (mole)

DERIVED or SECONDARY QUANTITIES:

- Those quantities which possess more than on fundamental dimension or quantity.

Derived quantity	Symbol	Relationship with base quantities	Derived unit	Unit in SI base unit
Area	A	Length × Length	m^2	$m \times m = m^2$
Volume	V	Length × Length × Length	m^3	$m \times m \times m = m^3$
Density	ρ	$\frac{\text{Mass}}{\text{Length} \times \text{Length} \times \text{Length}}$	$kg\ m^{-3}$	$\frac{kg}{m^3} = kg\ m^{-3}$
Velocity	v	$\frac{\text{Displacement}}{\text{Time}}$	$m\ s^{-1}$	$\frac{m}{s} = m\ s^{-1}$
Acceleration	a	$\frac{\text{Velocity}}{\text{Time}}$	$m\ s^{-2}$	$\frac{ms^{-1}}{s} = m\ s^{-2}$
Force	F	Mass × Acceleration	$kg\ m\ s^{-2}$	$kg \times m\ s^{-2} = kg\ m\ s^{-2}$

Exercise:

- Determine the dimension of

1. work
2. pressure
3. specific weight
4. kinematic viscosity
5. discharge
6. shear stress
7. angular acceleration
8. angular velocity

Exercise:

- Determine the dimension of

1.	work	$ML^{2}T^{-2}$
2.	pressure	$ML^{-1}T^{-2}$
3.	specific weight	$ML^{-2}T^{-2}$
4.	kinematic viscosity	$L^{2}T^{-1}$
5.	discharge	$L^{3}T^{-1}$
6.	shear stress	$ML^{-1}T^{-2}$
7.	angular acceleration	T^{-2}
8.	angular velocity	T^{-1}

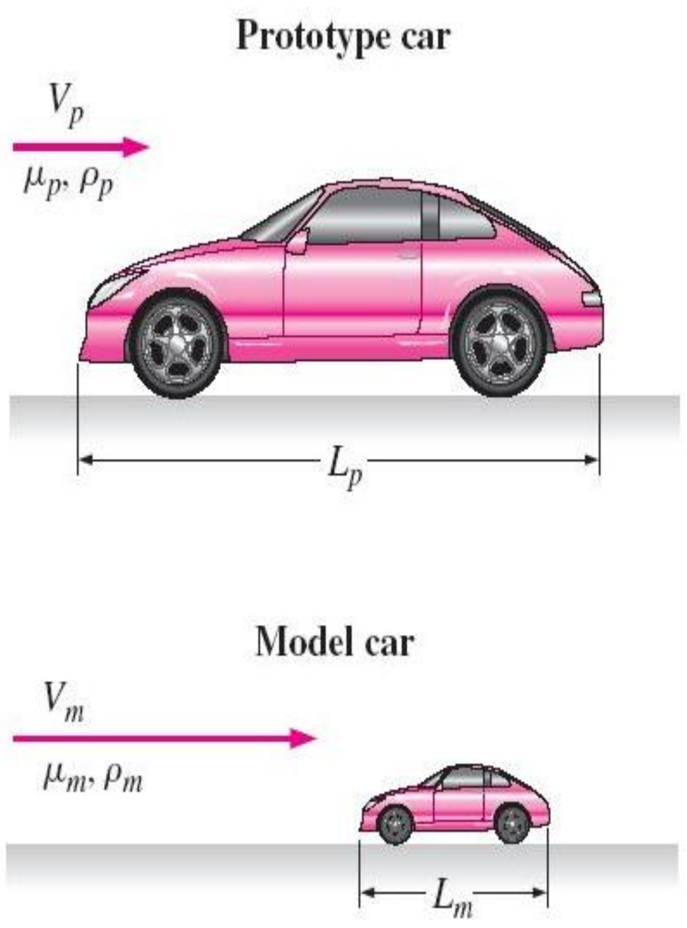
METHODS OF DIMENSIOTNAL ANALYSIS:

1. Rayleigh Method
2. Buckingham π- method

MODEL ANALYSIS:

- The **model** is the small scale replica of the actual structure or machine.
- The actual structure or machine is called **prototype**.
- The model analysis is the experimental technique of finding solution of complex problem.
- It has following advantages:

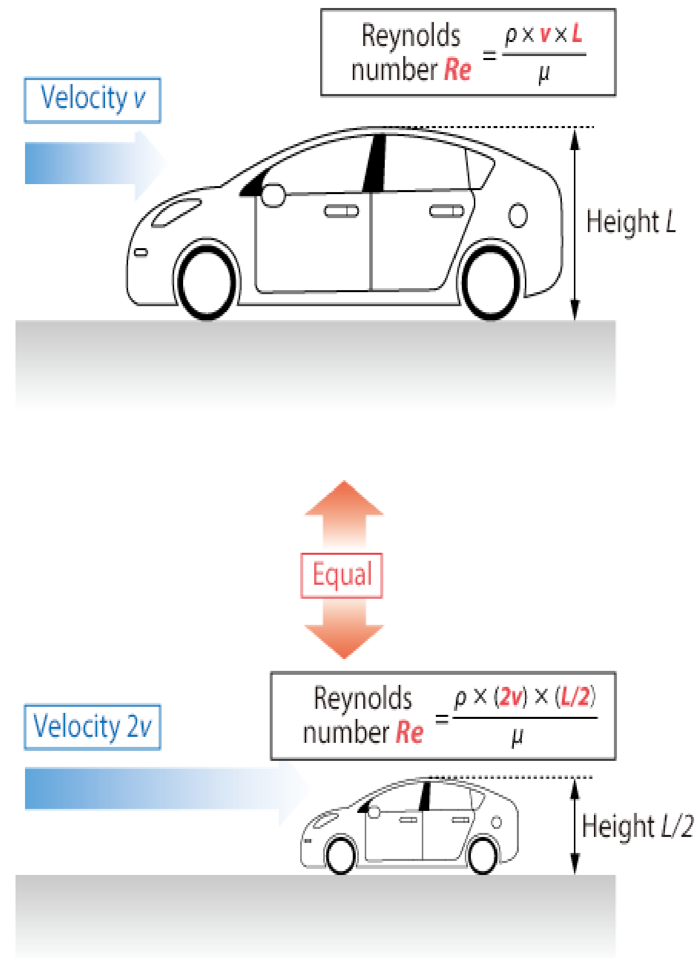
1. The performance of structure or machine can be easily predicted.
2. Relationship between the variables influencing a flow problem can be develop using dimension analysis.
3. To check similarity exists between model and prototype.



SIMILITUDE – TYPES OF SIMILARITIES:

- **Similitude** : similarity between model and prototype.
- Type of similarity:

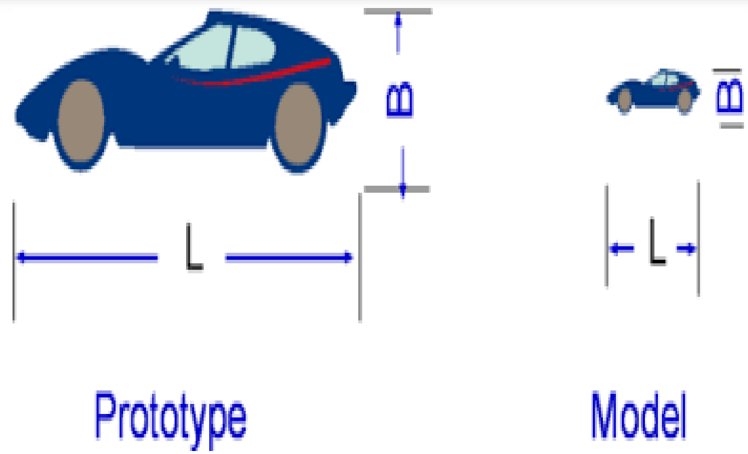
1. Geometric similarity
2. Kinematic similarity
3. Dynamic similarity



SIMILITUDE – TYPES OF SIMILARITIES:

1. Geometric similarity:

- It is exist if the ratio of all corresponding linear dimension in the model and prototype are equal.



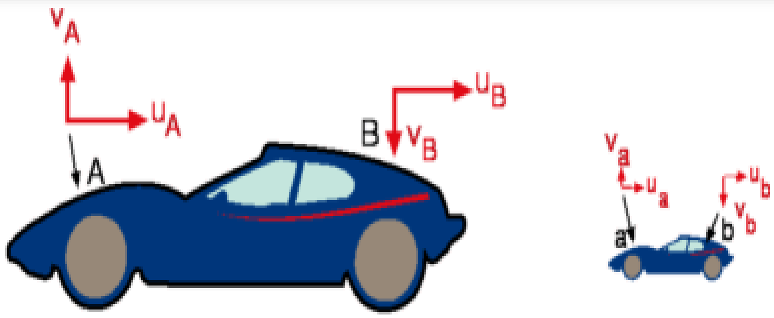
- $\frac{L_p}{L_m} = \frac{W_p}{W_m} = \frac{D_p}{D_m} = L_r$ = scale ratio

- $\frac{A_p}{A_m} = \frac{L_p}{L_m}\frac{W_p}{W_m} = L_r\,L_r$ = Area's ratio

- $\frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{W_p}{W_m}\right)^3$ = Volume's ratio

2. Kinematic Similarity:

- It means the similarity of motion between model and prototype.

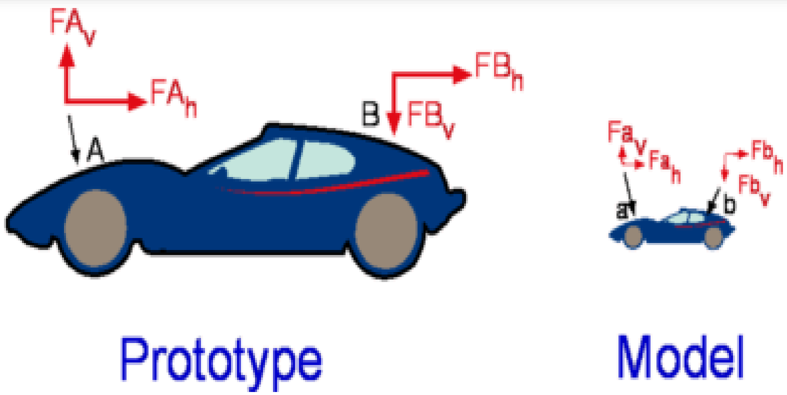


- $\frac{v_{p1}}{v_{m1}} = \frac{v_{p2}}{v_{m2}} =$ v_r = Velocity ratio
- $\frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} =$ a_r = Acceleration ratio

3. Dynamic Similarity:

- It means the similarity of forces between model and prototype.

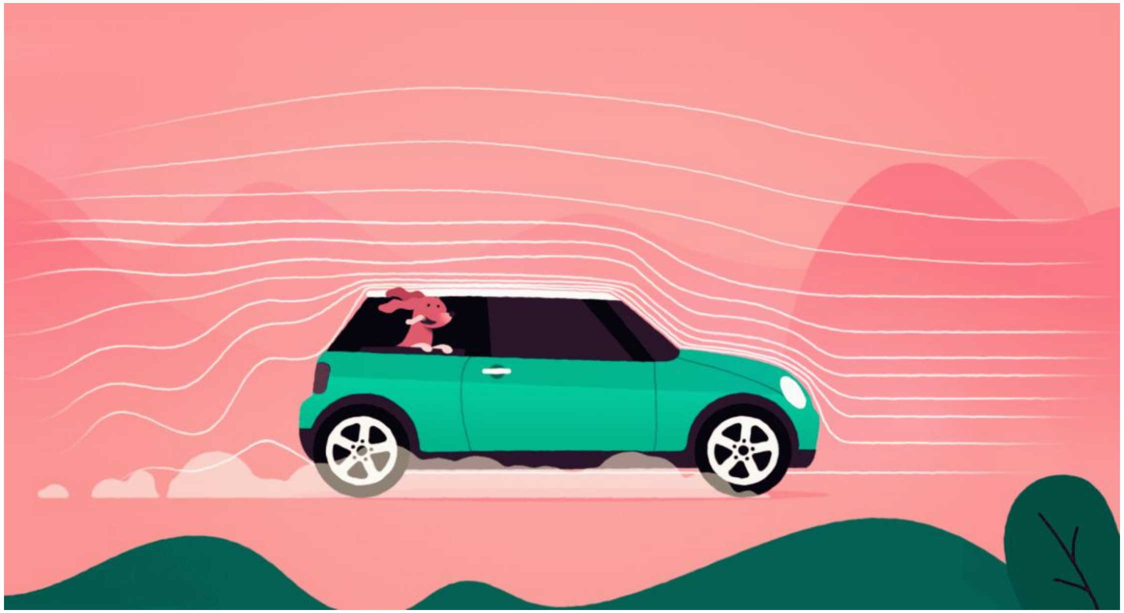
- $\frac{(Fi)_p}{(F_i)_m} = \frac{(Fv)_p}{(F_v)_m} = \frac{(Fg)_p}{(F_g)_m}$ = Force ratio



TYPES OF FORCES ACTING ON FLUID:

- The forces are

1. Inertia force, F_i
2. Viscous force, F_v
3. Gravity force, F_g
4. Pressure force, F_p
5. Surface tension force, F_s
6. Elastic force, F_e



TYPES OF FORCES ACTING ON FLUID:

1. **Inertia force, F_i :**
- It is equal to the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.

2. **Viscous force, F_v :**
- It is equal to the product of shear stress due to viscosity and surface area of the flow.

3. **Gravity force, F_g :**
- It is equal to the product of mass and acceleration due to gravity of the flowing fluid.

4. **Pressure force, F_p :**
- It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid.

5. **Surface tension force, F_s :**
- It is equal to the product of surface tension and length of surface of the flowing fluid.

6. **Elastic force, F_e :**
- It is equal to product of elastic stress and area of the flowing fluid.

DIMENSIONLESS NUMBERS:

1. Reynold’s number
2. Froude’s number
3. Euler’s number
4. Weber’s number
5. Mach’s number

1. Reynold Number:

- Ratio of inertia force of a flowing fluid and viscous force of the fluid.
- $Re = F_i / F_v$

- Inertia force = Mass * Acceleration of flowing fluid

  = Density * Volume * Velocity/time

  $= \rho * Q * v$

  $= \rho * A * v * v$

  $= \rho * A * v^2$

- Viscous force = Shear stress * Area ($\tau = \mu\ du/dy$)

  $= \mu\ du/dy * A$

  $= \mu\ v/L * A$

1. Reynold Number:

- Ratio of inertia force of a flowing fluid and viscous force of the fluid.

$$Re = F_i / F_v$$

$$= \frac{\rho A v^2}{\mu \left(\frac{v}{L}\right) A}$$

$$= \frac{\rho v L}{\mu}$$

- Higher the Re, greater the inertia effect. Smaller the Re, greater the viscous effect.
- **Examples of such situation**:

  I. Flow of incompressible fluid in a pipe

  II. Motion of submarine

2. Froude's Number:

- It is defined as the square root of the ratio of the inertia force and gravitational force.
- $$Fr = \sqrt{\frac{\text{Inertia force}}{\text{gravitational force}}}$$

$$= \sqrt{\frac{\rho A v^2}{\rho A L g}}$$

$$= \sqrt{\frac{v^2}{Lg}}$$

$$= \frac{v}{\sqrt{Lg}}$$

- **Example of such situation:**

  I. Flow of liquid jet from the orifice

  II. Flow over notches, weirs of a dam

3. Euler's Number:

- It is defined as the square root of the ratio of the inertia force and pressure force.
- $$Fr = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}$$

$$= \sqrt{\frac{\rho A v^2}{p\ A}}$$

$$= \sqrt{\frac{v^2}{p/\rho}}$$

$$= \frac{v}{\sqrt{p/\rho}}$$

- **Example of such situation:**

  I. Flow through pipe

  II. Discharge through orifice and mouthpieces

4. Weber's Number:

- It is defined as the square root of the ratio of the inertia force and surface tension force.
- Fr = $\sqrt{\dfrac{\text{Inertia force}}{\text{Surface tension force}}}$

  $= \sqrt{\dfrac{\rho A v^2}{\sigma L}}$

  $= \sqrt{\dfrac{\rho L^2 v^2}{\sigma L}}$

  $= \dfrac{v}{\sqrt{\sigma/\rho L}}$
- **Example of such situation:**

  I. Capillarity tube action

  II. Flow of blood in veins and arteries

5. Mach Number:

- It is defined as the square root of the ratio of the inertia force and elastic force.
- Fr = $\sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}}$

$$= \sqrt{\frac{\rho A v^2}{K\ L^2}}$$

$$= \sqrt{\frac{\rho L^2 v^2}{K\ L^2}}$$

$$= \frac{v}{\sqrt{K/\rho}}$$

$$= \frac{v}{C}$$, $\sqrt{K/\rho}$ = velocity of sound in the fluid

- **Example of such situation:**

  I. Aerodynamic testing

  II. Water hammer problem

MODEL LAWS OR SIMILARITY LAWS:

- The laws on which the models are designed for dynamic similarity are called **model laws** or **laws of similarity**.
- Followings are the model laws:

1. Reynold's model law
2. Froude model law
3. Euler model law
4. Weber model law
5. Mach model law

1. Reynold's Model Law:

- It is law in which models are based on Reynold's number.
- Model based on Reynold's number includes:
  1. Pipe flow
  2. Resistance experience by sub-marines, airplanes, fully immersed bodies etc.
- Fluid flow problems where **viscous forces** alone are predominant, the models are designed for dynamic similarity on Reynold's law.
- It states that **"The Reynold's number for the model must be equal to the Reynold's number for the prototype"** .

$$[Re]_m = [Re]_p$$

$$\frac{\rho_m v_m L_m}{\mu_m} = \frac{\rho_p v_p L_p}{\mu_p}$$

$$\frac{\rho_p v_p L_p}{\rho_m v_m L_m} * \frac{1}{\mu_p/\mu_m} = 1$$

$$\frac{\rho_r v_r L_r}{\mu_r} = 1$$

Where,

ρ_r = scale ratio for density
v_r = scale ratio for velocity
L_r = scale ratio for linear dimension
μ_r = scale ratio for viscosity

1. Reynold's Model Law:

- Time scale ratio is:

$$t_r = \frac{L_r}{v_r}$$

- Acceleration scale ratio is:

$$a_r = \frac{v_r}{t_r}$$

- Force scale ratio is:

$$F_r = m_r * a_r$$

$$= \rho_r A_r v_r * a_r$$

$$= \rho_r L^2{}_r v_r * a_r$$

- Discharge scale ratio is:

$$Q_r = \rho_r A_r v_r$$

$$= \rho_r L^2{}_r v_r$$

2. Froude Model Law:

- It is law in which models are based on Froude number.
- It is applicable to following fluid flow problems:
  1. Free surface flow such as flow over weirs, channels etc
  2. Flow of jet from an orifice or nozzle
  3. Where waves are likely to be formed over one another
  4. Where fluids of different densities flow over one another
- The law is applicable when the **gravity force** is only predominant force which controls the flow.

$$[Fe]_m = [Fe]_p$$

$$\frac{V_m}{\sqrt{L_m g_m}} = \frac{V_p}{\sqrt{L_p g_p}}$$

- If test performed on the same place then $g_m = g_p$ equation becomes

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}} \qquad \sqrt{\frac{L_p}{L_m}} = \frac{V_p}{V_m} \qquad \frac{V_p}{V_m} = V_r = \sqrt{L_r}$$

2. Froude Model Law:

- **Scale ratio** for various physical quantities based on Froude model law are:

1. **Scale ratio for time**:

$$T_r = \frac{T_p}{T_m} = \frac{\frac{L_p}{v_p}}{\frac{L_m}{v_p}} = \frac{L_p}{L_m} * \frac{v_m}{v_p} = L_r * \frac{1}{\sqrt{L_r}} = \sqrt{L_r}$$

2. **Scale ratio for acceleration**:

$$a_r = \frac{a_p}{a_m} = \frac{\frac{v_p}{T_p}}{\frac{v_m}{T_p}} = \sqrt{L_r} * \frac{1}{\sqrt{L_r}} = 1$$

3. **Scale ratio for discharge**:

$$Q = A * v = L^2 * \frac{L}{T} = \frac{L^3}{T}$$

$$Q_r = \frac{Q_p}{Q_m} = \frac{\left(\frac{L^3}{T}\right)_p}{\left(\frac{L^3}{T}\right)_m} = L^3{}_r * \frac{1}{\sqrt{L_r}} = L^{2.5}{}_r$$

2. Froude Model Law:

4. Scale ratio for force:

$$F = m * a = \rho * V * a = \rho L^3 * \frac{v}{T} = \rho L^2 * \frac{L}{T} * v = \rho L^2 v^2$$

$$F_r = \frac{F_P}{F_m} = \frac{\rho_p L_p{}^2 v_p{}^2}{\rho_m L_m{}^2 v_m{}^2}$$

$$= \frac{\rho_p}{\rho_m} * \left(\frac{L_p}{L_m}\right)^2 * \left(\frac{v_p}{v_m}\right)^2 \quad \{ \text{if fluid is same } \rho_p = \rho_m$$

$$= \left(\frac{L_p}{L_m}\right)^2 * \left(\frac{v_p}{v_m}\right)^2$$

$$= L_r{}^3$$

Similarly,

Scale ratio for pressure $p_r = L_r$

Torque $T_r = L_r{}^4$

Power $P_r = L_r{}^{3.5}$

Hydraulic model:

- There are two types (i) undistorted model (ii) distorted model
- If the models are geometrically similar to its prototype, the models are known as undistorted model.
- If the models are having different ratio for the horizontal and vertical dimensions, the models are known as distorted model.

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