*Last Updated on: **19-02-2020***

Applied Fluid Mechanics
(2160602)

Module-1

Turbulent Flow

Prof. Mehul Pujara

mehul.pujara@darshan.ac.in
98795 10743

Dedicated Faculty, Committed Education
Darshan Institute of Engineering & Technology

1. Reynold Number:

- Ratio of inertia force of a flowing fluid and viscous force of the fluid.
- $Re = F_i / F_v$

- Inertia force = Mass * Acceleration of flowing fluid

  = Density * Volume * Velocity/time

  $= \rho * Q * v$

  $= \rho * A * v * v$

  $= \rho * A * v^2$
- Viscous force = Shear stress * Area ($\tau = \mu\, du/dy$)

  $= \mu\, du/dy * A$

  $= \mu\, v/L * A$

1. Reynold Number:

- Ratio of inertia force of a flowing fluid and viscous force of the fluid.

$$Re = F_i / F_v$$

$$= \frac{\rho A v^2}{\mu \left(\frac{v}{L}\right) A}$$

$$= \frac{\rho v L}{\mu}$$

- Higher the Re, greater the inertia effect. Smaller the Re, greater the viscous effect.
- **Examples of such situation**:

I. Flow of incompressible fluid in a pipe

II. Motion of submarine

Shear Stress in Turbulent Flow:

- The shear stress in viscous flow is given by Newton's law of viscosity as

$$\tau_v = \mu \frac{du}{dy}, \qquad \tau_v = \text{shear stress due to viscosity}$$

- Similarly J. Boussinesq has given turbulent shear stress in mathematical form as

$$\tau_t = \eta \frac{d\bar{u}}{dy}, \qquad \tau_t = \text{shear stress due to turbulence}$$

$$\eta = \text{eddy viscosity}$$

- The ratio of η and ρ is known as kinematic eddy viscosity.

$$\varepsilon = \frac{\eta}{\rho}$$

Shear Stress in Turbulent Flow:

- Reynold in 1866 has suggested the turbulent shear stress as

$$\tau = \rho\, \acute{u}\, \acute{v},$$

$\acute{u}\ \acute{v}$ = fluctuating component of velocity in the direction of x and y due to turbulence

- The average shear stress is given by

$$\bar{\tau} = \overline{\rho u v}$$

The above turbulent shear stress is known as Renold stress.

Prandtl Mixing Length Theory:

- Turbulent stress can be calculated from the value of $\acute{u}\ \acute{v}$ is known.

$$\tau = \rho\, \acute{u}\, \acute{v},$$

- But it is very difficult to measure $\overline{uv}$

$$\bar{\tau} = \overline{\rho u v}$$

- The above difficulty is solved by L. Prandtl in 1925 by introducing mixing length hypothesis.
- **The mixing length l**, is that distance between two layers in the transverse direction such that the lumps of fluid particles from one layer could reach the other layer and the particles are mixed in the other layer in such a way that the momentum of the particles in the direction of x is same.
- The value of $\acute{u}\ \acute{v}$ is given as

$$\acute{u} = l \frac{du}{dy}$$

$$\acute{v} = l \frac{du}{dy}$$

Prandtl Mixing Length Theory:

- The value of $\overline{uv}$ is given as

$$\overline{uv} = l\,\frac{du}{dy} * l\,\frac{du}{dy}$$

$$= l^2\left(\frac{du}{dy}\right)^2$$

- The value of turbulent stress is

$$\bar{\tau} = \rho\, l^2\left(\frac{du}{dy}\right)^2$$

- The total shear stress at any point in turbulent flow is given by

$$\tau \text{ or } \bar{\tau} = \mu\frac{du}{dy} + \rho\, l^2\left(\frac{du}{dy}\right)^2$$

Velocity distribution in turbulent flow:

- In case of turbulent flow, the total shear stress at any point is the sum of viscous shear stress and turbulent shear stress.
- Also the viscous shear stress is negligible except near the boundary. So shear stress is calculated using equation,

$$\tau \text{ or } \bar{\tau} = \rho\, l^2 \left(\frac{du}{dy}\right)^2$$

- From this equation, the velocity distribution can be obtained if the relation between l, the mixing length and y is known.
- Prandtl assumed that the mixing length, l is a linear function of the distance y from the pipe wall i.e., **$l = ky$**, where k is a constant, known as Karman constant and = 0.4

$$\tau \text{ or } \bar{\tau} = \rho\, (ky)^2 \left(\frac{du}{dy}\right)^2$$

$$\tau \text{ or } \bar{\tau} = \rho\, k^2 y^2 \left(\frac{du}{dy}\right)^2$$

Velocity distribution in turbulent flow:

- Rewriting the below equation,

$$\tau \text{ or } \bar{\tau} = \rho\, k^2 y^2 \left(\frac{du}{dy}\right)^2$$

$$\left(\frac{du}{dy}\right)^2 = \tau / \rho\, k^2 y^2$$

$$\frac{du}{dy} = \sqrt{\tau / \rho\, k^2 y^2}$$

$$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau}{\rho}}$$

- For small values of y that is very close to the boundary of the pipe, Prandtl assumed shear stress τ to be constant and approximately equal to τ_0 which presents the turbulent shear stress at the pipe boundary. Substituting $\tau = \tau_0$ in above equation

$$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau_0}{\rho}}$$

Velocity distribution in turbulent flow:

$$\frac{du}{dy} = \frac{1}{ky}\sqrt{\frac{\tau_0}{\rho}}$$

- In above equation the dimension of term $\sqrt{\frac{\tau_0}{\rho}}$ is $\frac{L}{T}$ which is velocity dimension. So,

$$\sqrt{\frac{\tau_0}{\rho}} = \textbf{shear velocity} = u_*$$

- So,

$$\frac{du}{dy} = \frac{1}{ky} u_*$$

- Integrating above equation,

$$u = \frac{u_*}{k} \ln y + C$$

Velocity distribution in turbulent flow:

- Equation shows that in turbulent flow, the velocity varies directly with the logarithm of the distance from the boundary.
- In other words the velocity distribution in turbulent flow is logarithmic in nature.
- To determine the constant of integration, *C* the boundary condition that at y = *R* (radius of pipe), $u = u_{max}$ is substituted in equation

$$u_{max} = \frac{u_*}{k} \ln R + C$$

$$\therefore C = u_{max} - \frac{u_*}{k} \ln R$$

- Putting the value of C in equation $u = \frac{u_*}{k} \ln y + C$

$$u = \frac{u_*}{k} \ln y + u_{max} - \frac{u_*}{k} \ln R$$

$$= u_{max} + \frac{u_*}{k} \ln (y/R)$$

Velocity distribution in turbulent flow:

$$u = \frac{u_*}{k} \ln y + u_{max} - \frac{u_*}{k} \ln R$$

$$= u_{max} + \frac{u_*}{k} \ln (y/R)$$

- Putting the value of k= 0.4 = Karman constant

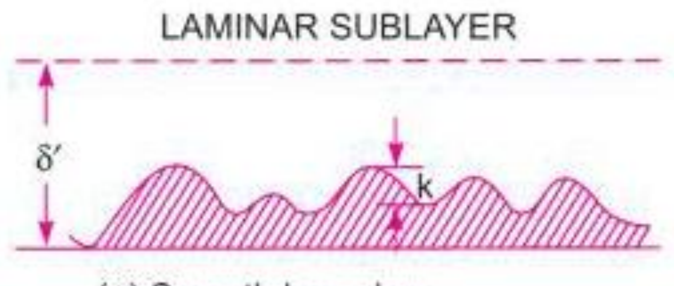
$$u = u_{max} + \frac{u_*}{0.4} \ln (y/R)$$

$$u = u_{max} + 2.5\, u_* \ln (y/R)$$

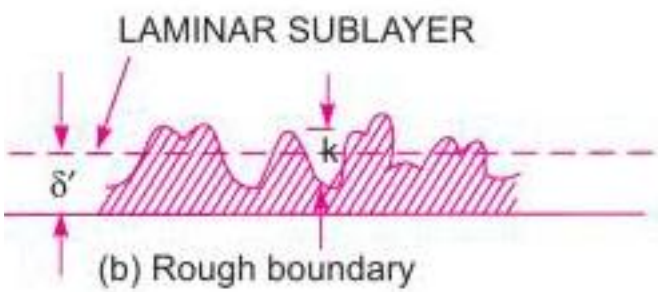
- Equation is called 'Prandtl's universal velocity distribution equation for turbulent flow in pipes. This equation is applicable to smooth as well as rough pipe boundaries.

Hydrodynamically Smooth and Rough Boundaries :

- Let k is the average height of the irregularities projecting from the surface of a boundary as shown in Figure
- If the value of k is large for a boundary then the boundary is called rough boundary
- if the value of k is less, then boundary is known as smooth boundary.
- in general. This is the classification of rough and smooth boundary based on boundary characteristics. But for proper classification, the flow and fluid characteristics are also to be considered.



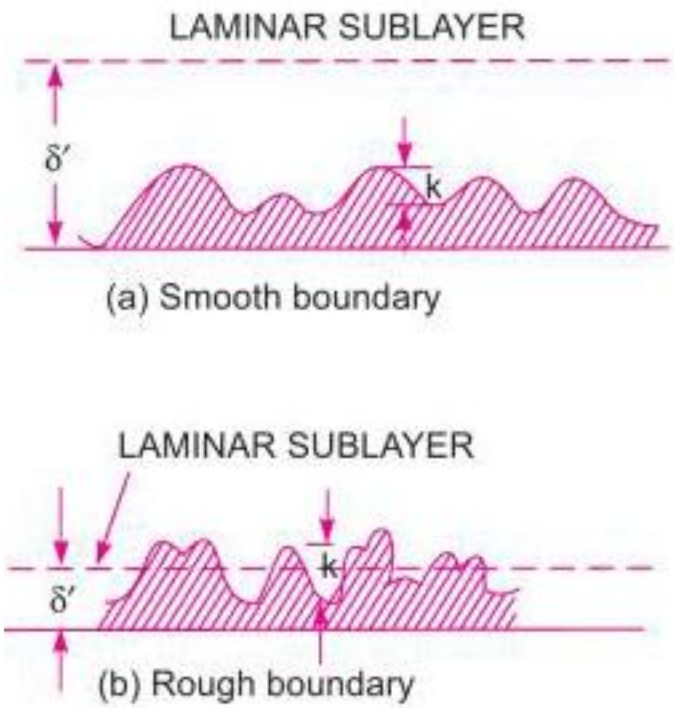
(a) Smooth boundary



(b) Rough boundary

Hydrodynamically Smooth and Rough Boundaries :

- For turbulent flow analysis along a boundary, the flow is divided in two portions.
- The first portion consists o f a thin layer of fluid in the immediate neighbourhood of the boundary, where viscous shear stress predominates while the shear stress due to turbulence is negligible. This portion is known as **laminar sub-layer**.
- The second portion of flow, where shear stress due to turbulence are large as compared to viscous stress is known as **turbulent zone**.



References:

1. Fluid Mechanics and Fluid Power Engineering by D.S. Kumar, S.K.Kataria & Sons
2. Fluid Mechanics and Hydraulic Machines by R.K. Bansal, Laxmi Publications
3. Fluid Mechanics and Hydraulic Machines by R.K. Rajput, S.Chand & Co
4. Fluid Mechanics; Fundamentals and Applications by John. M. Cimbala Yunus A. Cengel, McGraw-Hill Publication