**Single - Phase AC Circuits**

### 2.1 Equation for generation of alternating induce EMF

- An AC generator uses the principle of Faraday’s electromagnetic induction law. It states that when current carrying conductor cut the magnetic field then emf induced in the conductor.
- Inside this magnetic field a single rectangular loop of wire rotes around a fixed axis allowing it to cut the magnetic flux at various angles as shown below figure 2.1.

Where,
- \( N \) = No. of turns of coil
- \( A \) = Area of coil (m^2)
- \( \omega \) = Angular velocity (radians/second)
- \( \phi_m \) = Maximum flux (wb)

![Figure 2.2.1 Generation of EMF](image)

- When coil is along XX’ (perpendicular to the lines of flux), flux linking with coil = \( \phi_m \). When coil is along YY’ (parallel to the lines of flux), flux linking with the coil is zero. When coil is making an angle \( \theta \) with respect to XX’ flux linking with coil, \( \phi = \phi_m \cos \omega t [\theta = \omega t] \).

![Figure 2.2 Alternating Induced EMF](image)

- According to Faraday’s law of electromagnetic induction,

\[
e = -N \frac{d\phi}{dt} = -Nd \left( \frac{\phi_m \cos \omega t}{dt} \right)
\]

\[
e = -N\phi_m (-\sin \omega t )\times \omega
\]

\[
e = N\phi_m \omega \sin \omega t
\]

\[
e = E_m \sin \omega t
\]

Where,
- \( E_m = N\phi_m \omega \)
- \( N \) = no. of turns of the coil
- \( \phi_m = B_m A \)
- \( B_m = Maximum \ flux \ density \ (wb/m^2) \)
- \( A \) = Area of the coil (m^2)
- \( \omega = 2\pi f \)
\[ e = N B_m A 2\pi f \sin \omega t \]

- Similarly, an alternating current can be express as
  \[ i = I_m \sin \omega t \]
  Where, \( I_m \) = Maximum values of current

- Thus, both the induced emf and the induced current vary as the sine function of the phase angle \( \omega t = 0 \). Shown in figure 2.3.

\[ \omega t = 0^\circ \quad e = 0 \\
\omega t = 90^\circ \quad e = E_m \\
\omega t = 180^\circ \quad e = 0 \\
\omega t = 270^\circ \quad e = -E_m \\
\omega t = 360^\circ \quad e = 0 \]

![Figure 2.3 Waveform of Alternating Induced EMF](image)

### 2.2 Definitions

- **Waveform**
  It is defined as the graph between magnitude of alternating quantity (on Y axis) against time (on X axis).

![Figure 2.4 A.C. Waveforms](image)

- **Cycle**
  It is defined as one complete set of positive, negative and zero values of an alternating quantity.
- **Instantaneous value**
  It is defined as the value of an alternating quantity at a particular instant of given time. Generally denoted by small letters.
  - e.g. \( i \) = Instantaneous value of current
  - \( v \) = Instantaneous value of voltage
  - \( p \) = Instantaneous values of power

- **Amplitude/ Peak value/ Crest value/ Maximum value**
  It is defined as the maximum value (either positive or negative) attained by an alternating quantity in one cycle. Generally denoted by capital letters.
  - e.g. \( I_m \) = Maximum Value of current
  - \( V_m \) = Maximum value of voltage
  - \( P_m \) = Maximum values of power

- **Average value**
  It is defined as the average of all instantaneous value of alternating quantities over a half cycle.
  - e.g. \( V_{ave} \) = Average value of voltage
  - \( I_{ave} \) = Average value of current

- **RMS value**
  It is the equivalent dc current which when flowing through a given circuit for a given time produces same amount of heat as produced by an alternating current when flowing through the same circuit for the same time.
  - e.g. \( V_{rms} \) = Root Mean Square value of voltage
  - \( I_{rms} \) = Root Mean Square value of current

- **Frequency**
  It is defined as number of cycles completed by an alternating quantity per second. Symbol is \( f \). Unit is Hertz (Hz).

- **Time period**
  It is defined as time taken to complete one cycle. Symbol is \( T \). Unit is seconds.

- **Power factor**
  It is defined as the cosine of angle between voltage and current. Power Factor = \( pf = \cos \phi \), where \( \phi \) is the angle between voltage and current.

- **Active power**
  It is the actual power consumed in any circuit. It is given by product of rms voltage and rms current and cosine angle between voltage and current. (\( VI \cos \phi \)).
  - Active Power = \( P = I^2R = VI \cos \phi \).
  - Unit is Watt (W) or kW.
- **Reactive power**
  The power drawn by the circuit due to reactive component of current is called as reactive power. It is given by product of rms voltage and rms current and sine angle between voltage and current \((VI \sin \phi)\).

  Reactive Power \(Q = I^2X = VI\sin \phi\).
  Unit is VAR or kVAR.

- **Apparent power**
  It is the product of rms value of voltage and rms value of current. It is total power supplied to the circuit.

  Apparent Power \(S = VI\).
  Unit is VA or kVA.

- **Peak factor/ Crest factor**
  It is defined as the ratio of peak value (crest value or maximum value) to rms value of an alternating quantity.

  Peak factor \(Kp = 1.414\) for sine wave.

- **Form factor**
  It is defined as the ratio of rms value to average value of an alternating quantity. Denoted by \(Kf\). Form factor \(Kf = 1.11\) for sine wave.

- **Phase difference**
  It is defined as angular displacement between two zero values or two maximum values of the two-alternating quantity having same frequency.

- **Leading phase difference**
  A quantity which attains its zero or positive maximum value before the compared to the other quantity.

- **Lagging phase difference**
  A quantity which attains its zero or positive maximum value after the other quantity.
2.3 Derivation of average value and RMS value of sinusoidal AC signal

➢ Average Value

**Graphical Method**

![Graphical Method for Average Value](image)

**Analytical Method**

![Analytical Method for Average Value](image)

\[
V_{ave} = \frac{\text{Sum of All Instantaneous Values}}{\text{Total No. of Values}}
\]

\[
V_{ave} = \frac{v_1 + v_2 + v_3 + v_4 + v_5 + \ldots + v_{10}}{10}
\]

\[
V_{ave} = \frac{\int_0^\pi V_m \sin \omega t \, d\omega t}{\pi}
\]

\[
V_{ave} = \frac{V_m}{\pi} \left( -\cos \omega t \right)_0^{\pi}
\]

\[
V_{ave} = -\frac{V_m}{\pi} (\cos \pi - \cos 0)
\]

\[
V_{ave} = \frac{2V_m}{\pi}
\]

\[
V_{ave} = 0.637 \, V_m
\]
## RMS Value

### Graphical Method

![Graphical Method for RMS Value](image)

\[
V_{rms} = \sqrt{\frac{\text{Sum of all sq. of instantaneous values}}{\text{Total No. of Values}}}
\]

\[
V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2 + \ldots + V_{10}^2}{10}}
\]

### Analytical Method

![Analytical Method for RMS Value](image)

\[
V_{rms} = \sqrt{\frac{\text{Area under the sq. curve}}{\text{Base of the curve}}}
\]

\[
V_{rms} = \sqrt{\frac{\int_0^{2\pi} V_m^2 \sin^2 \omega t \, d\omega t}{2\pi}}
\]

\[
V_{rms} = \frac{V_m^2}{2} \int_0^{2\pi} (1 - \cos 2\omega t) \, d\omega t
\]

\[
V_{rms} = \frac{V_m^2}{4\pi} \left[ \omega t \int_0^{2\pi} - \frac{(\sin 2\omega t)^2}{2} \right]_0
\]

\[
V_{rms} = \frac{V_m}{\sqrt{2}} - 0
\]

\[
V_{rms} = 0.707 \times V_m
\]
2.4 Phasor Representation of Alternating Quantities

- Sinusoidal expression given as: \( v(t) = V_m \sin (\omega t \pm \Phi) \) representing the sinusoid in the time-domain form.
- Phasor is a quantity that has both "Magnitude" and "Direction".

![Figure 2.10 Phasor Representation of Alternating Quantities](image_url)

**Phase Difference of a Sinusoidal Waveform**

- The generalized mathematical expression to define these two sinusoidal quantities will be written as:
  \[
  v = V_m \sin \omega t \\
  i = I_m \sin (\omega t - \Phi)
  \]

![Figure 2.11 Wave Forms of Voltage & Current](image_url)

- As shown in the above voltage and current equations, the current, \( i \) is lagging the voltage, \( v \) by angle \( \Phi \).
- So, the difference between the two sinusoidal quantities representing in waveform shown in Fig. 2.11 & phasors representing the two sinusoidal quantities is angle \( \Phi \) and the resulting phasor diagram shown in Fig. 2.12.
2.5 Purely Resistive Circuit

- The Fig. 2.13 an AC circuit consisting of a pure resistor to which an alternating voltage $v_t = V_m \sin \omega t$ is applied.

Circuit Diagram

![Circuit Diagram](image)

Equations for Voltage and Current

- As show in the Fig. 2.13 voltage source
  
  $v_t = V_m \sin \omega t$

- According to ohm's law
  
  $i_t = \frac{v_t}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$

- From above equations it is clear that current is in phase with voltage for purely resistive circuit.

Waveforms and Phasor Diagram

- The sinewave and vector representation of $v_t = V_m \sin \omega t$ & $i_t = I_m \sin \omega t$ are given in Fig. 2.14 & 2.15.

![Waveform](image)

![Phasor Diagram](image)
Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

**Instantaneous power**

\[ p(t) = v \times i \]

\[ p(t) = V_m \sin \omega t \times I_m \sin \omega t \]

\[ p(t) = V_m I_m \sin^2 \omega t \]

\[ p(t) = \frac{V_m I_m (1 - \cos 2\omega t)}{2} \]

**Average Power**

\[
P_{\text{ave}} = \frac{1}{2} \int_{0}^{2\pi} V_m I_m (1 - \cos 2\omega t) \, d\omega t
\]

\[
P_{\text{ave}} = \frac{V_m I_m}{4\pi} \left[ (\omega t)_{2\pi} - (\frac{\sin 2\omega t}{2})_{0} \right]_{0}^{2\pi}
\]

\[
P_{\text{ave}} = \frac{V_m I_m}{4\pi} \left[ (2\pi - 0) - (0 - 0) \right]
\]

\[
P_{\text{ave}} = \frac{V_m I_m}{2}
\]

\[
P_{\text{ave}} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}
\]

\[
P_{\text{ave}} = V_{\text{rms}} I_{\text{rms}}
\]

\[
P_{\text{ave}} = VI
\]

- The average power consumed by purely resistive circuit is multiplication of \( V_{\text{rms}} \) & \( I_{\text{rms}} \).

2.6 **Purely Inductive Circuit**

- The Fig. 2.16 an AC circuit consisting of a pure Inductor to which an alternating voltage \( v(t) = V_m \sin \omega t \) is applied.

**Circuit Diagram**

![Figure 2.16 Pure Inductor Connected to AC Supply](image)
Equations for Voltage and Current

- As shown in the Fig. 2.16 voltage source
  \[ v_i = V_m \sin \omega t \]

- Due to self-inductance of the coil, there will be emf induced in it. This back emf will oppose the instantaneous rise or fall of current through the coil, it is given by
  \[ e_b = -L \frac{di}{dt} \]

- As, circuit does not contain any resistance, there is no ohmic drop and hence applied voltage is equal and opposite to back emf.

\[ v_i = -e_b \]
\[ v_i = -L \frac{di}{dt} \]
\[ v_i = L \frac{di}{dt} \]
\[ V_m \sin \omega t = L \frac{di}{dt} \]
\[ di = \frac{V_m \sin \omega t}{L} \]

- Integrate on both the sides,

\[ \int di = V_m \int \sin \omega t \ dt \]
\[ i_i = V_m \left( \frac{- \cos \omega t}{\omega} \right) \]
\[ i_i = -\frac{V_m}{\omega L} \cos \omega t \]
\[ i_i = I_m \sin(\omega t - 90^\circ) \quad (\because \frac{V_m}{\omega L} = I_m) \]

- From the above equations it is clear that the current lags the voltage by 90° in a purely inductive circuit.

Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

**Instantaneous Power**

\[ p_i = v \times i \]
\[ p_i = V_m \sin \omega t \times I_m \sin (\omega t - 90^\circ) \]
\[ p_i = V_m \sin \omega t \times (-I_m \cos \omega t) \]
\[ p_i = -2V_m I_m \sin \omega t \cos \omega t \]
\[ p_i = \]

- Figure 2.17 Waveform of Voltage & Current for Pure Inductor
- Figure 2.18 Phasor Diagram of Voltage & Current for Pure Inductor
\[ P_r = \frac{-V_m I_m \sin 2\omega t}{2} \]

### Average Power

\[
P_{ave} = \frac{1}{2\pi} \int_{0}^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t
\]

\[
P_{ave} = -\frac{V_m I_m}{4\pi} \left[ -\cos 2\omega t \right]_{0}^{2\pi}
\]

\[
P_{ave} = \frac{V_m I_m}{8\pi} \left[ \cos 4\pi - \cos 0 \right]
\]

\[
P_{ave} = 0
\]

- The average power consumed by purely inductive circuit is zero.

#### 2.7 Purely Capacitive Circuit

- The Fig. 2.19 shows a capacitor of capacitance \( C \) farads connected to an a.c. voltage supply \( v_t = V_m \sin \omega t \).

### Circuit Diagram

\[ v_t = V_m \sin \omega t \]

---

### Equations for Voltage & Current

- As show in the Fig. 2.19 voltage source

\[ v_t = V_m \, Sin \, \omega t \]

- A pure capacitor having zero resistance. Thus, the alternating supply applied to the plates of the capacitor, the capacitor is charged.

- If the charge on the capacitor plates at any instant is ‘\( q \)’ and the potential difference between the plates at any instant is ‘\( v_t \)’ then we know that,

\[ q = CV_t \]

\[ q = CV_m \sin \omega t \]

- The current is given by rate of change of charge.

\[ i_t = \frac{dq}{dt} \]

\[ i_t = \frac{dCV_m \sin \omega t}{dt} \]
\[i_i = \omega CV_m \sin \omega t\]
\[i_i = \frac{V_m}{1/\omega C} \cos \omega t\]
\[i_i = \frac{V_m}{X_c} \cos \omega t\]
\[i_i = I_m \sin(\omega t + 90^\circ) \quad (\because \frac{V_m}{X_c} = I_m)\]

- From the above equations it is clear that the current leads the voltage by 90° in a purely capacitive circuit.

**Waveform and Phasor Diagram**

\[\phi = +90^\circ\]

\[\text{Figure 2.20 Waveform of Voltage & Current for Pure Capacitor} \quad \text{Figure 2.21 Phasor Diagram of Voltage & Current for Pure Capacitor}\]

**Power**

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

**Instantaneous Power**

\[p_{(t)} = v \times i\]
\[p_{(t)} = V_m \sin \omega t \times I_m \sin (\omega t + 90^\circ)\]
\[p_{(t)} = V_m \sin \omega t \times I_m \cos \omega t\]
\[p_{(t)} = \frac{V_m I_m \sin \omega t \cos \omega t}{2}\]
\[p_{(t)} = \frac{V_m I_m}{2} \sin 2\omega t\]

**Average Power**

\[P_{ave} = \frac{1}{2\pi} \int_0^{2\pi} V_m I_m \sin 2\omega t \, d\omega t\]
The average power consumed by purely capacitive circuit is zero.

2.8 Series Resistance-Inductance (R-L) Circuit

- Consider a circuit consisting of a resistor of resistance \( R \) ohms and a purely inductive coil of inductance \( L \) henry in series as shown in the Figure 2.22.

In the series circuit, the current \( i_t \) flowing through \( R \) and \( L \) will be the same.

But the voltage across them will be different. The vector sum of voltage across resistor \( V_R \) and voltage across inductor \( V_L \) will be equal to supply voltage \( v_t \).

Waveforms and Phasor Diagram

- The voltage and current waves in R-L series circuit is shown in Fig. 2.23.

We know that in purely resistive the voltage and current both are in phase and therefore vector \( V_R \) is drawn superimposed to scale onto the current vector and in purely inductive circuit the current I lag the voltage \( V_L \) by 90°.

So, to draw the vector diagram, first I taken as the reference. This is shown in the Fig. 2.24. Next \( V_R \) drawn in phase with I. Next \( V_L \) is drawn 90° leading the I.

The supply voltage \( V \) is then phasor Addition of \( V_R \) and \( V_L \).
Thus, from the above, it can be said that the current in series R-L circuit lags the applied voltage V by an angle $\phi$. If supply voltage

$$v = V_m \sin \omega t$$

$$i = I_m \sin (\omega t - \phi) \quad \text{Where} \quad I_m = \frac{V_m}{Z}$$

**Voltage Triangle**

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I \sqrt{R^2 + X_L^2}$$

$$= IZ$$

where, $Z = \sqrt{R^2 + X_L^2}$

**Impedance Triangle**

$$Z = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1} \frac{X_L}{R}$$

**Power Triangle**

Real Power $P = V I \cos \phi$

$$= I^2 R$$

Reactive Power $Q = V I \sin \phi$

$$= I^2 X_L$$

Apparent Power $S = V I$

$$= I^2 Z$$
Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

**Instantaneous power**

\[ p_t = v \times i \]

\[ p_t = V_m \sin \omega t \times I_m \sin (\omega t - \phi) \]

\[ p_t = V_m I_m \sin \omega t \times \sin (\omega t - \phi) \]

\[ p_t = \frac{2 V_m I_m \sin \omega t \times \sin (\omega t - \phi)}{2} \]

\[ p_t = \frac{V_m I_m}{2} \left[ \cos \phi - \cos (2\omega t - \phi) \right] \]

- Thus, the instantaneous values of the power consist of two components.
- First component is constant w.r.t. time and second component vary with time.

**Average Power**

\[ P_{ave} = \frac{2\pi}{V_m I_m} \left[ \cos \phi - \cos (2\omega t - \phi) \right] \, d\omega t \]

\[ P_{ave} = \frac{V_m I_m}{2\pi} \left( \frac{1}{2} \left[ \cos \phi - \cos (2\omega t - \phi) \right] \right) \, d\omega t \]

\[ P_{ave} = \frac{V_m I_m}{4\pi} \left[ \cos \phi \int_0^{2\pi} d\omega t - \int_0^{2\pi} \cos (2\omega t - \phi) \, d\omega t \right] \]

\[ P_{ave} = \frac{V_m I_m}{4\pi} \cos \phi \left( \frac{2\pi}{0} \cos \phi - \frac{\sin (2\omega t - \phi)}{2} \right)_{0}^{2\pi} \]

\[ P_{ave} = \frac{V_m I_m}{4\pi} \cos \phi \left[ - \frac{\sin (4\pi - \phi)}{8\pi} - \sin (-\phi) \right] \]

\[ P_{ave} = \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{8\pi} \left[ - \sin \phi + \sin \phi \right] \]

\[ P_{ave} = \frac{V_m I_m}{2} \cos \phi - 0 \]

\[ P_{ave} = \frac{V_m I_m}{2} \cos \phi \]

\[ P_{ave} = \frac{V_m I_m}{\sqrt{2}} \cos \phi \]

\[ P_{ave} = VI \cos \phi \]
2.9 Series Resistance-Capacitance Circuit

- Consider a circuit consisting of a resistor of resistance \( R \) ohms and a purely capacitive of capacitance farad in series as in the Fig. 2.28.

\[
\begin{align*}
V_R & \quad \text{Resistor} \\
& \quad \text{Capacitor} \\
& \quad \text{i} \\
& \quad V = V_m \sin \omega t
\end{align*}
\]

*Figure 2.28 Circuit Diagram of Series R-C Circuit*

- In the series circuit, the current \( i \) flowing through \( R \) and \( C \) will be the same. But the voltage across them will be different.
- The vector sum of voltage across resistor \( V_R \) and voltage across capacitor \( V_C \) will be equal to supply voltage \( v_t \).

**Waveforms and Phasor Diagram**

\[
\begin{align*}
\phi & \quad \text{Phase} \\
& \quad \text{Vector Addition of} \ V_R \text{ and } \ V_C
\end{align*}
\]

*Figure 2.29 Waveform of Voltage and Current of Series R-C Circuit*

- We know that in purely resistive the voltage and current in a resistive circuit both are in phase and therefore vector \( V_R \) is drawn superimposed to scale onto the current vector and in purely capacitive circuit the current \( I \) lead the voltage \( V_C \) by 90°.
- So, to draw the vector diagram, first \( I \) taken as the reference. This is shown in the Fig. 2.30. Next \( V_R \) drawn in phase with \( I \). Next \( V_C \) is drawn 90° lagging the \( I \). The supply voltage \( V \) is then phasor Addition of \( V_R \) and \( V_C \).

*Figure 2.30 Phasor Diagram of Series R-C Circuit*
Thus, from the above equation it is clear that the current in series R-C circuit leads the applied voltage $V$ by an angle $\phi$. If supply voltage

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \phi)$$

Where, $I_m = \frac{V_m}{Z}$

### Voltage Triangle

![Figure 2.31 Voltage Triangle of Series R-C Circuit](image)

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I \sqrt{R^2 + X_C^2}$$

$$= IZ \text{ where, } Z = \sqrt{R^2 + X_C^2}$$

### Impedance Triangle

![Figure 2.32 Impedance Triangle Series R-L Circuit](image)

$$Z = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1} \frac{X_C}{R}$$

### Power Triangle

![Figure 2.33 Power Triangle Series R-L Circuit](image)

**Real Power, $P$**

$$P = VI \cos \phi = I^2 R$$

**Reactive Power, $Q$**

$$Q = VI \sin \phi = I^2 X_L$$

**Apparent Power, $S$**

$$S = VI = I^2 Z$$

### Power Factor

$$p.f. = \cos \phi = \frac{R}{Z} \text{ or } \frac{P}{S}$$

### Power

- The instantaneous value of power drawn by this circuit is given by the product of the instantaneous values of voltage and current.

### Instantaneous power

$$p_t = v \times i$$

$$p_t = V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$p_t = \frac{2 V_m I_m \sin \omega t \times \sin(\omega t + \phi)}{2}$$

$$p_t = \frac{V_m I_m}{2} \left[ \cos \phi - \cos(2\omega t + \phi) \right]$$

- Thus, the instantaneous values of the power consist of two components. First component remains constant w.r.t time and second component vary with time.
### Average Power

\[
P_{ave} = \frac{2\pi V_m I_m}{2} \left[ \cos \phi - \cos(2\omega t + \phi) \right] d\omega t
\]

\[
P_{ave} = \frac{V_m I_m}{2\pi} \left[ \frac{1}{2} \left[ \cos \phi - \cos(2\omega t + \phi) \right] d\omega t \right]
\]

\[
P_{ave} = \frac{V_m I_m}{4\pi} \left[ \cos \phi \left( \omega t \right) - \int_{0}^{2\pi} \cos(2\omega t + \phi) d\omega t \right]
\]

\[
P_{ave} = \frac{V_m I_m}{4\pi} \left[ \cos \phi \left(2\pi - \phi\right) - \int_{0}^{2\pi} \frac{\sin(2\omega t + \phi)}{2} d\omega t \right]
\]

\[
P_{ave} = \frac{V_m I_m}{2} \left[ \cos \phi \right] - \frac{V_m I_m}{8\pi} \left[ \sin(4\pi + \phi) - \sin(\phi) \right]
\]

\[
P_{ave} = \frac{V_m I_m}{2} \cos \phi - 0
\]

\[
P_{ave} = \frac{V_m I_m}{\sqrt{2}} \cos \phi
\]

\[
P_{ave} = VI \cos \phi
\]

### 2.10 Series RLC circuit

- Consider a circuit consisting of a resistor of R ohm, pure inductor of inductance L henry and a pure capacitor of capacitance C farads connected in series.

![Figure 2.34 Circuit Diagram of Series RLC Circuit](image)

**Figure 2.34 Circuit Diagram of Series RLC Circuit**

**Phasor Diagram**

- Current I is taken as reference.
- \(V_R\) is drawn in phase with current,
- \(V_L\) is drawn leading I by 90°,
- \(V_C\) is drawn lagging I by 90°

![Figure 2.35 Phasor Diagram of Series RLC Circuit](image)

**Figure 2.35 Phasor Diagram of Series RLC Circuit**
Since $V_L$ and $V_C$ are in opposition to each other, there can be two cases:

1. $V_L > V_C$
2. $V_L < V_C$

**Case-1**
When, $V_L > V_C$, the phasor diagram would be as in the figure 2.36

**Phasor Diagram**

![Figure 2.36 Phasor Diagram of Series R-L-C Circuit for Case $V_L > V_C$]

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{(IR)^2 + I(X_L - X_C)^2}$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$
$$= IZ \quad \text{where, } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- The angle $\phi$ by which $V$ leads $I$ is given by
  $$\tan \phi = \frac{(V_L - V_C)}{R}$$
  $$\therefore \phi = \tan^{-1} \left( \frac{V_L - V_C}{IR} \right)$$
  $$\therefore \phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$

- Thus, when $V_L > V_C$ the series current $I$ lags $V$ by angle $\phi$.

If $v_i = V_m \sin \omega t$

$$i_i = I_m \sin (\omega t - \phi)$$

- Power consumed in this case is equal to series RL circuit $P_{ave} = VI \cos \phi$.

**Case-2**
When, $V_L < V_C$, the phasor diagram would be as in the figure 2.37

**Phasor Diagram**

![Figure 2.37 Phasor Diagram of Series R-L-C Circuit for Case $V_L < V_C$]

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$
$$= \sqrt{(IR)^2 + I(X_C - X_L)^2}$$
$$= I\sqrt{R^2 + (X_C - X_L)^2}$$
$$= IZ \quad \text{where, } Z = \sqrt{R^2 + (X_C - X_L)^2}$$

- The angle $\phi$ by which $V$ lags $I$ is given by
  $$\tan \phi = \frac{(V_C - V_L)}{R}$$
  $$\therefore \phi = \tan^{-1} \left( \frac{V_C - V_L}{IR} \right)$$
  $$\therefore \phi = \tan^{-1} \left( \frac{X_C - X_L}{R} \right)$$

- Thus, when $V_L < V_C$ the series current $I$ leads $V$ by angle $\phi$.

If $v_i = V_m \sin \omega t$

$$i_i = I_m \sin (\omega t + \phi)$$

- Power consumed in this case is equal to series RC circuit $P_{ave} = VI \cos \phi$. 
2.11 Series resonance RLC circuit

- Such a circuit shown in the Fig. 2.38 is connected to an A.C. source of constant supply voltage $V$ but having variable frequency.

\[ v_t = V_m \sin \omega t \]

- The frequency can be varied from zero, increasing and approaching infinity. Since $X_L$ and $X_C$ are functions of frequency, at a particular frequency of applied voltage, $X_L$ and $X_C$ will become equal in magnitude and power factor become unity.

\[ X_L = X_C \]

\[ Z = \sqrt{R^2 + 0^2} = R \]

- The circuit, when $X_L = X_C$ and hence $Z = R$, is said to be in resonance. In a series circuit since current $I$ remain the same throughout we can write,

\[ I X_L = I X_C \quad \text{i.e.} \quad V_L = V_C \]

Phasor Diagram

- Shown in the Fig. 2.39 is the phasor diagram of series resonance RLC circuit.

\[ V_L = V_R \]

\[ V = V_R \]

- i.e. the supply voltage will drop across the resistor $R$.

Resonance Frequency

- At resonance frequency $X_L = X_C$

\[ 2\pi f_R L = \frac{1}{2\pi f_R C} \quad (f_R \text{ is the resonance frequency}) \]
\[
\therefore f_r^2 = \frac{1}{(2\pi)^2 LC}
\]
\[
\therefore f_r = \frac{1}{2\pi\sqrt{LC}}
\]

**Q- Factor**

- The Q- factor is nothing but the voltage magnification during resonance.
- It indicates as to how many times the potential difference across L or C is greater than the applied voltage during resonance.
- Q- factor = Voltage magnification

\[
Q - Factor = \frac{V_L}{V_S} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f_L}{R} \quad \text{But } f_r = \frac{1}{2\pi\sqrt{LC}}
\]

\[
\therefore Q - Factor = \frac{1}{R} \sqrt{\frac{L}{C}}
\]

**Graphical Representation of Resonance**

- **Resistance (R)** is independent of frequency. Thus, it is represented by straight line.
- **Inductive reactance (X_L)** is directly proportional to frequency. Thus, it increases linearly with the frequency.

\[
\therefore X_L = 2\pi fL
\]

\[
\therefore X_L \propto f
\]

- **Capacitive reactance (X_C)** is inversely proportional to frequency. Thus, it is show as hyperbolic curve in fourth quadrant

\[
\therefore X_C = \frac{1}{2\pi fC}
\]

\[
\therefore X_C \propto \frac{1}{f}
\]

- **Impedance (Z)** is minimum at resonance frequency.

\[
Z = \sqrt{R^2 + (X_L - X_C)^2}
\]

For, \( f = f_r \), \( Z = R \)

- **Current (I)** is maximum at resonance frequency.

\[
\therefore I = \frac{V}{Z}
\]

For \( f = f_r \), \( I = \frac{V}{R} \) is maximum, \( I_{\text{max}} \)
2 A.C. Circuits

- **Power factor** is unity at resonance frequency.

  \[
  \text{Power factor}=\cos\phi=\frac{R}{Z}
  \]

  For \( f = f_r \), \( p.f. = 1 \) (unity)

\[\cos\phi = 1\]

![Graphical Representation of Series Resonance RLC Circuit](image)

**Figure 2.40 Graphical Representation of Series Resonance RLC Circuit**

### 2.11 Parallel Resonance RLC Circuit

- Fig. 2.41 Shows a parallel circuit consisting of an inductive coil with internal resistance \( R \) ohm and inductance \( L \) henry in parallel with capacitor \( C \) farads.

![Circuit Diagram of Parallel Resonance RLC Circuit](image)

**Figure 2.41 Circuit Diagram of Parallel Resonance RLC Circuit**

- The current \( I_C \) can be resolved into its active and reactive components. Its active component \( I_L \cos\phi \) and reactive component \( I_L \sin\phi \).

![Circuit Diagram of Parallel Resonance RLC Circuit](image)

**Figure 2.42 Circuit Diagram of Parallel Resonance RLC Circuit**
A parallel circuit is said to be in resonance when the power factor of the circuit becomes unity. This will happen when the resultant current \( I \) is in phase with the resultant voltage \( V \) and hence the phase angle between them is zero.

In the phasor diagram shown, this will happen when \( I_C = I_L \sin \phi \) and \( I = I_L \cos \phi \).

**Resonance Frequency**

- To find the resonance frequency, we make use of the equation \( I_C = I_L \sin \phi \).

\[
I_C = I_L \sin \phi \\
V = V_X L \\
X_C = \frac{V}{Z_L Z_L} \\
Z_L^2 = X_L X_C \\
Z_L^2 = 2\pi f_r L \\
\frac{I}{2\pi f_r C} = \frac{L}{C} \\
(R^2 + \omega_r^2 L^2) = \frac{L}{C} \\
\omega_r^2 = \frac{L}{C(\frac{1}{L^2})} - \frac{R^2}{L^2} \\
(2\pi f_r)^2 = \frac{L}{C(\frac{1}{L^2})} - \frac{R^2}{L^2} \\
f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}
\]

- If the resistance of the coil is negligible,

\[
f_r = \frac{1}{2\pi \sqrt{LC}}
\]

**Impedance**

- To find the resonance frequency, we make use of the equation \( I = I_L \cos \phi \) because, at resonance, the supply current \( I \) will be in phase with the supply voltage \( V \).

\[
I = I_L \cos \phi \\
V = V_R \\
Z = \frac{V}{Z_L Z_L} \\
Z = \frac{Z_L^2}{R} \\
But 
Z_L^2 = \frac{L}{C} \\
Z = \frac{L}{RC}
\]

- The impedance during parallel resonance is very large because of \( L \) and \( C \) has a very large value at that time. Thus, impedance at the resonance is maximum.

\[
I = \frac{V}{Z} \text{ will be minimum.}
\]
Q-Factor

- Q-factor = Current magnification

\[ Q - \text{Factor} = \frac{I_q}{I} = \frac{I_L \sin \phi}{I_L \cos \phi} = \frac{\sin \phi}{\cos \phi} = \tan \phi = \frac{\omega L}{R} = \frac{2\pi f_L L}{R} \]

\[ \therefore Q - \text{Factor} = \frac{1}{R \sqrt{C}} \]

Graphical representation of Parallel Resonance

- **Conductance (G)** is independent of frequency. Hence it is represented by straight line parallel to frequency.
- **Inductive Susceptance (B_L)** is inversely proportional to the frequency. Also, it is negative.
  \[ B_L = \frac{1}{jX_L} = \frac{1}{j2\pi fL}, \quad \therefore B_L \propto \frac{1}{f} \]
- **Capacitive Susceptance (B_C)** is directly proportional to the frequency.
  \[ B_C = \frac{1}{-jX_C} = \frac{j}{X_C} = j2\pi fC, \quad \therefore B_C \propto f \]

---

**Figure 2.43 Graphical Representation of Parallel Resonance RLC Circuit**
2. A.C. Circuits

- **Admittance (Y)** is minimum at resonance frequency.
  \[ Y = \sqrt{G^2 + (B_L - B_C)^2} \]
  For, \( f = f_r, Y = G \)

- **Current (I)** is minimum at resonance frequency.
  \[ I = VY \]

- **Power factor** is unity at resonance frequency.
  \[ \text{Power factor} = \cos \phi = \frac{G}{Y} \]

### 2.12 Comparison of Series and Parallel Resonance

<table>
<thead>
<tr>
<th>Sr.No.</th>
<th>Description</th>
<th>Series Circuit</th>
<th>Parallel Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Impedance at resonance</td>
<td>Minimum ( Z = R )</td>
<td>Maximum ( Z = \frac{L}{RC} )</td>
</tr>
<tr>
<td>2</td>
<td>Current</td>
<td>Maximum ( I = \frac{V}{R} )</td>
<td>Minimum ( I = \frac{V}{L/RC} )</td>
</tr>
<tr>
<td>3</td>
<td>Resonance Frequency</td>
<td>( f_r = \frac{1}{2\pi\sqrt{LC}} )</td>
<td>( f_r = \frac{1}{2\pi\sqrt{LC}} )</td>
</tr>
<tr>
<td>4</td>
<td>Power Factor</td>
<td>Unity</td>
<td>Unity</td>
</tr>
<tr>
<td>5</td>
<td>Q- Factor</td>
<td>( f_r = \frac{L}{R\sqrt{C}} )</td>
<td>( f_r = \frac{L}{R\sqrt{C}} )</td>
</tr>
<tr>
<td>6</td>
<td>It magnifies at resonance</td>
<td>Voltage</td>
<td>Current</td>
</tr>
</tbody>
</table>
## Three - Phase AC Circuits

### 2.13 Comparison between single phase and three phase

<table>
<thead>
<tr>
<th>Basis for Comparison</th>
<th>Single Phase</th>
<th>Three Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition</strong></td>
<td>The power supply through one conductor.</td>
<td>The power supply through three conductors.</td>
</tr>
<tr>
<td><strong>Wave Shape</strong></td>
<td><img src="image" alt="Single Phase Wave Shape" /></td>
<td><img src="image" alt="Three Phase Wave Shape" /></td>
</tr>
<tr>
<td><strong>Number of wire</strong></td>
<td>Require two wires for completing the circuit</td>
<td>Requires four wires for completing the circuit</td>
</tr>
<tr>
<td><strong>Voltage</strong></td>
<td>Carry 230V</td>
<td>Carry 415V</td>
</tr>
<tr>
<td><strong>Phase Name</strong></td>
<td>Split phase</td>
<td>No other name</td>
</tr>
<tr>
<td><strong>Network</strong></td>
<td>Simple</td>
<td>Complicated</td>
</tr>
<tr>
<td><strong>Loss</strong></td>
<td>Maximum</td>
<td>Minimum</td>
</tr>
<tr>
<td><strong>Power Supply Connection</strong></td>
<td><img src="image" alt="Single Phase Connection" /></td>
<td><img src="image" alt="Three Phase Connection" /></td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
<td>Less</td>
<td>High</td>
</tr>
<tr>
<td><strong>Economical</strong></td>
<td>Less</td>
<td>More</td>
</tr>
<tr>
<td><strong>Uses</strong></td>
<td>For home appliances.</td>
<td>In large industries and for running heavy loads.</td>
</tr>
</tbody>
</table>

### 2.14 Generation of three phase EMF

According to Faraday's law of electromagnetic induction, we know that whenever a coil is rotated in a magnetic field, there is a sinusoidal emf induced in that coil.
Now, we consider 3 coil $C_1$(R-phase), $C_2$(Y-phase) and $C_3$(B-phase), which are displaced $120^0$ from each other on the same axis. This is shown in fig. 2.44.

The coils are rotating in a uniform magnetic field produced by the N and S pols in the counter clockwise direction with constant angular velocity.

According to Faraday's law, emf induced in three coils. The emf induced in these three coils will have phase difference of $120^0$, i.e. if the induced emf of the coil $C_1$ has phase of $0^0$, then induced emf in the coil $C_2$ lags that of $C_1$ by $120^0$ and $C_3$ lags that of $C_2$ $120^0$.

$$e_R = E_m \sin \omega t$$
$$e_Y = E_m \sin (\omega t - 120^0)$$
$$e_B = E_m \sin (\omega t - 240^0)$$

Thus, we can write,

The above equation can be represented by their phasor diagram as in the Fig. 2.46.

2.15 Important definitions

- **Phase Voltage**
  It is defined as the voltage across either phase winding or load terminal. It is denoted by $V_{ph}$. Phase voltage $V_{RN}$, $V_{YN}$ and $V_{BN}$ are measured between R-N, Y-N, B-N for star connection and between R-Y, Y-B, B-R in delta connection.
- **Line voltage**
  It is defined as the voltage across any two-line terminal. It is denoted by $V_L$.
  Line voltage $V_{RY}$, $V_{YB}$, $V_{BR}$ measure between R-Y, Y-B, B-R terminal for star and delta connection both.

- **Phase current**
  It is defined as the current flowing through each phase winding or load. It is denoted by $I_{ph}$.
  Phase current $I_{R(ph)}$, $I_{Y(ph)}$ and $I_{B(ph)}$ measured in each phase of star and delta connection respectively.

- **Line current**
  It is defined as the current flowing through each line conductor. It denoted by $I_L$.
  Line current $I_{R(line)}$, $I_{Y(line)}$, and $I_{B(line)}$ are measured in each line of star and delta connection.

- **Phase sequence**
  The order in which three coil emf or currents attain their peak values is called the phase sequence. It is customary to denoted the 3 phases by the three colours. i.e. red (R), yellow (Y), blue (B).

- **Balance System**
  A system is said to be balance if the voltages and currents in all phase are equal in magnitude and displaced from each other by equal angles.

- **Unbalance System**
  A system is said to be unbalance if the voltages and currents in all phase are unequal in magnitude and displaced from each other by unequal angles.

- **Balance load**
  In this type the load in all phase are equal in magnitude. It means that the load will have the same power factor equal currents in them.

- **Unbalance load**
  In this type the load in all phase have unequal power factor and currents.
2.16 Relation between line and phase values for voltage and current in case of balanced delta connection.

- Delta (Δ) or Mesh connection, starting end of one coil is connected to the finishing end of another phase coil and so on which giving a closed circuit.

Circuit Diagram

![Circuit Diagram](image)

- Let, 
  - Line voltage, \( V_{RY} = V_{YB} = V_{BR} = V_L \)
  - Phase voltage, \( V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph} \)
  - Line current, \( I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line} \)
  - Phase current, \( I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph} \)

Relation between line and phase voltage
- For delta connection line voltage \( V_L \) and phase voltage \( V_{ph} \) both are same.
  - \( V_{RY} = V_{R(ph)} \)
  - \( V_{YB} = V_{Y(ph)} \)
  - \( V_{BR} = V_{B(ph)} \)
  - \( \therefore V_L = V_{ph} \)

Line voltage = Phase Voltage

Relation between line and phase current
- For delta connection,
  - \( I_{R(line)} = I_{R(ph)} - I_{B(ph)} \)
  - \( I_{Y(line)} = I_{Y(ph)} - I_{R(ph)} \)
  - \( I_{B(line)} = I_{B(ph)} - I_{Y(ph)} \)
  - i.e. current in each line is vector difference of two of the phase currents.
• So, considering the parallelogram formed by $I_R$ and $I_B$.

\[
I_{R(line)} = \sqrt{I_{B(\text{ph})}^2 + I_{B(\text{ph})}^2 + 2I_{B(\text{ph})}I_{B(\text{ph})} \cos \theta}
\]

\[
\therefore I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2 \cos 60^\circ}
\]

\[
\therefore I_L = \sqrt{3} I_{ph}
\]

• Similarly, $I_{Y(line)} = I_{B(line)} = \sqrt{3} I_{ph}$

• Thus, in delta connection Line current = $\sqrt{3}$ Phase current

**Power**

\[
P = V_{ph}I_{ph} \cos \phi + V_{ph}I_{ph} \cos \phi + V_{ph}I_{ph} \cos \phi
\]

\[
P = 3V_{ph}I_{ph} \cos \phi
\]

\[
P = 3V_L \left( \frac{1}{\sqrt{3}} \right) \cos \phi
\]

\[
\therefore P = \sqrt{3}VLI_L \cos \phi
\]
2.17 Relation between line and phase values for voltage and current in case of balanced star connection.

- In the Star Connection, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point.

Circuit Diagram

![Circuit Diagram of Three Phase Star Connection](image)

- Let,
  - line voltage, \( V_{RY} = V_{BY} = V_{BR} = V_L \)
  - phase voltage, \( V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph} \)
  - line current, \( I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line} \)
  - phase current, \( I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph} \)

Relation between line and phase voltage

- For star connection, line current \( I_L \) and phase current \( I_{ph} \) both are same.
  \[ I_{R(line)} = I_{R(ph)} \]
  \[ I_{Y(line)} = I_{Y(ph)} \]
  \[ I_{B(line)} = I_{B(ph)} \]
  \[ \therefore I_L = I_{ph} \]

Line Current = Phase Current

Relation between line and phase voltage

- For delta connection,
\[ V_{RY} = V_{R(ph)} - V_{Y(ph)} \]
\[ V_{YB} = V_{Y(ph)} - V_{B(ph)} \]
\[ V_{BR} = V_{B(ph)} - V_{R(ph)} \]

- i.e. line voltage is vector difference of two of the phase voltages. Hence,

![Phasor Diagram of Three Phase Star Connection](image)

**Figure 2.52 Phasor Diagram of Three Phase Star Connection**

From parallelogram,
\[
V_{RY} = \sqrt{V_{R(ph)}^2 + V_{Y(ph)}^2 + 2V_{R(ph)}V_{Y(ph)} \cos \theta}
\]

\[
V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph} \cos 60^\circ}
\]

\[
V_L = \sqrt{3V_{ph}^2}
\]

\[
V_L = \sqrt{3}V_{ph}
\]

- Similarly, \( V_{YB} = V_{BR} = \sqrt{3} V_{ph} \)
- Thus, in star connection Line voltage = \( \sqrt{3} \) Phase voltage

**Power**
\[
P = V_{ph}I_{ph} \cos \phi + V_{ph}I_{ph} \cos \phi + V_{ph}I_{ph} \cos \phi
\]
\[
P = 3V_{ph}I_{ph} \cos \phi
\]
\[
P = \frac{V_L}{\sqrt{3}} I_L \cos \phi
\]

\[
\therefore P = \sqrt{3}V_L I_L \cos \phi
\]
2.18 Measurement of power in balanced 3-phase circuit by two-watt meter method

- This is the method for 3-phase power measurement in which sum of reading of two wattmeter gives total power of system.

Circuit Diagram

![Circuit Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection](image1)

- The load is considered as an inductive load and thus, the phasor diagram of the inductive load is drawn below in Fig. 2.54.

![Phasor Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection](image2)
The three voltages \( V_{RN}, V_{YN} \) and \( V_{BN} \), are displaced by an angle of 120° electrical as shown in the phasor diagram. The phase current lag behind their respective phase voltages by an angle \( \phi \). The power measured by the Wattmeter, \( W_1 \) and \( W_2 \).

Reading of wattmeter, \( W_1 = V_{by}I_r \cos \phi_1 = V_l I_l \cos(30 + \phi) \)

Reading of wattmeter, \( W_2 = V_{by}I_b \cos \phi_2 = V_l I_l \cos(30 - \phi) \)

Total power, \( P = W_1 + W_2 \)

\[
\therefore P = V_l I_l \cos(30 + \phi) + V_l I_l \cos(30 - \phi)
\]

\[
= V_l I_l \left[ \cos(30 + \phi) + \cos(30 - \phi) \right]
\]

\[
= V_l I_l \left[ \cos30\cos\phi + \sin30\sin\phi + \cos30\cos\phi - \sin30\sin\phi \right]
\]

\[
= V_l I_l \left[ 2\cos30\cos\phi \right]
\]

\[
= V_l I_l \left[ 2 \left( \frac{\sqrt{3}}{2} \right) \cos\phi \right]
\]

\[
= \sqrt{3}V_l I_l \cos\phi
\]

Therefore, the sum of the readings of the two wattmeter is equal to the power absorbed in a 3-phase balanced system.

**Determination of Power Factor from Wattmeter Readings**

- As we know that

\[
W_1 + W_2 = \sqrt{3}V_l I_l \cos \phi
\]

Now,

\[
W_1 - W_2 = V_l I_l \cos(30 + \phi) - V_l I_l \cos(30 - \phi)
\]

\[
= V_l I_l \left[ \cos\phi \cos30 + \sin\phi \sin30 - \cos\phi \cos30 + \sin\phi \sin30 \right]
\]

\[
= V_l I_l \left[ 2\sin30\sin\phi \right]
\]

\[
= V_l I_l \left[ 2 \left( \frac{1}{2} \right) \sin\phi \right] = V_l I_l \sin\phi
\]

\[
\therefore \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3}V_l I_l \sin \phi}{\sqrt{3}V_l I_l \cos \phi} = \tan \phi
\]

\[
\therefore \tan \phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}
\]

- Power factor of load given as,

\[
\therefore \cos \phi = \cos \left( \tan^{-1} \left( \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \right) \right)
\]
Effect of power factor on wattmeter reading:

- From the Fig. 2.54, it is clear that for lagging power factor $\cos \phi$, the wattmeter readings are
  
  $W_1 = V_L I_L \cos (30 + \phi)$
  
  $W_2 = V_L I_L \cos (30 - \phi)$

- Thus, readings $W_1$ and $W_2$ will vary depending upon the power factor angle $\phi$.

<table>
<thead>
<tr>
<th>p.f</th>
<th>$\phi$</th>
<th>$W_1 = V_L I_L \cos (30 + \phi)$</th>
<th>$W_2 = V_L I_L \cos (30 - \phi)$</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos \phi = 1$</td>
<td>0°</td>
<td>$\frac{\sqrt{3}}{2} V_L I_L$</td>
<td>$\frac{\sqrt{3}}{2} V_L I_L$</td>
<td>Both equal and +ve</td>
</tr>
<tr>
<td>$\cos \phi = 0.5$</td>
<td>60°</td>
<td>0</td>
<td>$\frac{\sqrt{3}}{2} V_L I_L$</td>
<td>One zero and second total power</td>
</tr>
<tr>
<td>$\cos \phi = 0$</td>
<td>90°</td>
<td>$-\frac{1}{2} V_L I_L$</td>
<td>$\frac{1}{2} V_L I_L$</td>
<td>Both equal but opposite</td>
</tr>
</tbody>
</table>

********************

Remark for $\cos 1\phi$:

- $\cos 0\phi = 0$
- $\cos 0.5\phi = 60°$
- $\cos 90\phi = 90°$

Both equal and +ve

One zero and second total power

Both equal but opposite

***************