CHAPTER – 1 BALANCING

Theory

1. Attempt the following questions.
   I. Need of balancing
   II. Primary unbalanced force in reciprocating engine.
   III. Explain clearly the terms static balancing and dynamic balancing.

2. Explain the methods of Static and Dynamic balancing using balancing machines in the industry.

3. How and why are reciprocating masses balanced in a piston-cylinder assembly? Why reciprocating masses are partially balanced?

4. For an uncoupled two cylinder locomotive engine, derive the expressions of ‘variation in tractive force’, ‘swaying couple’ and ‘hammer blow’.

5. Discuss the method of Balancing of V-engines and determine the expression for magnitude and direction of resultant primary force.

6. Explain the concept of direct and reverse crank for balancing of radial engines.

Examples

BALANCING OF ROTATING MASSES

1. A circular disc mounted on a shaft carries three attached masses of 4 kg, 3 kg and 2.5 kg at radial distances of 75 mm, 85 mm and 50 mm and at the angular positions of 45°, 135° and 240° respectively. The angular positions are measured counterclockwise from the reference line along the x-axis. Determine the amount of the countermass at a radial distance of 75 mm required for the static balance.

2. Four masses m₁, m₂, m₃ and m₄ are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

3. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.
4. Four masses A, B, C and D carried by a rotating shaft are at radii 110, 140, 210 and 160 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the masses of B, C and D are 16 kg, 10 kg and 8 kg respectively. Find the required mass A and the relative angular positions of the four masses so that shaft is in complete balance.

5. Four masses 150 kg, 200 kg, 100 kg and 250 kg are attached to a shaft revolving at radii 150 mm, 200 mm, 100 mm and 250 mm; in planes A, B, C and D respectively. The planes B, C and D are at distances 350 mm, 500 mm and 800 mm from plane A. The masses in planes B, C and D are at an angle 105°, 200° and 300° measured anticlockwise from mass in plane A. It is required to balance the system by placing the balancing masses in the planes P and Q which are midway between the planes A and B, and between C and D respectively. If the balancing masses revolve at radius 180 mm, find the magnitude and angular positions of the balance masses.

6. A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine:
   (i) The magnitude of the masses at A and D;
   (ii) The distance between planes A and D; and
   (iii) The angular position of the mass at D.

7. A rotating shaft carries four masses A, B, C and D which are radially attached to it. The mass centers are 30 mm, 40 mm, 35 mm and 38 mm respectively from the axis of rotation. The masses A, C and D are 7.5 kg, 5 kg and 4 kg respectively. The axial distances between the planes of rotation of A and B is 400 mm and between B and C is 500 mm. The masses A and C are at right angles to each other. Find for a complete balance,
   (i) the angles between the masses B and D from mass A,
   (ii) the axial distance between the planes of rotation of C and D, and
   (iii) the magnitude of mass B.

8. The four masses A, B, C and D revolve at equal radii are equally spaces along the shaft. The mass B is 7 kg and radii of C and D makes an angle of 90° and 240° respectively (counterclockwise) with radius of B, which is horizontal. Find the magnitude of A, C and D and angular position of A so that the system may be completely balance. Solve problem by analytically.
9. Four masses A, B, C and D are completely balanced. Masses C and D make angles of 90° and 195° respectively with B in the same sense. The rotating masses have following properties.
\[ m_b = 25 \text{ kg, } m_c = 40 \text{ kg, } m_d = 35 \text{ kg, } r_a = 150 \text{ mm, } r_b = 200 \text{ mm, } r_c = 100, \text{ rd } = 180 \text{ mm,} \]
Planes B and C are 250 mm apart.
Determine: (a) The mass A and its angular position,
(b) The position of planes A and D.

10. A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their centre of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance. Determine:
(a) Relative angular positions of the pulleys.
(b) Dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m.

11. A shaft with 3 meters span between two bearings carries two masses of 10 kg and 20 kg acting at the extremities of the arms 0.45 m and 0.6 m long respectively. The planes in which these masses rotate are 1.2 m and 2.4 m respectively from the left end bearing supporting the shaft. The angle between the arms is 60°. The speed of rotation of the shaft is 200 r.p.m. If the masses are balanced by two counter-masses rotating with the shaft acting at radii of 0.3 m and placed at 0.3 m from each bearing centres, estimate the magnitude of the two balance masses and their orientation with respect to the X-axis, i.e. mass of 10 kg.

**BALANCING OF RECIPIROCATING MASSES**

1. The following data relate to a single - cylinder reciprocating engine:
Mass of reciprocating parts = 40 kg
Mass of revolving parts = 30 kg at crank radius
Speed = 150 rpm
Stroke = 350 mm
If 60% of the reciprocating parts and all the revolving parts are to be balanced, determine (i) balance mass required at a radius of 320 mm
(ii) unbalanced force when crank has turned 45° from top dead centre.

2. A single cylinder reciprocating engine has speed 240 rpm, stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 30 kg. If all the mass of revolving parts and two-third of the mass of reciprocating parts are to be balanced, find the balance mass required at radius of 400 mm and the residual unbalanced force when the crank has rotated 60° from IDC.
BALANCING OF LOCOMOTIVES

1. An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles.

   The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

   Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m.

2. The following data refers to two-cylinder uncoupled locomotive:

   Rotating mass per cylinder = 280 kg
   Reciprocating mass per cylinder = 300 kg
   Distance between wheels = 1400 mm
   Distance between cylinder centers = 600 mm
   Diameter of treads of driving wheels = 1800 mm
   Crank radius = 300 mm
   Radius of centre of balance mass = 620 mm
   Locomotive speed = 50 km/hr
   Angle between cylinder cranks = 90°
   Dead load on each wheel = 3.5 tonne

   Determine:
   (i) Balancing mass required in planes of driving wheels if whole of the revolving and 2/3 of reciprocating mass are to be balanced
   (ii) Swaying couple
   (iii) Variation in tractive force
   (iv) Maximum and minimum pressure on the rails
   (v) Maximum speed of locomotive without lifting the wheels from rails.

3. The three cranks of a three cylinder locomotive are all on the same axle and are set at 120°. The pitch of the cylinders is 1 meter and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank. If 40% of the reciprocating parts are to be balanced, find :

   (a) The magnitude and the position of the balancing masses required at a radius of 0.6 m and
   (b) The hammer blow per wheel when the axle makes 6 r.p.s.
4. A two cylinder locomotive has the following specifications:
   - Reciprocating mass per cylinder = 306 Kg
   - Crank radius = 300 mm
   - Angle between cranks = 90°
   - Driving wheels diameter = 1800 mm
   - Distance between cylinder centers = 650 mm
   - Distance between driving wheel planes = 1550 mm

   Determine
   (a) The fraction of reciprocating masses to be balanced, if the hammer blow is not to exceed 46 KN at 96.5 Km/hr.
   (b) The variation in tractive force.
   (c) The maximum swaying couple.

5. The following data apply to an outside cylinder uncoupled locomotive:
   - Mass of rotating parts per cylinder = 360 kg
   - Mass of reciprocating parts per cylinder = 300 kg
   - Angle between cranks = 90°
   - Crank radius = 0.3 m
   - Cylinder centres = 1.75 m
   - Radius of balance masses = 0.75 m
   - Wheel centres = 1.45 m.

   If whole of the rotating and two-thirds of reciprocating parts are to be balanced in planes of the driving wheels, find:
   (i) Magnitude and angular positions of balance masses,
   (ii) Speed in kilometres per hour at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and
   (iii) Swaying couple at speed arrived at in (ii) above.

**BALANCING OF MULTICYLINDER ENGINES**

1. A four crank engine has the two outer cranks set at 120° to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks. If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force?
2. The intermediate cranks of a four cylinder symmetrical engine, which is in complete primary balance, are 90° to each other and each has a reciprocating mass of 300 kg. The centre distance between intermediate cranks is 600 mm and between extreme cranks it is 1800 mm. Lengths of the connecting rod and cranks are 900 mm and 300 mm respectively. Calculate the masses fixed to the extreme cranks with their relative angular positions. Also find the magnitudes of secondary forces and couples about the centre line of the system if the engine speed is 1500 rpm.

3. The cranks and connecting rods of a 4-cylinder in-line engine running at 1800 r.p.m. are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg. Determine: (i) Unbalanced primary and secondary forces, if any, and (ii) Unbalanced primary and secondary couples with reference to central plane of the engine.

4. The successive cranks of a five cylinder in-line engine are at 144° apart. The spacing between cylinder centre lines is 400 mm. The lengths of the crank and the connecting rod are 100 mm and 450 mm respectively and the reciprocating mass for each cylinder is 20 kg. The engine speed is 630 r.p.m. Determine the maximum values of the primary and secondary forces and couples and the position of the central crank at which these occur.

5. A four stroke five cylinder in-line engine has a firing order of 1-4-5-3-2-1. The centers lines of cylinders are spaced at equal intervals of 15 cm, the reciprocating parts per cylinder have a mass of 15 kg, the piston stroke is 10 cm and the connecting rods are 17.5 cm long. The engine rotates at 600 rpm. Determine the values of maximum primary and secondary unbalanced forces and couples about the central plane.

6. In an in-line six cylinder engine working on two stroke cycle, the cylinder centre lines are spaced at 600 mm. In the end view, the cranks are 60° apart and in the order 1-4-5-2-3-6. The stroke of each piston is 400 mm and the connecting rod length is 1 m. The mass of the reciprocating parts is 200 kg per cylinder and that of rotating parts 100 kg per crank. The engine rotates at 300 r.p.m. Examine the engine for the balance of primary and secondary forces and couples. Find the maximum unbalanced forces and couples.
7. The firing order in a 6 cylinder vertical four stroke in-line engine is 1-4-2-6-3-5. The piston stroke is 100 mm and the length of each connecting rod is 200 mm. The pitch distances between the cylinder centre lines are 100 mm, 100 mm, 150 mm, 100 mm, and 100 mm respectively. The reciprocating mass per cylinder is 1 kg and the engine runs at 3000 r.p.m. Determine the out-of-balance primary and secondary forces and couples on this engine, taking a plane midway between the cylinder 3 and 4 as the reference plane.

**BALANCING OF V-ENGINES**

1. Reciprocating mass per cylinder in 60° V-twin engine is 1.5 kg. The stroke and connecting rod length are 100 mm and 250 mm respectively. If the engine runs at 2500 rpm, determine the maximum and minimum values of primary and secondary forces.

2. A V-twin engine has the cylinder axes at right angles and the connecting rods operate a common crank. The reciprocating mass per cylinder is 11.5 kg and the crank radius is 75 mm. The length of the connecting rod is 0.3 m. Show that the engine may be balanced for primary forces by means of a revolving balance mass. If the engine speed is 500 rpm, what is the value of maximum resultant secondary force?

3. For a twin V-engine the cylinder centerlines are set at 90°. The mass of reciprocating parts per cylinder is 2.5 kg. Length of crank is 100 mm and length of connecting rod is 400 mm. determine the primary and secondary unbalanced forces when the crank bisects the lines of cylinder centerlines. The engine runs at 1000 rpm.
CHAPTER – 2 MECHANICAL VIBRATIONS

INTRODUCTION TO VIBRATIONS

**Theory**

1. What is vibration? Explain the phenomenon of vibration.

2. What are the causes of vibrations? How the effects of undesirable vibrations can be reduced?

3. What are the advantage and disadvantages of vibration?

4. Define the following terms: Periodic motion, Time period, Frequency, Amplitude, Natural frequency, Fundamental mode of vibration, Degree of freedom, Simple harmonic motion, Phase difference, spring stiffness, Damping, Damping coefficient, Resonance

5. What are the various types of vibrations?

**UNDAMPED FREE VIBRATIONS**

**Theory**

1. Define following terms: Free vibration, Undamped free vibration and natural frequency of vibration.

2. What are different approaches to get equations of motion of a vibratory system? Explain any one in brief. (Three methods: Equilibrium method, Energy method and Rayleigh’s method)

3. Derive equation of natural frequency for undamped free transverse vibration of single degree of freedom system.

4. Derive equation of natural frequency for undamped free torsional vibration of single degree of freedom system.

**Examples**

1. A compound pendulum shown in fig. 1 is suspended from a point O and is free to oscillate. G is the centre of gravity, m is the mass and k is the radius of gyration about an axis through the center of gravity G. Determine the equation of motion and equation for natural frequency of vibration of the system.
2. A pendulum consists of a stiff weightless rod of length ‘l’ carrying a mass ‘m’ on its end as shown in figure below. Two springs each of stiffness ‘K’ are attached to the rod at a distance ‘a’ from the upper end. Determine the frequency for small oscillation.

3. Find the natural frequency of a system shown in fig.1, Take K=1000 N/m, M =10 kg, m = 2 kg, R = 50 mm and r = 30 mm

4. Derive the equation of natural frequency of free vibration for the single degree of freedom system shown in Figure 1. Also find the natural frequency for the system if the mass of the semi-cylinder is 5 kg and radius is 60 mm.

5. Find the natural frequency of vibration of the half solid cylinder shown in Figure 1, when slightly displaced from the equilibrium position and released.

6. Two simple pendulums are connected in parallel by a spring as shown in Figure 2. Derive the equations of motion for two masses and expression of ratio of amplitudes.

7. Derive an expression for finding natural frequency of a cylindrical object having a radius r, which is rolling without slipping and vibrating inside a circular surface of radius R.

8. A double pendulum consists of point masses $m_1$ and $m_2$ suspended by strings of length $l_1$ and $l_2$. Derive equations of motion for two masses. Also get an expression for amplitude ratio.

9. A cantilever shaft of 50 mm diameter and 300 mm long has a disc of mass 100 Kg at its free end. The Young’s modulus for the shaft material is 200 GPa. Determine the frequency of longitudinal and transverse vibration of the shaft.
DAMPED FREE VIBRATIONS

Theory

1. What is damped vibration? What are the different types of damping methods?

2. Derive an expression for the displacement of spring-mass-damper system in case of under damping, critical damping and over damping.


4. Derive an expression for logarithmic decrement. What is the significance of logarithmic decrement?

Examples

1. A disc of torsion pendulum has a moment of inertia of 0.05 kg·m² is immersed in a viscous fluid. During vibration of pendulum, the observed amplitudes on the same side of the neutral axis for successive cycles are found to decay 50% of the initial value. Determine (i) Logarithmic decrement. (ii) Damping torque per unit velocity. (iii) The periodic time of vibration. Assume \( G = 4.5 \times 10^{10} \) N/m² for the material of shaft. For shaft \( d = 0.10 \) m and \( l = 0.50 \) m.

2. The electric motor is supported on a spring and a dashpot. The spring has the stiffness 6400 N/m and the dashpot offers resistance of 500 N at 4.0 m/sec. The unbalanced mass 0.5 kg rotates at 50 mm radius and the total mass of vibratory system is 20 kg. The motor runs at 400 r.p.m. Determine: damping factor, amplitude of vibration and phase angle, resonant speed and amplitude.

3. In a single-degree damped vibration system, a suspended mass of 8 kg makes 30 oscillations in 18 seconds. The amplitude decreases to 0.25 of the initial value after 5 oscillations. Find (1) Spring stiffness, (2) Logarithmic decrement, (3) Damping factor and (4) Damping coefficient.

4. In a single degree viscously damped vibrating system, the suspended mass of 16 kg makes 45 oscillations in 27 seconds. The amplitude of natural vibration decreases to one fourth of initial value after 5 oscillations. Determine: (i) The logarithmic decrement (ii) The damping factor and damping coefficient (iii) The stiffness of the spring.

5. The successive amplitudes of vibrations of a vibratory system as obtained under free vibrations are 0.69, 0.32, 0.19 and 0.099 units respectively. Determine the damping ratio of the system. If the damping ratio is doubled what would be the amplitude ratio then?

6. For a spring mass system, compute circular frequency, damping factor and displacement after 0.01 sec. The mass of the system is 10 kg, stiffness \( k=16 \) kN/m.
and C=1600 Ns/m. To determine constants make use of the initial conditions. The mass is displaced by 0.01 m and released with a velocity of 2 m/sec in the direction of return motion.

7. A machine having mass of 100 kg is supported on a spring which deflects 20 mm under the dead load of machine. A dashpot is fitted to reduce the amplitude of free vibration to 10% of its initial value in two complete oscillations. Determine the stiffness of the spring, critical damping coefficient, logarithmic decrement, damping factor and frequency of damped-free vibration.

8. A mass of 20 kg is mounted on two slabs of isolators placed one over the other. One of the isolator is of rubber having stiffness 5000 N/m and damping coefficient of 200 N-sec/m while the other isolator is of felt with stiffness of 15000 N/m and damping coefficient of 400 N-sec/m. If the system is set in motion in vertical direction, determine: Damped natural frequency, Damping factor, Logarithmic decrement and undamped natural frequency of the system.

9. A damped vibrating system consisting of 40 kg mass executes 20 oscillations in 5 sec. The amplitude of vibration decreases to one-eighth of the initial value after 8 complete oscillations. Determine: Logarithmic decrement, Damping factor, Damping co-efficient and spring stiffness.

10. A steel bridge structure is deflected at midspan by winching the bridge down and then releasing it. It was observed that the amplitude of frequency decays exponentially from 9 mm to 4 mm at the end of 3 cycles. The frequency of decay is observed to be 1.7 Hz. The test was once again repeated by placing a vehicle of 35000 Kg at midspan and the frequency was observed as 1.52 Hz. Find (i) the damping factor of the structure (ii) the effective mass and stiffness of the structure.

11. A mass of 85 kg is supported on a spring which deflects 18 mm under the weight of the mass. The vibrations of the mass are constrained to be linear and vertical. A dashpot is provided which reduces the amplitude to one-quarter of its initial value in two complete oscillations. Calculate magnitude of the damping force at unit velocity and periodic time of damped vibrations.

12. A motor car moving with a speed of 100 km/hr has a gross mass of 1500 kg. It passes over rough road which has a sinusoidal surface with amplitude of 75 mm and a wavelength of 5 m. The suspension system has a spring constant of 500 N/mm and damping ratio of 0.5. Determine the displacement amplitude of the car and time lag.

13. A machine weighs 18 kg and is supported on springs and dashpots. The total stiffness of the springs in 12 N/mm and damping is 0.2 N/mm/s. the system is initially at rest and a velocity of 120 mm/s is imparted to the mass. Determine: (1) The displacement and velocity of mass as a function of time (2) The displacement and velocity after 0.4 s.
14. A coil of spring stiffness 4 N/mm supports vertically a mass of 20 kg at the free end. The motion is resisted by the oil dashpot. It is found that the amplitude at the beginning of the fourth cycle is 0.8 times the amplitude of the previous vibration. Determine the damping force per unit velocity. Also find the ratio of the frequency of damped and undamped vibrations.

**FORCED DAMPED VIBRATIONS**

*Theory*

1. What are the various sources of external excitations?

2. Derive an expression for amplitude of steady state vibrations and phase angle for a spring-mass-damper system subjected to a sinusoidal force \( F_0 \sin \omega t \). (Two methods: Analytical and Graphical)

3. What do you mean by transient vibrations and steady state vibrations?

4. Explain the term magnification factor (Dynamic magnifier) and obtain expression for it.

5. Explain salient feature of frequency response curve and phase frequency curve.

6. Derive an expression for forced vibration due to rotating unbalance system. Plot the amplitude frequency response curves for different damping factors.

7. Derive an expression for forced vibration due to reciprocating unbalance system.

8. Write short note on forced vibration due to base excitation with characteristic curves. (Two methods: Relative amplitude and absolute amplitude)

**VIBRATION ISOLATION AND TRANSMISSIBILITY**

*Theory*

5. What is vibration isolation? What is importance of it?

6. What are the various materials used for vibration isolation?

7. Write short note on Force transmissibility and motion transmissibility.

8. Explain with neat sketch salient features of Frequency response curve of force transmissibility.

**Examples**

1. A single cylinder engine has a mass of 100 kg and is acted upon by a vertical unbalanced force of 400\sin (13\pi t) N. The engine block is supported on a spring having a stiffness 60 kN/m and a damper which gives a damping force of 700 N per unit velocity. Find the damping ratio and force transmitted to the foundation.
2. A seismic instrument is used to find the magnitude of vibration of a machine tool structure. It gives a reading of relative displacement of 0.4 μm. The natural frequency of the seismic instrument is 5 Hz. The machine tool structure is subjected to a kinematic excitation at a frequency of 2 Hz. Find the magnitude of acceleration of the vibrating machine tool structure. Assume that the damping of the seismic instrument is negligible.

3. A vehicle has a mass of 1200 kg. The suspension system has a spring constant of 400 kN/m and a damping ratio of 0.5. The road surface varies sinusoidal with amplitude of 0.05 m and a wavelength of 6 m. If the vehicle speed is 100 km/hr, determine the displacement of the vehicle.

4. A vibratory body of mass 150 kg supported on springs of total stiffness 1050 kN/m has a rotating unbalance force of 525 N at a speed of 6000 r.p.m. If the damping factor is 0.3, Determine (i) the amplitude caused by the unbalance and its phase angle (ii) the transmissibility.

5. A machine having mass of 100 kg is mounted on four springs of combined stiffness 1500 kN/m with an estimated damping factor of 0.25. A piston within the machine has a mass of 2 kg which reciprocates with stroke of 80 mm at a speed of 3000 rpm. Assuming the motion of the piston to be SHM, determine: (i) The amplitude of the steady state vibration. (ii) The force transmitted to the foundation. (iii) Magnification factor.

6. A body of mass 80 kg is suspended from a spring which deflects 20 mm due to the mass. If the body is subjected to a periodic disturbing force of 700 N and of frequency equal to 0.63 times natural frequency, find: Amplitude of forced vibration, Transmissibility, Dynamic Magnification Factor & Force transmitted to support.

7. A machine having mass of 1000 kg is mounted on the rubber pad having, stiffness of 2000 kN/m and equivalent viscous damping coefficient of 1050 N·sec/m. The machine is subjected to external disturbing harmonic force of 0.6 kN at the frequency of 6π rad/sec. Determine: Amplitude of vibration of machine, Maximum force transmitted to the foundation because of unbalance force, Transmissibility and Magnification factor.

8. A machine of mass 100 kg is supported on an elastic support of total stiffness 800 kN/m and has rotating unbalanced element which results in disturbing force of 400 N at a speed of 3000 rpm. Assuming the damping ratio as 0.25, determine the amplitude of vibrations due to unbalance and the force transmitted to the support.

9. A machine of mass 1000 kg is acted upon by an external force of 2450 N at 1500 r.p.m. To reduce the effect of vibration, isolators of rubber having a static deflection of 2 mm under machine weight and an estimated damping factor of 0.2 are used. Determine: (i) Amplitude of vibration of machine (ii) Force transmitted to the foundation (iii) Phase lag and Phase angle between transmitted force and exciting force (iv) Speed at which the maximum amplitude of vibration would occur.
10. A refrigerator weighing 30 kg is to be supported by three springs, each having stiffness of $K$ (N/m). If the unit operates at 580 rpm, find $K$, if only 10% of the shaking force is to be transmitted to the supporting structure. Neglect damping.

11. A refrigerator unit having mass of 35 kg is to be supported on three springs, each having a spring stiffness $s$. The unit operates at 480 rpm. Find the value of stiffness $s$ if only 10% of the shaking force is allowed to be transmitted to the supported.

12. A 30 kg weight of motor mounted on a damper which deflects by 2 mm due to motor weight. The weight of the rotor is 8 kg and has an eccentricity of 0.2 mm. The motor rotates at 1800 rpm. Find the amplitude of vibration of the motor and force transmitted to the foundation.

CRITICAL (WHIRLING) SPEED OF SHAFT

**Theory**

1. Define briefly whirling speed of shaft with its significance.

2. Derive an expression for critical speed of a shaft carrying rotor and without damping.

3. Why shafts are run above critical speed in case of high speed application?

4. Derive an expression to determine deflection of a shaft simply supported at two ends, carrying single rotor at the center, rotating at an angular speed $\omega$, considering damping.

5. Explain the term half frequency whirl. Derive the expression for it.

6. Discuss the methods to determine critical speed of shafts carrying multiple rotors. (Two methods: Rayleigh's method and Dunkerley's method)

**Examples**

1. A single cylinder engine has a mass of 100 kg and is acted upon by a vertical unbalanced force of $400\sin(13\pi t)$ N. The engine block is supported on a spring having a stiffness 60 kN/m and a damper which gives a damping force of 700 N per unit velocity. Find the damping ratio and force transmitted to the foundation.

2. A rotor has a mass of 12 kg mounted midway on a 24 mm diameter horizontal shaft supported at the ends by two bearings which are 1 m apart. The shaft rotates at 2400 rpm. If the centre of mass m of the rotor is 0.11 mm away from the geometric centre of the rotor due to certain manufacturing defects, find the natural frequency, amplitude of the steady state vibration and dynamic force transmitted to the bearings. The shaft is assumed to be simply supported. Take modulus of elasticity as 200 GPa.
3. A shaft of negligible weight 6 cm diameter and 5 meters long is simply supported at the ends and carries four weights 50 kg each at equal distance over the length of the shaft. Find the frequency of vibration by Dunkerley’s method. Take $E = 2 \times 10^6$ kg/cm$^2$.

4. A shaft 40 mm diameter and 2.5 m long is supported between two short bearings at its ends. It carries three rotors of masses 90 kg, 140 kg and 60 kg at 0.8 m, 1.5 m and 2 m from the left bearing respectively. Take Young’s modulus of the shaft material as $2\times10^5$ N/mm$^2$ and neglecting the mass of the shaft determine the critical speed of the shaft by using Dunkerley’s method.

5. A shaft 100 mm diameter is simply supported in two bearings 4 m apart carrying three discs having masses 125 kg, 200 kg and 100 kg situated at 1.5 m, 2 m and 3 m from one of the bearings respectively. Determine the frequency of transverse vibration of the beam by Dunkerley’s method. Neglect mass of the beam. Assume $E = 2 \times 10^5$ MPa.

6. A shaft 50 mm diameter and 3 m long is simply supported at the ends carries three loads of 100 kg, 150 kg and 75 kg at 1 m, 2 m and 2.5 m from the left support. The modulus of elasticity of the shaft material is $2\times10^5$ MPa. Find the critical speed of the shaft by using Dunkerley’s method.

7. A shaft of 50 mm diameter and 3 m length has a mass of 10 kg per meter length. It is simply supported at the ends and carries three masses of 70 kg, 90 kg and 50 kg situated at 1 m, 2 m and 2.5 m respectively from the left support. Find the natural frequency of transverse vibrations by using Dunkerley’s method. Consider value of $E=200$ GPa.

8. A horizontal shaft of 10 mm diameter is simply supported at both ends by bearings. A rotor of mass 5 Kg is attached at middle of the horizontal shaft. The span between two bearings is 500 mm. The center gravity of the rotor is 2.5 mm offset from the geometric center of the rotor. The equivalent viscous damping at the center of the rotor-shaft may be taken as 52 Ns/m. Find the deflection of the shaft and critical speed of the shaft.

**TORSIONAL VIBRATION**

**Theory**

1. How the natural frequency of torsional vibration of two rotor system is determined?

2. How the natural frequency of torsional vibration of three rotor system is determined?

3. Define following terms: Zero frequency and Node point.

4. Derive an expression for Torsionally Equivalent Shaft System.
5. Discuss free torsional vibrations of geared system and derive the natural frequency relationships considering a three rotor system in conventional notations. (Neglecting inertia of gears)

6. Discuss free torsional vibrations of geared system and derive the natural frequency relationships considering a three rotor system in conventional notations. (Considering inertia of gears)

**Examples**

1. Two rotors A and B are attached to the end of a shaft 50 cm long. Weight of the rotor A is 300 N and its radius of gyration is 30 cm and the corresponding values of B are 500 N and 45 cm respectively. The shaft is 7 cm in diameter for the first 25 cm, 12 cm diameter for the next 10 cm and 10 cm diameter for the remainder of its length. Modulus of rigidity for the shaft material is $8 \times 10^6$ kg/cm$^2$. Find (i) the position of the node and (ii) the frequency of torsional vibration.

2. Two rotors, A and B are attached to the ends of the shaft 600 mm long. The mass and radius of gyration of rotor A is 40 kg and 400 mm respectively and that of rotor B are 50 kg and 500 mm respectively. The shaft is 80 mm diameter for first 250 mm, 120 mm for next 150 mm and 100 mm for the remaining length from the rotor A. Assume the modulus of rigidity of the shaft material $0.8\times10^5$ N/mm$^2$. Find the position of node on equivalent shaft of diameter 80 mm and on the actual shaft. Also find the natural frequency of the torsional vibrations.

3. Two rotors A & B are attached to the ends of a shaft 1.6 m long. The mass of rotor A is 2500 kg and its radius of gyration is 0.8 m. The corresponding values for rotor B are 500 kg and 0.5 m respectively. The diameter of shaft is 180 mm for first 0.5 m, 220 mm for next 0.4 m and 100 mm for the remaining length, measuring from rotor A. Assuming $G = 0.8\times10^5$ MPa, for the shaft material, find position of node and natural frequency of torsional vibration.

4. In a geared system shown in Figure the mass of moment of inertia of rotor A and B are 2 kg-m$^2$ and 0.3 kg-m$^2$ respectively. The gear ratio between rotor B and A is 3. Calculate the node position and natural frequency of torsional oscillations. Ignore the inertia of the gears and shafts. Take modulus of rigidity of shaft material as $80 \times 10^9$ N/m$^2$. 

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MULTI DEGREE VIBRATIONS

Theory

1. Write down the step by step procedure of stodola method to find out fundamental natural frequency of system having three degree of freedom.

2. Explain hozler's method to determine natural frequency of multi rotor system

3. Discuss the Rayleigh's method, to obtain fundamental natural frequency of vibration of a multi-degree of freedom system, with suitable example.

4. Derive the generalized equation of transverse vibrations of a beam of uniform cross section carrying uniformly distributed load.

5. Discuss the effect of inertia of constraint in longitudinal and transverse vibrations.

Examples

1. The vibrations of a cantilever are given by \( y = y_1 [1 - \cos (\pi x/2)] \). Calculate the natural frequency with following data for the cantilever using Rayleigh's method. Modulus of elasticity of the material \( 2 \times 10^{11} \) N/m². Second moment of area about bending axis \( 0.02 \text{ m}^4 \), Mass = \( 6 \times 10^4 \) kg, Length = 30 m.

2. A simply supported beam is subjected to three point loads of masses 20 kg, 50 kg and 40 kg located at 1 m, 2.5 m and 4 m from the left hand end respectively. The beam span is 5 m. Find the lower natural frequency of the transverse vibrations by using Rayleigh's method. Take \( E = 2.1 \times 10^5 \) N/mm² and \( I = 3.33 \times 10^6 \) mm⁴.
3. A four cylinder engine whose shaft is coupled to a damper at one end and a generator at the other end has a flywheel mounted on the shaft between the engine and the generator. A schematic of the system is shown in figure below with the values of the rotor inertias and the stiffness of the shafts. Estimate the two lowest natural frequencies using Holzer’s method.

\[
\begin{align*}
J_1 &= 10 \text{ kg-m}^2 \\
J_2 &= J_3 = J_4 = J_5 = 1.5 \\
J_6 &= 20 \\
J_7 &= 120 \\
k_{t1} &= 40 \times 10^6 \text{ N-m/rad} \\
k_{t2} &= k_{t3} = k_{t4} = 30 \times 10^6 \\
k_{t5} &= 60 \times 10^6 \\
k_{t6} &= 10 \times 10^6
\end{align*}
\]

**VIBRATION MEASUREMENT**

**Theory**

1. Why the measurement of vibration is necessary? What do you mean by vibration monitoring of machine? Enlist different vibration measuring instruments. Explain any one in detail.

2. What are various frequency measuring instruments? Explain any one in detail.

3. Explain about vibro meter, accelerometer and seismometer in brief.