1

INTRODUCTION TO MACHINE DESIGN

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1.1 Introduction to Machine Design

- The subject Machine Design is the creation of new and better machines and improving the existing ones.
- A new or better machine is one which is more economical in the overall cost of production and operation.
- The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived.
- The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings.
- In designing a machine component, there is no rigid rule. The problem may be attempted in several ways.
- However, the general procedure to solve a design problem is discussed below.

1.1.1 General Procedure in Machine Design

![General Procedure in Machine Design](image)

1. **Recognition of need:**
   - First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.
2. **Synthesis (Mechanisms):**
   - Select the possible mechanism or group of mechanisms which will give the desired motion.
3. **Analysis of forces:**
   - Find the forces acting on each member of the machine and the energy transmitted by each member.
4. Material selection:
   - Select the material best suited for each member of the machine.
5. Design of elements (Size and Stresses):
   - Find the size of each member of the machine by considering the force
     acting on the member and the permissible stresses for the material used.
   - It should be kept in mind that each member should not deflect or deform
     than the permissible limit.
6. Modification:
   - Modify the size of the member to agree with the past experience and
     judgment to facilitate manufacture.
   - The modification may also be necessary by consideration of
     manufacturing to reduce overall cost.
7. Detailed drawing:
   - Draw the detailed drawing of each component and the assembly of the
     machine with complete specification for the manufacturing processes
     suggested.
8. Production:
   - The component, as per the drawing, is manufactured in the workshop.

1.2 Standardization
   - Standardization is defined as obligatory (or compulsory) norms, to which various
     characteristics of a product should comply (or agree) with standards.
   - The characteristics include materials, dimensions and shape of the component,
     method of testing and method of marking, packing and storing of the product.
   - There are two words – “standard and code” which are often used in standards.
   - A standard is defined as a set of specifications for parts, materials or processes. The
     objective of, a standard is to reduce the variety and limit the number of items to a
     reasonable level.
   - On the other hand, a code is defined as a set of specifications for the analysis, design,
     manufacture, testing and erection of the product. The purpose of a code is to
     achieve a specified level of safety.
   - There are three types of standards used in design office. They are as follows:
     (i) Company Standards: They are used in a particular company or a group of
         sister concerns.
     (ii) National standards:
         - India - BIS (Bureau of Indian Standards),
         - Germany - DIN (Deutsches Institut für Normung),
         - USA - AISI (American Iron and Steel Institute) or SAE (Society of Automotive
           Engineers),
         - UK - BS (British Standards)
(iii) **International standards**: These are prepared by the International Standards Organization (ISO).

- The following standards are used in mechanical engineering design:

  (i) **Standards for Materials, their chemical compositions, Mechanical properties and Heat Treatment**:

    - For example, Indian standard IS 210 specifies seven grades of grey cast iron designated as FG 150, FG 200, FG 220, FG 260, FG 300, FG 350 and FG 400. The number indicates ultimate tensile strength in N/mm².

  (ii) **Standards for Shapes and dimensions of commonly used Machine Elements**:

    - The machine elements include bolts, screws and nuts, rivets, belts and chains, ball and roller bearings, wire ropes, keys and splines, etc.
    - For example, IS 2494 (Part 1) specifies dimensions and shape of the cross-section of endless V-belts for power transmission.
    - The dimensions of the trapezoidal cross-section of the belt, viz. width, height and included angle are specified in this standard.

  (iii) **Standards for Fits, Tolerances and Surface Finish of Component**:

    - For example, selection of the type of fit for different applications is illustrated in IS 2709 on 'Guide for selection of fits'.
    - The tolerances or upper and lower limits for various sizes of holes and shafts are specified in IS 919 on 'Recommendations for limits and fits for engineering'.
    - IS 10719 explains method for indicating surface texture on technical drawings.

  (iv) **Standards for Testing of Products**:

    - These standards, sometimes called 'codes', give procedures to test the products such as pressure vessel, boiler, crane and wire rope, where safety of the operator is an important consideration.
    - For example, IS 807 is a code of practice for design, manufacture, erection and testing of cranes and hoists.
    - The method of testing of pressure vessels is explained in IS 2825 on 'Code for unfired pressure vessels'.

  (v) **Standards for Engineering of Components**:

    - For example, there is a special publication SP46 prepared by Bureau of Indian Standards on 'Engineering Drawing Practice for Schools and Colleges' which covers all standards related to engineering drawing.

**1.2.1 Benefits of Standardization**

- Reductions in types and dimensions of identical components (inventory control).
- Reduction in manufacturing facilities.
- Easy to replace (Interchangeability).
1. INTRODUCTION TO MACHINE DESIGN

- No need to design or test the elements.
- Improves quality and reliability.
- Improves reputation of the company which manufactures standard components.
- Sometimes it ensures the safety.
- It results in overall cost reduction.

1.3 Preferred Numbers

- With the acceptance of standardization, there is a need to keep the standard sizes or dimensions of any component or product in discrete steps.
- The sizes should be spread over the wide range, at the same time these should be spaced properly.
- For example, if shaft diameters are to be standardized between 10 mm and 25 mm, then sizes should be like: 10 mm, 12.5 mm, 16 mm, 20 mm, 25 mm and not like: 10 mm, 11 mm, 13 mm, 18 mm, 25 mm.
- This led to the use of geometric series known as series of preferred numbers or preferred series.
- Preferred series are series of numbers obtained by geometric progression and rounded off.
- There are five basic series with step ratios of:
  \[ \sqrt[5]{10}, \sqrt[10]{10}, \sqrt[20]{10}, \sqrt[40]{10}, \text{and} \sqrt[80]{10} \]
- These ratios are approximately equal to 1.58, 1.26, 1.12, 1.06 and 1.03.
- The five basic series of preferred numbers (known as preferred series) are designated as: R5, R10, R20, R40, and R80.
- These series were first introduced by the French engineer Renard hence denoted by the symbol R.
- Each series is established by taking the first number one and multiplying it by a constant (or step or G.P.) ratio to get the second number.
- The second number is then multiplied by a step ratio to get the third number. The procedure is continued until the complete series is built up.
- The examples of preferred number series are: standard shaft diameters, power rating of coupling, centre distances of standard gear boxes, etc.
- The other series called derived series may be obtained.
- Series R 10/3 (1, ..., 10) indicates a derived series comprising of every third term of the R10 series and having the lower limit as 1 and higher limit as 10.
- The advantages of preferred series are as follows:
  (i) The difference in two successive terms has a fixed percentage.
  (ii) It provides small steps for small quantities and large steps for large quantities.
  (iii) The product range is covered with minimum number of sizes without restricting the choice of the customers.
Following table shows basic series of preferred numbers according to IS: 1076 (Part I) – 1985 (Reaffirmed 1990).

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Example 1.1  Find out the numbers of the R5 basic series from 1 to 10.
Solution:
The series factor for the R5 series is given by,
\[ \sqrt[5]{10} = 1.5849 \]
First number  = 1
Second number  = 1 \times 1.5849 = 1.5849 = (1.6)
Third number  = (1.5849) \times 1.5849 = 2.51 = (2.5)
Fourth number  = (1.5849)^2 \times 1.5849 = 3.98 = (4.0)
Fifth number  = (1.5849)^3 \times 1.5849 = 6.31 = (6.3)
Sixth number  = (1.5849)^4 \times 1.5849 = 10 = (10)
In above calculations, the rounded numbers are shown in brackets.
The complete series is given by,
1, 1.6, 2.5, 4.0, 6.3 and 10.0

Example 1.2  Find out series R 20/4 for 100 rpm to 1000 rpm.
Solution:
The series factor for the R20 series is given by,
\[ \sqrt[20]{10} = 1.122 \]
Since every fourth term of the R20 series is selected, the ratio factor (\(\phi\)) is given by,
\[ \phi = (1.122)^4 = 1.5848 \]
First number  = 100
Second number  = 100 \times 1.5848 = 158.48 = (160)
Third number  = 100 \times (1.5848)^2 = 251.16 = (250)
Fourth number  = 100 \times (1.5848)^3 = 398.04 = (400)
Fifth number  = 100 \times (1.5848)^4 = 630.81 = (630)
Sixth number  = 100 \times (1.5848)^5 = 999.71 = (1000)
In above calculations, the rounded numbers are shown in brackets.
The complete series is given by,
100, 160, 250, 400, 630 and 1000

Example 1.3  Standardize six speeds between 250 to 1400 rpm and State the series of torque for 0.5 kW drive.
Solution:
Date Given:
Maximum speed = 1400 rpm,
Minimum speed = 250 rpm,
Power = 0.5 kW,
No. of speeds = 6
\[ \frac{N_{\text{max}}}{N_{\text{min}}} = \phi^{z-1} \]
\[ \frac{1400}{250} = \phi^{6-1} \]
\[ 5.6 = \phi^5 \]
\[ \phi = (5.6)^{\frac{1}{5}} \]
\[ \phi = 1.411 \]
Series of speed:
\[N_1 = 250 \text{ rpm}\]
\[N_2 = 250 \times (1.411) = 352.75 = (350 \text{ rpm})\]
\[N_3 = 250 \times (1.411) \times (1.411) = 250 \times (1.411)^2 = 497.73 = (500 \text{ rpm})\]
\[N_4 = 250 \times (1.411)^2 \times (1.411) = 250 \times (1.411)^3 = 702.29 = (700 \text{ rpm})\]
\[N_5 = 250 \times (1.411)^3 \times (1.411) = 250 \times (1.411)^4 = 990.94 = (1000 \text{ rpm})\]
\[N_6 = 250 \times (1.411)^4 \times (1.411) = 250 \times (1.411)^5 = 1398.22 = (1400 \text{ rpm})\]

\[P = \frac{2\pi NT}{60000}\]
\[\therefore T = \frac{P \times 60000}{2\pi N}\]
\[\therefore T = \frac{0.5 \times 60000}{2\pi N}\]
\[\therefore T = \frac{4774.65}{N}\]

Series of torque:
\[T_1 = \frac{4774.65}{N_1} = \frac{4774.65}{250} = 19.1 \text{ N.m}\]
\[T_2 = \frac{4774.65}{N_2} = \frac{4774.65}{350} = 13.65 \text{ N.m}\]
\[T_3 = \frac{4774.65}{N_3} = \frac{4774.65}{500} = 9.55 \text{ N.m}\]
\[T_4 = \frac{4774.65}{N_4} = \frac{4774.65}{700} = 6.82 \text{ N.m}\]
\[T_5 = \frac{4774.65}{N_5} = \frac{4774.65}{1000} = 4.77 \text{ N.m}\]
\[T_6 = \frac{4774.65}{N_6} = \frac{4774.65}{140} = 3.41 \text{ N.m}\]

1.4 Aesthetic Considerations

- In a present days of buyer's market, with a number of products available in the market are having most of the parameters identical, the appearance of product is often a major factor in attracting the customer.

- This is particularly true for consumer durables like: automobiles, domestic refrigerators, television sets, etc.

- *Aesthetics* is defined as a set of principles of appreciation of beauty. It deals with the appearance of the product.

- *Appearance* is an outward expression of quality of the product and is the first communication of the product with the user.

- For any product, there exists a relationship between the functional requirement and the appearance of a product.

- The aesthetic quality contributes to the performance of the product, though the extent of contribution varies from the product to product.
• For example, the chromium plating of the automobile components improves the corrosion resistance along with the appearance.
• Similarly, the aerodynamic shape of the car improves the performance as well as gives the pleasing appearance.
• The following guidelines may be used in aesthetic design (*design for appearance*):
  (i) The appearance should contribute to the performance of the product.
  • For example, the aerodynamic shape of the car will have a lesser air resistance, resulting in the lesser fuel consumption.
  (ii) The appearance should reflect the function of the product.
  • For example, the aerodynamic shape of the car indicates the speed.
  (iii) The appearance should reflect the quality of the product.
  • For example, the robust and heavy appearance of the hydraulic press reflects its strength and rigidity.
  (iv) The appearance should not be at too much of extra cost unless it is a prime requirement.
  (v) The appearance should be suitable to the environment in which the product is used.
• At any stage in the product life, the aesthetic quality cannot be separated from the product quality.
• The growing importance of the aesthetic considerations in product design has given rise to a separate discipline, known as ‘industrial design’.
• The job of an industrial designer is to create new shapes and forms for the product which are aesthetically appealing

1.4.1 Form (Shape):
• There are five basic forms of the products, namely, step, taper, shear, streamline and sculpture, as shown in Figure.
• The external shape of any product is based on one or combination of these basic forms.
  (i) Step form:
  • The step form is a stepped structure having vertical accent.
  • It is similar to the shape of a multistorey building.
  (ii) Taper form:
  • The taper form consists of tapered blocks or tapered cylinders.
  (iii) Shear form:
  • The shear form has a square outlook.
  (iv) Streamline form:
  • The streamline form has a streamlined shape having a smooth flow as seen in automobile and aeroplane structures.
  (v) Sculpture form:
  • The sculpture form consists of ellipsoids, paraboloids and hyperboloids.
The sculpture and stream forms are suitable for mobile products like vehicles, while step and shear forms are suitable for stationary products.

1.4.2 Colour
- Colour is one of the major contributors to the aesthetic appeal of the product.
- Many colours are linked with different moods and conditions.
- The selection of the colour should be compatible with the conventions.
- Morgan has suggested the colour code given in the following Table.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>Danger, Hazard, Hot</td>
</tr>
<tr>
<td>Orange</td>
<td>Possible Danger</td>
</tr>
<tr>
<td>Yellow</td>
<td>Caution</td>
</tr>
<tr>
<td>Green</td>
<td>Safety</td>
</tr>
<tr>
<td>Blue</td>
<td>Caution-Cold</td>
</tr>
<tr>
<td>Grey</td>
<td>Dull</td>
</tr>
</tbody>
</table>

1.4.3 Material and Surface Finish
- The material and surface finish of the product contribute significantly to the appearance.
- The material like, stainless steel gives better appearance than the cast irons, plain carbon steels or low alloy steels.
- The brass or bronze give richness to the appearance of the product.
- The products with better surface finish are always aesthetically pleasing.
- The surface coating processes like: spray painting, anodizing, electroplating, etc. greatly enhances the aesthetic appeal of the product.
1.5 Ergonomic Considerations

- **Ergonomics** is defined as the scientific study of the man-machine-working environment relationship and the application of anatomical, physiological and psychological principles to solve the problems arising from this relationship.
- The word ‘ergonomics’ is formed from two Greek words: ‘ergon’ (work) and ‘nomos’ (natural laws).
- The final objective of the ergonomics is to make the machine fit for user rather than to make the user adapt himself or herself to the machine.
- It aims at decreasing the physical and mental stresses to the user.
- *Psychology* - Experimental psychologists who study people at work to provide data on such things as: Human sensory capacities, psychomotor performance, Human decision making, Human error rates, Selection tests and procedures, Learning and training.
- *Anthropometry* - An applied branch of anthropology concerned with the measurement of the physical features of people. Measures how tall we are, how far we can reach, how wide our hips are, how our joints flex, and how our bodies move.
- *Applied Physiology* - Concerns the vital processes such as cardiac function, respiration, oxygen consumption, and electromyography activity, and the responses of these vital processes to work, stress, and environmental influences.

1.5.1 Communication Between Man (User) and Machine

![Man-Machine Closed Loop System](image)

- Figure shows the man-machine closed loop system.
- The machine has a display unit and a control unit.
- A man (user) receives the information from the machine display through the sense organs.
- He (or she) then takes the corrective action on the machine controls using the hands or feet.
- This man-machine closed loop system is influenced by the working environmental factors such as: lighting, noise, temperature, humidity, air circulation, etc.
1.5.2 **Ergonomic Considerations in Design of Displays:**
The basic objective in the design of the displays is to minimize the fatigue to the user. The ergonomic considerations in the design of the displays are as follows:

- The scale should be clear and legible.
- The size of the numbers or letters on the scale should be taken appropriate.
- The pointer should have a knife-edge with a mirror in a dial to minimize the parallax error while taking the readings.
- The scale should be divided in a linear progression such as 0 – 10 – 20 – 30… and not as 0 – 5 – 25 – 45….
- The number of subdivisions between the numbered divisions should be as less as possible.
- The numbering should be in clockwise direction on a circular scale, from left to right on a horizontal scale and from bottom to top on a vertical scale.

![Examples of Displays](image)

1.5.3 **Ergonomic Considerations in Design of Controls:**
The ergonomic considerations in the design of the controls are as follows:

- The control devices should be logically positioned and easily accessible.
- The control operation should involve minimum and smooth moments.
- The control operation should consume minimum energy.
- The portion of the control device which comes in contact with user’s hand should be in conformity with the anatomy of human hands.
- The proper colours should be used for control devices and backgrounds so as to give the required psychological effect.
- The shape and size of the control device should be such that the user is encouraged to handle it in such a way as to exert the required force, but not excessive force, damaging the control or the machine.

1.5.4 **Working Environment**
- The working environment affects significantly the man-machine relationship.
- It affects the efficiency and possibly the health of the operator. The major working environmental factors are discussed below:
  
  **Lighting:**
  - The amount of light that is required to enable a task to be performed effectively depends upon the nature of the task, the cycle time, the reflective characteristics of the equipment involved and the vision of the operator.
  - The intensity of light in the surrounding area should be less than that at the task area. This makes the task area the focus of attention.
• Operators will become less tired if the lighting and colour schemes are arranged so that there is a gradual change in brightness and colour from the task area to the surroundings.
• The task area should be located such that the operator can occasionally relax by looking away from the task area towards a distinct object or surface.
• The distinct object or surface should not be so bright that the operator’s eyes take time to adjust to the change when he or she again looks at the task.
• Glare often causes discomfort and also reduces visibility, and hence it should be minimized or if possible eliminated by careful design of the lighting sources and their positions.

Noise:
• The noise at the work place causes annoyance, damage to hearing and reduction of work efficiency.
• The high pitched noises are more annoying than the low pitched noises.
• Noise caused by equipment that a person is using is less annoying than that caused by the equipment, being used by another person, because the person has the option of stopping the noise caused by his own equipment, at least intermittently.
• The industrial safety rules specify the acceptable noise levels for different work places.
• If the noise level is too high, it should be reduced at the source by maintenance, by the use of silencers and by placing vibrating equipment on isolating mounts.
• Further protection can be obtained by placing the sound-insulating walls around the equipment.
• If required, ear plugs should be provided to the operators to reduce the effect of noise.

Temperature:
• For an operator to perform the task efficiently, he should neither feel hot nor cold.
• When the heavy work is done, the temperature should be relatively lower and when the light work is done, the temperature should be relatively higher.
• The optimum required temperature is decided by the nature of the work.
• The deviation of the temperature from the optimum temperature required reduces the efficiency of the operator.

Humidity and air circulation:
• Humidity has little effect on the efficiency of the operator at ordinary temperatures. However, at high temperatures, it affects significantly the efficiency of the operator.
At high temperatures, the low humidity may cause discomfort due to drying of throat and nose and high humidity may cause discomfort due to sensation of stuffiness and over sweating in an ill-ventilated or crowded room.

The proper air circulation is necessary to minimize the effect of high temperature and humidity.

1.6 Manufacturing considerations in Design or Design for Manufacturing and Assembly (DFMA)

- The design effort makes up only 5% of the total cost of a product.
- However, it usually determines more than 70% of the manufacturing cost of the product.
- Hence, only 30% of the product’s cost can be changed once the design is finalised and drawings are prepared.
- So the best strategy to lower product cost is to recognize the importance of manufacturing early in the design stage.
- It starts with the formation of the design team which tends to be multi-disciplinary, including engineers, manufacturing managers, cost accountants, and marketing and sales professionals.
- The most basic approach to design for manufacturing and assembly is to apply design guidelines.
- We should use design guidelines with an understanding of design goals.
- Make sure that the application of a guideline improves the design concept on those goals.

Benefits of DFMA

Design for manufacturing and assembly are simple guidelines formulated to get the below benefits.

- It simplifies the design.
- It simplifies the production processes and decreases the product cost.
- It improves product quality and reliability (because if the production process is simplified, then there is less opportunity for errors).
- It decreases the assembly cost.
- It decreases the assembly time.
- It reduces time required to bring a new product into market.

1.6.1 Design for Assembly or DFA Guidelines (Assembly Considerations in Design)

1. Reduce the Part Counts:

- Design engineers should try for product design that uses the minimum number of parts.
- Fewer parts result in lower costs.
- It also makes assembly simpler and fewer chances of defects.
- Minimize part count by incorporating multiple functions into single parts.
• Several parts could be fabricated by using different manufacturing processes (sheet metal forming, injection molding etc.).

![Fig 1.5 Reduce the Part Counts](image1)

2. Use modular designs:

![Fig 1.6 Use modular designs](image2)

• Modularize multiple parts into single sub-assemblies.
• Modular design reduces the number of parts being assembled at any one time and also simplifies final assembly.
• Field service becomes simple, fast and cheap because dismantling is faster and requires fewer tools.

3. Assemble in the open:
• Design to allow assembly in open spaces, not confined spaces.
• Assembly operation should be carried out in clear view. This is important in manual assembly.
4. Optimize part handling:
   - Design parts so they do not tangle or stick to each other or require special handling prior to assembly.

5. Do not fight gravity:
   - Design products so that they can be assembled from the bottom to top along vertical axis.
   - Design the first part large and wide to be stable and then assemble the smaller parts on top of it sequentially.

6. Design for part identity (symmetry):
   - Symmetric parts are easy to assemble.
   - Maximizing part symmetry will make orientation unnecessary.
   - Features should be added to enhance symmetry wherever required.
7. Eliminate Fasteners:
   - Fasteners are a major obstacle to efficient assembly and should be avoided wherever possible.
   - They are difficult to handle and can cause jamming, if defective.
   - If the use of fasteners cannot be avoided, limit the number of different types of fasteners used.

   ![Fig 1.10 Eliminate Fasteners](image)

8. Design parts for simple assembly:
   - Design parts with orienting features to make alignment easier.

   ![Fig 1.11 Design parts for simple assembly](image)

9. Parts should easily indicate orientation for insertion:
   - Parts should have self-locking features so that the precise alignment during assembly is not required or provide marks (color) to make orientation easier.

   ![Fig 1.12 Parts should easily indicate orientation for insertion](image)
10. Standardize parts to reduce variety:
   - Using the same commodity items such as fasteners can avoid errors.
   - It also reduces the cost.

![Fig 1.13 Standardize parts to reduce variety](image1.png)

11. Color code parts that are different but shaped similarly:
   - Distinguish different parts that are shaped similarly by non-geometric means, such as color coding.

12. Design the mating features for easy insertion:
   - Add chamfers or other features to make parts easier to insert.

![Fig 1.14 Design the mating features for easy insertion](image2.png)

13. Provide alignment features:
14. Place fasteners away from obstructions:
   - It is better to locate fasteners in place where one has access to the fastener.

*Fig 1.16  Place fasteners away from obstructions*

15. Deep channels should be sufficiently wide to provide access to fastening tools:

*Fig 1.17  Deep channels should be sufficiently wide to provide access to fastening tools*

16. Providing flats for uniform fastening and fastening ease:
   - Do not fasten against angled surfaces.

*Fig 1.18  Providing flats for uniform fastening and fastening ease*
1.6.2 Design of components for casting

Why casting?
- Complex parts which are difficult to machine, are made by the casting process.
- Almost any metal can be melted and cast. Most of the sand cast parts are made of cast iron, aluminum alloys and brass.
- The size of the sand casting can be as small as 10 g and as large as 200 x 10^3 kg.
- Sand castings have irregular and grainy surfaces and machining is required if the part is moving with respect to some other part or structure.
- Cast components are stable, rigid and strong compared with machined or forged parts.
- Typical examples of cast components are machine tool beds and structures, cylinder blocks of internal combustion engines, pumps and gear box housings.

Basic considerations of casting
- Always keep the stressed areas of the parts in compression
- Round all external corners
- Wherever possible, the section thickness throughout should be held as uniform as compatible with overall design considerations
- Avoid concentration of metal at the junctions
- Avoid very thin sections
- The wall adjacent to the drilled hole should have a thickness equivalent to the thickness of the main body
- Oval-shaped holes are preferred with larger dimensions along the direction of forces
- To facilitate easy removal, the pattern must have some draft
- Outside bosses should be omitted to facilitate a straight pattern draft

(1) Always keep the stressed areas of the parts in compression
- Cast iron has more compressive strength than its tensile strength.
- The castings should be placed in such a way that they are subjected to compressive rather than tensile stresses.
• When tensile stresses are unavoidable, a clamping device such as a tie rod or a bearing cap should be considered.
• The clamping device relieves the cast iron components from tensile stresses.

![Fig 1.20](image)

*Fig 1.20 (a) Original component (b) Use of Tie-rod (c) Use of Bearing-cap*

2. Round all external corners
• It increases the endurance limit of the component and reduces the formation of brittle chilled edges.
• When the metal in the corner cools faster than the metal adjacent to the corner, brittle chilled edges are formed.
• Appropriate fillet radius reduces the stress concentration.

![Fig 1.21](image)

*Fig 1.21 Round all external corners*

3. Wherever possible, the section thickness throughout should be held as uniform as compatible with overall design considerations
• Abrupt changes in the cross-section result in high stress concentration.
• If the thickness is to be varied at all, the change should be gradual
(4) Avoid concentration of metal at the junctions

- At the junction, there is a concentration of metal.
- Even after the metal on the surface solidifies, the central portion still remains in the molten stage, with the result that a shrinkage cavity or blowhole may appear at the centre.
- There are two ways to avoid the concentration of metal.
  - One is to provide a cored opening in webs and ribs.
  - Alternatively, one can stagger the ribs and webs.

Fig 1.22 Uniform thickness throughout

Fig 1.23 Cored Holes

Fig 1.24 Staggered ribs
(5) Avoid very thin sections
- It depends upon the process of casting such as sand casting, permanent mold casting or die casting

(6) The wall adjacent to the drilled hole should have a thickness equivalent to the thickness of the main body
- The inserted stud will not restore the strength of the original thickness.

Fig 1.25 Wall adjacent to the drilled hole should have optimum thickness

(7) Oval-shaped holes are preferred with larger dimensions along the direction of forces

Fig 1.26 Oval-shaped holes

(8) To facilitate easy removal, the pattern must have some draft
- A minimum draft of 3° should be provided.

Fig 1.27 Draft allowance
1.6.3 Design of components for Forging

Why forging?

- A properly designed forging is not only sound with regard to strength but it also helps reduce the forging forces, improves die life and simplifies die design.

- Forged components are usually made of steels and non-ferrous metals.

- They can be as small as a gudgeon pin and as large as a crankshaft.

- Forged components are used under the following circumstances:
  
  i. Moving components requiring light weight to reduce inertia forces, e.g. connecting rod of I. C. engines.
  
  ii. Components subjected to excessive stresses, e.g. aircraft structures.
  
  iii. Small components that must be supported by other structures or parts, e.g. hand tools and handles.
  
  iv. Components requiring pressure tightness where the part must be free from internal cracks, e.g. valve bodies.
  
  v. Components whose failure would cause injury and expensive damage are forged for safety.

Basic considerations of Forging

- While designing a forging, advantage should be taken of the direction of fibre lines.

- The forged component should be provided with an adequate draft.

- The parting line should be in one plane as far as possible and it should divide the forging into two equal parts.

- The forging should be provided with adequate fillet and corner radius.

- Thin sections and ribs should be avoided in forged components.

(9) Outside bosses should be omitted to facilitate a straight pattern draft

Fig 1.28 Outside bosses should be omitted
(1) While designing a forging, advantage should be taken of the direction of fibre lines

- There are no fibre lines in the cast component and the grains are scattered.
- In case of a component prepared by machining methods, such as turning or milling, the original fibre lines of rolled stock are broken.
- It is only in case of forged parts that the fibre lines are arranged in a favorable way to withstand stresses due to external load.
- While designing a forging, the profile is selected in such a way that fibre lines are parallel to tensile forces and perpendicular to shear forces.

(2) The forged component should be provided with an adequate draft

- The draft angle is provided for an easy removal of the part from the die impressions.

(3) The parting line should be in one plane as far as possible and it should divide the forging into two equal parts

- When the parting line is broken, it results in unbalanced forging forces, which tends to displace the two die halves.
(4) The forging should be provided with adequate fillet and corner radius
- Sharp corners result in increasing difficulties in filling the material, excessive forging forces, and poor die life.
- The magnitude of fillet radius depends upon the material, the size of forging and the depth of the die cavity.

(5) Thin sections and ribs should be avoided in forged components
- A thin section cools at a faster rate in the die cavity and requires excessive force for plastic deformation.
- It reduces the die life, and the removal of the component from the die cavities becomes difficult.

1.6.4 Design of components for Welding
Why welding?
- Welding is the most important method of joining the parts into a complex assembly.

Basic considerations of Welding
- Select the Material with High Weldability
- Use Minimum Number of Welds
- Use Standard Components
- Select Proper Location for the Weld
- Prescribe Correct Sequence of Welding
- Reduce Edge Preparation
- Reduce the Scrap
- Avoid Weld Accumulation

(1) Select the Material with High Weldability
- In general, low carbon steel is more easily welded than high carbon steel.
- Higher carbon content tends to harden the welded joint, as a result of which the weld is susceptible to cracks.

(2) Use Minimum Number of Welds
- Only the adjoining area of the joint is heated up, which has no freedom to expand or contract.
• Uneven expansion and contraction in this adjoining area and parent metal results in distortion.

• Since distortion always occurs in welding, the design should involve a minimum number of welds and avoid over welding.

• It will not only reduce the distortion but also the cost.

(3) Use Standard Components
• The designer should specify standard sizes for plates, bars and rolled sections.
• Non-standard sections involve flame cutting of plates and additional welding.
• As far as possible, the designer should select plates of equal thickness for a butt joint.

(4) Select Proper Location for the Weld
• The welded joint should be located in an area where stresses and deflection are not critical.
• Also, it should be located at such a place that the welder and welding machine has unobstructed access to that location.

(5) Prescribe Correct Sequence of Welding
• The designer should consider the sequence in which the parts should be welded together for minimum distortion.
• This is particularly important for a complex job involving a number of welds.
• An incorrect sequence of welding causes distortion and sometimes cracks in the weld metal due to stress concentration at some point in fabrication.
• A correct welding sequence distributes and balances the forces and stresses induced by weld contraction.

(6) Reduce edge preparation
• It is necessary to prepare bevel edges for the components prior to welding operation.
• This preparatory work can be totally eliminated by making a slight change in the arrangement of components.

---

Fig 1.32  Reduce edge preparation
(7) Reduce the scrap
- Many times, fabrication is carried out by cutting steel plates followed by welding.
- The aim of the designer is to minimize scrap in such process.
- The circular top plate and annular bottom plate are cut from two separate plates resulting in excess scrap as shown in Figure.
- Making a slight change in design, the top plate and annual bottom plate can be cut from one plate reducing scrap and material cost.

![Diagram showing incorrect and correct methods of cutting steel plates](image)

*Fig 1.33  Reduce the scrap*

(8) Avoid Weld Accumulation
- Accumulation of welded joints results in shrinkage stresses.

![Diagram showing incorrect and correct methods of welding](image)

*Fig 1.34  Avoid Weld Accumulation*

### 1.6.5 Design of components for Machining

Why machining?
- Machined components are widely used in all industrial products.
- They are usually made from ferrous and non-ferrous metals.
- They are as small as a gear in a wristwatch and as large as huge turbine housing.
- Metal-cutting operations: Turning, Milling, drilling, shaping, boring, reaming etc.
- Surface finishing operations: Grinding, buffing etc.
- Machined components are used under the following circumstances:
  i. Components requiring precision and high dimensional accuracy
  ii. Components requiring flatness, roundness, parallelism or circularity for their proper functioning
  iii. Components of interchangeable assembly
  iv. Components, which are in relative motion with each other or with some fixed part
Basic considerations of Machining

- Avoid Machining
- Specify Liberal Tolerances
- Avoid Sharp Corners
- Use Stock Dimensions
- Design Rigid Parts
- Avoid Shoulders and undercuts
- Avoid Hard Materials
- Design machined parts with features that can be produced in a minimum number of setups
- Machine only functional surface
- Holes should be parallel or perpendicular to the axis of the part
- Design holes with conical ends
- Use minimum number of machines
- Design the product for existing machining facilities
- Machining should be completed in minimum machining operations
- Use standard size tooling
- Avoid parts with very large L/D ratios
- Machined surfaces should be parallel or perpendicular to each other as well as to base

1. Avoid Machining
   - Machining operations increase cost of the component.
   - Components made by casting or forming methods are usually cheaper.
   - Therefore, as far as possible, the designer should avoid machined surfaces.

2. Specify Liberal Tolerances
   - The secondary machining operations like grinding or reaming are costly.
   - Therefore, depending upon the functional requirement of the component, the designer should specify the most liberal dimensional and geometric tolerances.
   - Closer the tolerance, higher is the cost.

3. Avoid Sharp Corners
   - The Sharp corners result in stress concentration. Therefore, the designer should avoid shapes that require sharp corners.

4. Use Stock Dimensions
   - Raw materials like bars are available in standard sizes.
   - Using stock dimensions eliminates machining operations.
   - For example, a hexagonal bar can be used for a bolt and only the threaded portion can be machined. This will eliminate machining of hexagonal surfaces.

5. Design Rigid Parts
   - Any machining operation such as turning or shaping induces cutting forces on the components.
• The component should be rigid enough to withstand these forces.
• In this respect, components with thin walls or webs should be avoided.

(6) Avoid Shoulders and undercuts
• Shoulders and undercuts usually involve separate operations and separate tools, which increase the cost of machining.

(7) Avoid Hard Materials
• Hard materials are difficult to machine.
• They should be avoided unless such properties are essential for the functional requirement of the Product.

(8) Design machined parts with features that can be produced in a minimum number of setups

![Fig 1.35 Minimum number of setups](image)

(9) Machine only functional surface

![Fig 1.36 Machine only functional surface](image)

1.6.6 Design for Creep (Thermal Considerations in Design)
• Creep is defined as slow and progressive deformation of the material with time under a constant stress.
• Creep deformation is a function of stress level and temperature.
• Therefore, creep deformation is higher at higher temperature and creep becomes important for components operating at high temperatures.
• Deformation due to creep must remain within permissible limit and rupture must not occur during the service life.
• Most of the machine elements are used in engineering applications which operate at ordinary temperatures.
• Some applications where machine elements are subjected to high temperatures: I. C. Engines, Turbines, Boilers, Pressure Vessels in process industries etc.
• The rate of deformation is called the creep rate.
• It is the slope of the line in a curve of Creep strain vs. Time.
• Strain is deformation per unit length.
An idealized creep curve is shown in the above figure.

When the load is applied at the beginning of the creep test, the instantaneous elastic deformation OA occurs.

This elastic deformation is followed by the creep curve ABCD.

Creep occurs in three stages.

(i) Primary Creep

(ii) Secondary Creep

(iii) Tertiary Creep

Primary Creep

The first stage called primary creep is shown by AB on the curve.

During this stage, the creep rate, i.e., the slope of the creep curve from A to B progressively decreases with time.

Deformation becomes more and more difficult as strain increases, i.e. the material experiences strain hardening.

The metal strain hardens to support the external load.

Secondary Creep

The second stage called secondary creep is shown by BC on the curve.

During this stage, the creep rate is constant.

This stage occupies a major portion of the life of the component. The designer is mainly concerned with this stage.

The occurrence of a constant strain-rate is explained in terms of a balance between strain hardening and structure recovery (a softening process determined by the high temperature).

Tertiary Creep

The third stage called tertiary creep is shown by CD on the creep curve.

During this stage, the creep rate is accelerated due to necking and also due to formation of voids along the grain boundaries.

Therefore, creep rate rapidly increases and finally results in fracture at the point D.
Creep properties are determined by experiments and these experiments involve very long periods stretching into months.

Design Considerations to avoid Creep
1. Select material which gives good performance at high temperatures.
2. The component must be designed considering operating temperatures.
3. Use large grains or mono-crystals (small grains increase grain motion at the grain boundaries)
4. Addition of solid solutions to eliminate vacancies.
5. Consult creep test data during materials selection.
1.6.7 Design for Wear (Wear Considerations in Design)

Wear
- Wear can be defined as the progressive loss or removal of material from the surfaces in contact, as a result of the relative motion.
- Wear is not a material property but it is a response of the engineering system.

Effects of Wear
- It distorts the original geometry and surface finish of the machine elements.
- It increases the clearance between the mating parts, resulting in additional load, vibrations and noise.
- It reduces the functionality of the machine elements.
- It reduces the life of the machine elements and machine.
- It damages the machine elements and machine.
- Therefore, wear is one of the important design considerations while designing any machine element.

Applications where Wear is Undesirable
- Gears, Brakes, Clutches, Tyres, Piston and cylinder, Cam and follower, Bearings etc.

Applications where Wear is Desirable
- Machining, Grinding, Writing with pencils etc.

Design Considerations for Wear
1. Proper lubrication
2. Surface coating
3. Surface hardening
4. Reducing the surface roughness of the contacting surfaces
5. Sealing the contact areas to avoid the foreign abrasive particles.

1.6.8 Contact Stresses (or Hertz contact stress)

Hertz Contact Stress
- The theoretical contact area of two spheres is a point.
- The theoretical contact area of two parallel cylinders is a line.
- In reality, a small contact area is being created through elastic deformation, thereby inducing the stresses considerably.
- These contact stresses are called Hertz contact stresses.
• Two design cases will be considered.
  1. Sphere – Sphere Contact

(a) Two spheres held in contact by force F
(b) Contact stress has an elliptical distribution across contact over zone of diameter 2a

Fig 1.41  Sphere – Sphere Contact

• The theoretical contact area of two spheres is a point.
• Consider two solid elastic spheres held in contact by a force F such that their point of contact expands into a circular area of radius a, given as, \( a = k_a \frac{3}{4} \sqrt[3]{F} \)

\[
K_a = \left[ \frac{3}{8} \times \frac{(1-v^2)}{E_1} \frac{(1-v^2)}{E_2} \frac{1}{d_1 d_2} \right]^{\frac{1}{3}}
\]

F = applied force
\( v_1, v_2 \) = Poisson’s ratio for spheres 1 and 2 respectively
\( E_1, E_2 \) = Young’s modulus for spheres 1 and 2 respectively
\( d_1, d_2 \) = Diameters for spheres 1 and 2 respectively

• The maximum contact stress \( (\sigma_{CH}) \) occurs at the center point of the contact area and it is given by, \( \sigma_{CH} = \frac{3F}{2\pi a^2} \)

2. Cylinder – Cylinder Contact

(a) Two right circular cylinders held in contact by force F uniformly distributed along length l
(b) Contact stress has an elliptical distribution across contact over zone of width 2b

Fig 1.42  Cylinder – Cylinder Contact
• The theoretical contact area of two parallel cylinders is a line.
• Consider two solid elastic cylinders held in contact by forces \( F \) uniformly distributed along the cylinder length \( l \).
• The resulting pressure causes the line of contact to become a rectangular contact zone of half width \( b \) given as, \( b = k_b \sqrt{F} \)

\[
K_b = \left[ \frac{2}{\pi l} \times \left( \frac{(1-v_1^2)}{E_1} + \frac{(1-v_2^2)}{E_2} \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \]

where
- \( F \) = applied force
- \( v_1, v_2 \) = Poisson’s ratio for cylinders 1 and 2 respectively
- \( E_1, E_2 \) = Young’s modulus for cylinders 1 and 2 respectively
- \( d_1, d_2 \) = Diameters for cylinders 1 and 2 respectively
- \( l \) = Length for cylinders (\( l_1 = l_2 \) assumed)

• The maximum contact stress \( (\sigma_{CH}) \) between the cylinders acts along a longitudinal line at the center of the rectangular contact area, and is computed as

\[
\sigma_{CH} = \frac{2F}{\pi b l}
\]

1.7 Material Selection in Machine Design

• Selection of a proper material for the machine component is one of the most important steps in the process of machine design.
• The best material is one which will serve the desired purpose at minimum cost.
• It is not always easy to select such a material and the process may involve the trial and error method.
• The factors which should be considered while selecting the material for a machine component are as follows:
  1. Availability
  2. Cost
  3. Mechanical Properties
  4. Manufacturing Considerations

1. Availability:
• The material should be readily available in the market, in large enough quantities to meet the requirement.
• Cast iron and aluminium alloys are always available in abundance while shortage of lead and copper alloys is a common experience.

2. Cost:
• Cost: For every application, there is a limiting cost beyond which the designer cannot go.
• When the limit is exceeded, the designer has to consider other alternative materials.
• In cost analysis, there are two factors, namely cost of material and cost of processing the material into finished goods.
1. INTRODUCTION TO MACHINE DESIGN

• It is likely that the cost of material might be low, but the processing may involve costly manufacturing operations.

3. Mechanical Properties:
• Mechanical properties are the most important technical factor governing the selection of material.
• They include strength under static and fluctuating loads, elasticity, plasticity, stiffness, resilience, toughness, ductility, malleability and hardness.
• Depending upon the conditions and the functional requirement, different mechanical properties are considered and a suitable material is selected.
• The piston rings should have a hard surface to resist wear due to rubbing action with the cylinder surface, and surface hardness is the selection criterion.
• In case of bearing materials, a low coefficient of friction is desirable while clutch or brake requires a high coefficient of friction.

4. Manufacturing Considerations:
• In some applications, machinability of material is an important consideration in selection.
• Sometimes, an expensive material is more economical than a low priced one, which is difficult to machine.
• Free cutting steels have excellent machinability, which is an important factor in their selection for high strength bolts, axles and shafts.
• Where the product is of complex shape, castability or ability of the molten metal to flow into intricate passages is the criterion of material selection.
• In fabricated assemblies of plates and rods, weldability becomes the governing factor.
• The manufacturing processes, such as casting, forging, extrusion, welding and machining govern the selection of material.

1.8 Mechanical Properties of Metals
• The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load.

1. Strength: It is the ability of a material to resist the externally applied forces without breaking or yielding.
• The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness: It is the ability of a material to resist deformation under stress.
• The modulus of elasticity is the measure of stiffness.

3. Elasticity: It is the property of a material to regain its original shape after deformation when the external forces are removed.
• This property is desirable for materials used in tools and machines.
• It may be noted that steel is more elastic than rubber.
4. Plasticity: It is property of a material which retains the deformation produced under load permanently.
   • This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility: It is the property of a material enabling it to be drawn into wire with the application of a tensile force.
   • The ductility is usually measured by the terms, percentage elongation and percentage reduction in area.
   • The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Malleability: It is a special case of ductility which permits materials to be rolled or hammered into thin sheets.
   • The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

7. Britteness: It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion.
   • Brittle materials when subjected to tensile loads, it snaps off without giving any sensible elongation.
   • Cast iron is a brittle material.

8. Toughness: It is the property of a material to resist fracture due to high impact loads like hammer blows.
   • The toughness of the material decreases when it is heated.
   • It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed up to the point of fracture.
   • This property is desirable in parts subjected to shock and impact loads.

9. Machinability: It is the property of a material which refers to a relative case with which a material can be cut.

10. Resilience: It is the property of a material to absorb energy and to resist shock and impact loads.
    • It is measured by the amount of energy absorbed per unit volume within elastic limit.
    • This property is essential for spring materials.

11. Creep: When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep.
    • This property is considered in designing internal combustion engines, boilers and turbines.
12. Fatigue: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue.
- The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size.
- This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness: It is a very important property of the metals and has a wide variety of meanings.
- It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc.
- It also means the ability of a metal to cut another metal.
- The hardness is usually expressed in numbers which are dependent on the method of making the test.
- The hardness of a metal may be determined by the following tests:
  (a) Brinell hardness test, (b) Rockwell hardness test, (c) Vickers hardness test and (d) Shore scleroscope.

1.9 Effect of Impurities on Steel
- The following are the effects of impurities like silicon, sulphur, manganese and phosphorus on steel.

1. Silicon: The amount of silicon in the finished steel usually ranges from 0.05 to 0.30%.
- Silicon is added in low carbon steels to prevent them from becoming porous.
- It removes the gases and oxides, prevent blow holes and thereby makes the steel tougher and harder.

2. Sulphur: It occurs in steel either as iron sulphide or manganese sulphide.
- Iron sulphide because of its low melting point produces red shortness, whereas manganese sulphide does not affect so much.
- Therefore, manganese sulphide is less objectionable in steel than iron sulphide.

3. Manganese: It serves as a valuable deoxidizing and purifying agent in steel.
- Manganese also combines with sulphur and thereby decreases the harmful effect of this element remaining in the steel.
- When used in ordinary low carbon steels, manganese makes the metal ductile and of good bending qualities.
- In high speed steels, it is used to toughen the metal and to increase its critical temperature.

4. Phosphorus: It makes the steel brittle.
- It also produces cold shortness in steel.
1. In low carbon steels, it raises the yield point and improves the resistance to atmospheric corrosion.

2. The sum of carbon and phosphorus usually does not exceed 0.25%.

1.10 Effects of Alloying Elements on Steel

- Alloy steel may be defined as steel to which elements other than carbon are added in sufficient amount to produce an improvement in properties.
- The alloying is done for specific purposes to increase wearing resistance, corrosion resistance and to improve electrical and magnetic properties, which cannot be obtained in plain carbon steels.
- The chief alloying elements used in steel are nickel, chromium, molybdenum, cobalt, vanadium, manganese, silicon and tungsten.
- These elements may be used separately or in combination to produce the desired characteristic in steel.

1. Nickel: It increases the strength and toughness of the steel.
   - These steels contain nickel from 2 to 5% and carbon from 0.1 to 0.5%.
   - In this range, nickel contributes great strength and hardness with high elastic limit, good ductility and good resistance to corrosion.
   - An alloy containing 25% nickel possesses maximum toughness and offers the greatest resistance to rusting, corrosion and burning at high temperature.
   - It has proved to be of advantage in the manufacture of boiler tubes, valves for use with superheated steam, valves for I. C. engines and spark plugs for petrol engines.
   - A nickel steel alloy containing 36% of nickel is known as invar. It has nearly zero coefficient of expansion. So it is in great demand for measuring instruments and standards of lengths for everyday use.

2. Chromium: It is used in steels as an alloying element to combine hardness with high strength and high elastic limit.
   - It also imparts corrosion-resisting properties to steel.
   - The most common chrome steels contains from 0.5 to 2% chromium and 0.1 to 1.5% carbon.
   - The chrome steel is used for balls, rollers and races for bearings.
   - A nickel chrome steel containing 3.25% nickel, 1.5% chromium and 0.25% carbon is much used for armor plates.
   - Chrome nickel steel is extensively used for motor car crankshafts, axles and gears requiring great strength and hardness.

3. Vanadium: It aids in obtaining a fine grain structure in tool steel.
   - The addition of a very small amount of vanadium (less than 0.2%) produces a marked increase in tensile strength and elastic limit in low and medium carbon steels without a loss of ductility.
• The chrome-vanadium steel, containing about 0.5 to 1.5% chromium, 0.15 to 0.3% Vanadium and 0.13 to 1.1% carbon have extremely good tensile strength, elastic limit, endurance limit and ductility.
• These steels are frequently used for parts such as springs, shafts, gears, pins and many drop forged parts.

4. Tungsten: It prohibits grain growth, increases the depth of hardening of quenched steel and confers the property of remaining hard even when heated to red colour.
• It is usually used in conjunction with other elements.
• Steel containing 3 to 18% tungsten and 0.2 to 1.5% carbon is used for cutting tools.
• The principal uses of tungsten steels are for cutting tools, dies, valves, taps and permanent magnets.

5. Cobalt: It gives red hardness by retention of hard carbides at high temperatures.
• It tends to decarburise steel during heat-treatment.
• It increases hardness and strength and also residual magnetism and coercive magnetic force in steel for magnets.

6. Manganese. It improves the strength of the steel in both the hot rolled and heat treated condition.
• The manganese alloy steels containing over 1.5% manganese with a carbon range of 0.40 to 0.55% are used extensively in gears, axles, shafts and other parts where high strength combined with fair ductility is required.
• The principal use of manganese steel is in machinery parts subjected to severe wear. These steels are all cast and ground to finish.

7. Silicon: The silicon steels behave like nickel steels.
• These steels have a high elastic limit as compared to ordinary carbon steel.
• Silicon steels containing from 1 to 2% silicon and 0.1 to 0.4% carbon and other alloying elements are used for electrical machinery, valves in I. C. engines, springs and corrosion resisting materials.

8. Molybdenum: A very small quantity (0.15 to 0.30%) of molybdenum is generally used with chromium and manganese (0.5 to 0.8%) to make molybdenum steel.
• These steels possess extra tensile strength and are used for air-plane fuselage and automobile parts.
• It can replace tungsten in high speed steels.

1.11 Heat Treatment of Steels
• It can be defined as an operation or a combination of operations, involving the heating and cooling of a metal or an alloy in the solid state for the purpose of obtaining certain desirable conditions or properties without change in chemical composition.
• The aim of heat treatment is to achieve one or more of the following objects:
  1. To increase the hardness of metals.
2. To relieve the stresses set up in the material after hot or cold working.
3. To improve machinability.
4. To soften the metal.
5. To modify the structure of the material to improve its electrical and magnetic properties.
6. To change the grain size.

- Following are the various heat treatment processes commonly employed in engineering practice are as follow:
  1. Normalising
  2. Annealing
     (a) Full annealing
     (b) Process annealing
  3. Spheroidising
  4. Hardening
  5. Tempering
  6. Surface hardening or case hardening
DESIGN AGAINST FLUCTUATING LOADS

Course Contents

2.1 Stress Concentration
2.2 Fatigue
2.3 Factors Affecting Endurance Strength
2.4 Design for Reversed Stresses
2.5 Design for Fluctuating Stresses
2.6 Modified Goodman Diagrams

Examples
2.1 Stress Concentration

- It is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross-section.
- A plate with a small circular hole, subjected to tensile stress is shown in Fig. 2.1.
- The localized stresses in the neighborhood of the hole are far greater.

![Stress Concentration Diagram](image)

**Fig 2.1 Stress Concentration**

2.1.1 Causes of Stress Concentration

1. Variation in Properties of Materials:
   - In design of machine components, it is assumed that the material is homogeneous throughout the component.
   - In practice, there is variation in material properties from one end to another due to the following factors:
     (a) Internal cracks and flaws like blow holes;
     (b) Cavities in welds;
     (c) Air holes in steel components; and
     (d) Nonmetallic or foreign inclusions.
   - These variations act as discontinuities in the component and cause stress concentration.

2. Load Application:
   - Machine components are subjected to forces.
   - These forces act either at a point or over a small area of the component.
   - Since the area is small, the pressure at these points is excessive. This results in stress concentration.
   - The examples of these load applications are as follows:
     (a) Contact between the meshing teeth of the driving and the driven gear
(b) Contact between the cam and the follower
(c) Contact between the balls and the races of ball bearing
(d) Contact between the rail and the wheel
(e) Contact between the crane hook and the chain
• In all these cases, the concentrated load is applied over a very small area which results in stress concentration.

3. Abrupt Changes in Section:
• In order to mount gears, sprockets, pulleys and ball bearings on a transmission shaft, steps are cut on the shaft and shoulders are provided from assembly considerations.
• Although these features are essential, they create change of the cross-section of the shaft.
• This results in stress concentration at these cross-sections.

4. Discontinuities in the Component:
• Certain features of machine components such as oil holes or oil grooves, keyways and splines, and screw threads result in discontinuities in the cross-section of the component.
• There is stress concentration in the area of these discontinuities.

5. Machining Scratches:
• Machining scratches, stamp marks or inspection marks are surface irregularities, which cause stress concentration.

2.1.2 Methods to Reduce Stress Concentration
1. Additional Notches and Holes in Tension Member
• A flat plate with a V-notch subjected to tensile force is shown in Fig. 2.3 (a).
• It is observed that a single notch results in a high degree of stress concentration.
• The severity of stress concentration is reduced by three methods: (a) Use of multiple notches, (b) Drilling additional holes; and (c) Removal of undesired material
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Fig 2.3  Reduction of Stress Concentration due to V-notch: (a) Original Notch (b) Multiple Notches (c) Drilled Holes (d) Removal of Undesirable Material
- These methods are illustrated in Fig. 2.3 (b), (c) and (d) respectively.
- In these three methods, the sharp bending of a force flow line is reduced and it follows a smooth curve.

2. Fillet Radius, Undercutting and Notch for Member in Bending:

Fig 2.4  Reduction of stress Concentration due to Abrupt change in Cross-section: (a) Original Component (b) Fillet Radius (c) Undercutting (d) Additional Notch
- A bar of circular cross-section with a shoulder and subjected to bending moment is shown in Fig. 2.4 (a).
- Ball bearings, gears or pulleys are seated against this shoulder.
- The shoulder creates a change in cross-section of the shaft, which results in stress concentration.
- There are three methods to reduce stress concentration at the base of this shoulder.
- Fig. 2.4 (b) shows the shoulder with a fillet radius r. This results in gradual transition from small diameter to a large diameter.
• The fillet radius should be as large as possible in order to reduce stress concentration.
• In practice, the fillet radius is limited by the design of mating components.
• The fillet radius can be increased by undercutting the shoulder as illustrated in Fig. 2.4 (c).
• A notch results in stress concentration.
• Cutting an additional notch is an effective way to reduce stress concentration. This is illustrated Fig. 2.4 (d).

3. Drilling Additional Holes for Shaft:

![Diagram of shaft with keyway showing fillet radius and drilled holes](image)

**Fig 2.5 Reduction of Stress Concentration in Shaft with Keyway:** (a) Original Shaft (b) Drilled Holes (c) Fillet Radius

- A transmission shaft with a keyway is shown in Fig. 2.5 (a).
- The keyway is a discontinuity and results in stress concentration at the corners of the keyway and reduces torsional shear strength.
- In addition to giving fillet radius at the inner corners of the keyway (as shown in Fig. 2.5 (c)), there is another method of drilling two symmetrical holes on the sides of the keyway.
- These holes press the force flow lines and minimise their bending in the vicinity of the keyway. This method is illustrated in Fig. 2.5 (b).

4. Reduction of Stress Concentration in Threaded Members:

- A threaded component is shown in Fig. 2.6 (a).
- It is observed that the force flow line is bent as it passes from the shank portion to threaded portion of the component. This results in stress concentration in the transition plane.
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Fig 2.6  Reduction of Stress Concentration in Threaded Components: (a) Original Component (b) Undercutting (c) Reduction in Shank Diameter

- In Fig. 2.6 (b), a small undercut is taken between the shank and the threaded portion of the component and a fillet radius is provided for this undercut.
- This reduces bending of the force flow line and consequently reduces stress concentration.
- An ideal method to reduce stress concentration is illustrated in Fig. 2.6 (c), where the shank diameter is reduced and made equal to the core diameter of the thread.
- In this case, the force flow line is almost straight and there is no stress concentration.

2.1.3 Theoretical Stress Concentration factor

- In order to consider the effect of stress concentration and find out localized stresses, a factor called theoretical stress concentration factor is used.
- It is denoted by $K_t$ and defined as,

$$K_t = \frac{\text{Highest value of actual stress near discontinuity}}{\text{Nominal stress obtained by elementary equations}}$$

$$\therefore K_t = \frac{\sigma_{\text{max}}}{\sigma_0} = \frac{\tau_{\text{max}}}{\tau_0}$$

where $\sigma_0$ and $\tau_0$ are stresses determined by elementary equations and $\sigma_{\text{max}}$ and $\tau_{\text{max}}$ are localized stresses at the discontinuities.

- Elementary equations are:

$$\sigma_t = \frac{P}{A}, \quad \sigma_b = \frac{M_b y}{I} \quad \text{and} \quad \tau = \frac{M_r R}{J}$$

To find out value of stress concentration factor following charts are used.

- The chart for the stress concentration factor for a rectangular plate with a transverse hole loaded in tension or compression is shown in Fig. 2.7

- The nominal stress $\sigma_0$ in this case is given by,

$$\sigma_0 = \frac{P}{(w - d)t}$$

where $t$ is plate thickness.
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Fig 2.7  Stress Concentration Factor (Rectangular Plate with Transverse Hole in Tension or Compression)

- The chart for the stress concentration factor for a flat plate with shoulder fillet in tension or compression is shown in Fig. 2.8.
- The nominal stress $\sigma_0$ in this case is given by,

$$\sigma_0 = \frac{P}{d \ t}$$

Fig 2.8  Stress Concentration Factor (Flat Plate with Shoulder Fillet in Tension or Compression)
• The charts for stress concentration factor for a round shaft with shoulder fillet subjected to tensile force, bending moment and torsional moment are shown in Fig. 2.9, 2.10 and 2.11 respectively.
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Fig 2.11 Stress Concentration Factor (Round Shaft with Shoulder Fillet in Torsional Moment)

- The nominal stresses in these three cases are as follows:

Tensile Force:

\[ \sigma_0 = \frac{P}{\left(\frac{\pi}{4}d^2\right)} \]

Bending Moment:

\[ \sigma_0 = \frac{M_y}{I} \]

Torsional Moment:

\[ \tau_0 = \frac{M_t R}{J} \]

2.2 Fatigue

- Fatigue is a phenomenon associated with variable loading or more precisely to cyclic stressing or straining of a material.
- Just as we human beings get fatigue when a specific task is repeatedly performed, in a similar manner metallic components subjected to variable loading get fatigue, which leads to their premature failure under specific conditions.
- Fatigue loading is primarily the type of loading which causes cyclic variations in the applied stress or strain on a component. Thus any variable loading is basically a fatigue loading.
- A rotating shaft with a bending load applied to it is a good example of fully reversible load.
- In order to visualize the fully-reversing nature of the load, consider the shaft in a fixed position (not rotating) but subjected to an applied bending load.
The outermost fibers on the shaft surface lying on the convex side of the deflection (upper surface) will be loaded in tension, and the fibers on the opposite side will be loaded in compression.

Now, rotate the shaft 180° in its bearings, with the loads remaining the same. The shaft stress level is the same, but now the fibers which were loaded in compression before you rotated it are now loaded in tension, and vice-versa.

Thus if the shaft is rotated let us say at 900 revolutions per minute then the shaft is cyclically stressed 900 times a minute.

2.2.1 Fatigue failure

- Fatigue failure is defined as time delayed –fracture under cyclic loading.
- Examples of parts in which fatigue failures are common: transmission shafts, connecting rods, gears, vehicle suspension springs and ball bearings.
- The materials fail under fluctuating stresses at a stress magnitude which is lower than the ultimate tensile strength of the material.
- It has been found that the magnitude of the stress causing fatigue failure decreases as the number of stress cycles increase.
- This phenomenon of decreased resistance of the materials to fluctuating stresses is the main characteristic of fatigue failure.
- Three stages are involved in fatigue failure:
  1. Crack initiation
  2. Crack propagation
  3. Fracture
     1. Crack initiation:
        - Areas of localized stress concentrations such as fillets, notches, key ways, bolt holes and even scratches or tool marks are potential zones for crack initiation.
        - Cracks also generally originate from a geometrical discontinuity.
     2. Crack propagation:
        - This further increases the stress levels and the process continues, propagating the cracks across the grains or along the grain boundaries, slowly increasing the crack size.
        - As the size of the crack increases the cross sectional area resisting the applied stress decreases and reaches a level at which it is insufficient to resist the applied stress.
     3. Final fracture:
        - As the area becomes too insufficient to resist the induced stresses any further a sudden fracture results in the component.
        - Fatigue cracks are not visible till they reach the surface of the component and by that time the failure have already taken place.
        - The fatigue failure is sudden and total.
- The fatigue failure, however, depends upon a number of factors, such as the number of cycles, mean stress, stress amplitude, stress concentration, residual stresses, corrosion and creep.
- This makes the design of components subjected to fluctuating stresses more complex.

2.2.2 **Cyclic Stresses**
- Cyclic stresses may be classified as below:
  1. Fluctuating stresses
  2. Repeated stresses
  3. Reversed stresses

1. **Fluctuating Stresses**:

   ![Fluctuating Stresses Diagram](image)

   *Fig 2.12  Fluctuating Stresses*

   - In many applications, the components are subjected to forces, which are not static, but vary in magnitude with respect to time.
   - The stresses induced due to such forces are called fluctuating stresses. These fluctuating stresses result in fatigue failure.
   - It varies in sinusoidal manner with respect to time and fluctuates between maximum and minimum value.
   - It has some mean value as well as some amplitude value.

     \[
     Mean\ Stress, \sigma_m = \frac{1}{2} (\sigma_{max} + \sigma_{min})
     \]

     \[
     Amplitude\ Stress, \sigma_a = \frac{1}{2} (\sigma_{max} - \sigma_{min})
     \]

2. **Repeated Stresses**:

   - It varies in sinusoidal manner with respect to time but the variation is from zero to maximum value.

   \[\therefore \sigma_{min} = 0\]

   And therefore

   \[\sigma_{mean} = \sigma_{amplitude}\]
3. Reversed Stresses:

- It varies in sinusoidal manner with respect to time but it has zero mean value.
  \[ \sigma_{\text{mean}} = 0 \]
- Half portion of the cycle consists of tensile stress and the remaining half of compressive stress.

2.2.3 Endurance Limit

- Fatigue or endurance limit of a material is defined as the maximum amplitude of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure.

- Fatigue or endurance limit can also be defined as the magnitude of stress amplitude value at or below which no fatigue failure will occur, no matter how large the number of stress reversals are, in other words leading to an infinite life to the component.
- Since the fatigue test cannot be conducted for unlimited or infinite number of cycles, $10^6$ (10 million) cycles is considered as a sufficient number of cycles to define the endurance limit.
- Most non-ferrous metals have no endurance limit.
- For such materials, which do not have an endurance limit, it is customary to define fatigue strength.
- Fatigue or endurance strength is the stress level that produces failure after a specified number of cycles (usually $5 \times 10^6$ or $10^7$).

### 2.2.4 Fatigue life

#### Fig 2.16 Specimen for Fatigue Test

- Fatigue life is defined as the number of stress cycles that the standard specimen can complete during the test before the appearance of the first fatigue crack.
- This specimen is carefully machined and polished.
- The dimensions of the standard test specimen are in mm as shown in Fig. 2.16.
- In the laboratory, the endurance limit is determined by means of a rotating beam machine developed by R. R. Moore.

#### Fig 2.17 Rotating Beam Fatigue Testing Machine

- The principle of a rotating beam is illustrated in Fig. 2.17.
- A beam of circular cross-section is subjected to bending moment $M_b$.
- It is subjected to a completely reversed stress cycle.
- Changing the bending moment by addition or deletion of weights can vary the stress amplitude.
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- The specimen is rotated by an electric motor.
- The number of revolutions before the appearance of the first fatigue crack is recorded on a revolution counter.
- In each test, two readings are taken, viz., stress amplitude (Sf) and number of stress cycles (N).
- These readings are used as two coordinates for plotting a point on the S-N diagram.
- This point is called failure point.
- To determine the endurance limit of a material, a number of tests are to be carried out.

2.2.5 S – N curve

The S-N curve is the graphical representation of stress amplitude (Sf) versus the number of stress cycles (N) before the fatigue failure on a log-log graph paper.

The S-N curve for steels is illustrated in Fig. 2.18.

Each test on the fatigue testing machine gives one failure point on the S-N diagram.

In practice, the points are scattered in the figure and an average curve is drawn through them.

For ferrous materials like steels, the S-N curve becomes straight at 10⁶ cycles, which indicates the stress amplitude corresponding to infinite number of stress cycles.

The magnitude of this stress amplitude at 10⁶ cycles represents the endurance limit of the material.

The S-N curve shown in figure is valid only for ferrous metals.

For nonferrous metals like aluminum alloys, the S-N curve slopes gradually even after 10⁷ cycles.
2.2.6 Low-Cycle and High-Cycle Fatigue

The body of knowledge available on fatigue failure from N=1 to N=1000 cycles is generally classified as low-cycle fatigue.

High-cycle fatigue, then, is concerned with failure corresponding to stress cycles greater than 1000 cycles.

2.2.7 Notch Sensitivity

- It is observed that the actual reduction in the endurance limit of a material due to stress concentration is less than the amount indicated by the theoretical stress concentration factor $K_t$.
- $K_t$ is the theoretical stress concentration factor, which is applicable to ideal materials that are homogeneous, isotropic and elastic.
- $K_f$ is the fatigue stress concentration factor. This factor $K_f$ is applicable to actual materials and depends upon the grain size of the material.
- Notch sensitivity is defined as the susceptibility of a material to succumb (fail to resist) to the damaging effects of stress raising notches in fatigue loading.
- The notch sensitivity factor is denoted by $q$.

\[
q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}
\]

Since $\sigma_0$ = nominal stress as obtained by elementary equations

\[\therefore \text{Actual stress} = K_f \sigma_0 \text{ and theoretical stress} = K_t \sigma_0\]

Increase of actual stress over nominal stress = $(K_f \sigma_0 - \sigma_0)$

Increase of theoretical stress over nominal stress = $(K_t \sigma_0 - \sigma_0)$

\[
\therefore q = \frac{(K_f \sigma_0 - \sigma_0)}{(K_t \sigma_0 - \sigma_0)} = \frac{(K_f - 1)}{(K_t - 1)}
\]

\[\therefore K_f = 1 + q(K_t - 1)\]
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- When \( q = 0 \), \( K_f = 1 \). (No sensitivity to notches)
- When \( q = 1 \), \( K_f = K_t \). (Full sensitivity to notches)
- In general, the magnitude of the notch sensitivity factor \( q \) varies from 0 to 1.
- In case of doubt, the designer should use \( q = 1 \) and the design will be on the safe side.
- It depends upon material and notch radius.
- The notch sensitivity factors for various materials for reversed bending or axial stresses and reversed torsional shear stresses are obtained from Fig. 2.20 and 2.21 respectively.

![Fig 2.20 Notch Sensitivity Charts (for Reversed Bending and Reversed Axial Stresses)](image1)

![Fig 2.21 Notch Sensitivity Charts (for Reversed Torsional Shear Stresses)](image2)
2.3 Factors Affecting Endurance Strength

- The endurance strength ($S_e$) of a component is different from the endurance strength ($S_e'$) of a rotating beam specimen due to number of factors.
- The difference arises due to the fact that there are standard specifications and working conditions for the rotating beam specimen, while the actual components have different specifications and work under different conditions.
- Different modifying factors are used in practice to account for this difference. These factors are, sometimes, called “derating factors”.
- The purpose of derating factors is to 'derate' or reduce the endurance limit of a rotating beam specimen to suit the actual component.
- The relationship between ($S_e'$) and ($S_e$) is as follows:

$$S_e = K_a K_b K_c K_d S_e'$$

where,

- $K_a = $ Surface finish factor
- $K_b = $ Size factor
- $K_c = $ Reliability factor
- $K_d = $ Modifying factor to account for stress concentration

$S_e'$ = Endurance limit stress of a rotating beam specimen subjected to reversed bending stress (N/mm$^2$)

$S_e$ = Endurance limit stress of a particular mechanical component subjected to reversed bending stress (N/mm$^2$)

Endurance limit:
- There is an approximate relationship between the endurance limit and the ultimate tensile strength ($S_{ut}$) of the material.
  - For steels, $S_e' = 0.5 S_{ut}$
  - For cast iron and cast steels, $S_e' = 0.4 S_{ut}$
  - For wrought aluminum alloys, $S_e' = 0.4 S_{ut}$
  - For cast aluminum alloys, $S_e' = 0.3 S_{ut}$

Surface finish factor ($K_a$):

![Surface finish factor diagram](image)
Rotating beam specimen is polished to mirror finish.

- It is impractical to provide such an expensive surface finish for the actual component. This poor surface finish increases stress concentration.
- It depends upon ultimate tensile strength and process used to manufacture the component.
- It should be noted that ultimate tensile strength is also a parameter affecting the surface finish factor.
- High strength materials are more sensitive to stress concentration introduced by surface irregularities. Therefore, as the ultimate tensile strength increases, the surface finish factor decreases.

**Size factor** ($K_b$):

- In rotating beam specimen, the diameter is small as 7.5 mm. Larger the component, greater the probability of variation in size. Therefore, the endurance limit reduces with increase in size of component.
- For bending and Torsion, the values of size factor ($K_b$) are given in Table 2.1.

<table>
<thead>
<tr>
<th>Diameter (d) (mm)</th>
<th>$K_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d \leq 7.5$</td>
<td>1.00</td>
</tr>
<tr>
<td>$7.5 &lt; d \leq 50$</td>
<td>0.85</td>
</tr>
<tr>
<td>$d &gt; 50$</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Reliability factor** ($K_c$):

- There is considerable variation of the data when number of tests is conducted even when using the same material.
- The greater the likelihood that a part will survive, the more is the reliability and lower is the reliability factor.
- The reliability factor is one for 50% reliability. This means that 50% of the components will survive in the given set of conditions.
- To ensure that more than 50% of the parts will survive, the stress amplitude on the component should be lower than the tabulated value of the endurance limit.
- The reliability factor is used to achieve this reduction.
- The tests are considered to be 50% reliable. Hence for 50% reliability, $K_c = 1$.

<table>
<thead>
<tr>
<th>Reliability (R) (%)</th>
<th>$K_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.00</td>
</tr>
<tr>
<td>90</td>
<td>0.897</td>
</tr>
<tr>
<td>95</td>
<td>0.868</td>
</tr>
<tr>
<td>99</td>
<td>0.814</td>
</tr>
<tr>
<td>99.9</td>
<td>0.753</td>
</tr>
<tr>
<td>99.99</td>
<td>0.702</td>
</tr>
<tr>
<td>99.999</td>
<td>0.659</td>
</tr>
</tbody>
</table>
Modifying factor to account for stress concentration ($K_d$):

- Due to notch sensitivity, fatigue stress concentration factor is less than the theoretical stress concentration factor.
- To apply the effect of stress concentration, the designer can reduce the endurance limit by $K_d$.
- The modifying factor $K_d$ to account for the effect of stress concentration is defined as,

$$K_d = \frac{1}{K_f}$$

- The above mentioned four factors are used to find out the endurance limit of the actual component.
- All the above factors are considered for reversed bending stresses only.
- The endurance limit ($S_{se}$) of a component subjected to fluctuating torsional shear stresses is obtained from the endurance limit in reversed bending ($S_e$) using theories of failures.

According to the maximum shear stress theory,

$$S_{se} = 0.5 S_e$$

According to distortion – energy theory,

$$S_{se} = 0.577 S_e$$

- When the component is subjected to an axial fluctuating load, the conditions are different.
- In axial loading, the entire cross-section is uniformly stressed to the maximum value.
- Therefore, endurance limit in axial loading is lower than the rotating beam test.

For axial loading,

$$S_{ea} = 0.8 S_e$$

### 2.4 Design for Reversed Stresses

- There are two types of problems in fatigue design:
  - Components subjected to completely reversed stresses, and
  - Components subjected to fluctuating stresses.
- The mean stress is zero in case of completely reversed stresses.
- The stress distribution consists of tensile stresses for the first half cycle and compressive stresses for the remaining half cycle and the stress cycle passes through zero.
- In case of fluctuating stresses, there is always a mean stress, and the stresses can be purely tensile, purely compressive or mixed depending upon the magnitude of the mean stress. Such problems are solved with the help of the modified Goodman diagram.
- The design problems for completely reversed stresses are further divided into two groups:
  - a) Design for infinite life, and
  - b) Design for finite life.
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a) Design for infinite life:
- When the component is to be designed for infinite life, the endurance limit becomes the criterion of failure.
- The amplitude stress induced in such components should be lower than the endurance limit in order to withstand the infinite number of cycles.
- Such components are designed with the help of the following equations:
  \[
  \sigma_a = \frac{S_e}{f} \quad \tau_a = \frac{S_{te}}{f}
  \]
  where \((\sigma_a)\) and \((\tau_a)\) are stress amplitudes in the component and \(S_e\) and \(S_{te}\) are corrected endurance limits in reversed bending and torsion respectively.

b) Design for finite life:
- When the component is to be designed for finite life, the S-N curve as shown in Fig. 2.23 can be used.

The curve is valid for steels.
- It consists of a straight line AB drawn from \((0.9 \ S_{ut})\) at \(10^5\) cycles to \((S_e)\) at \(10^6\) cycles on a log-log paper.
- The design procedure for such problems is as follows:
  (i) Locate the point A with coordinates \([3, \log_{10} (0.9 \ S_{ut})]\) since \(\log_{10} (10^5) = 3\)
  (ii) Locate the point B with coordinates \([6, \log_{10} (S_e)]\) since \(\log_{10} (10^6) = 6\)
  (iii) Join AB, which is used as a criterion of failure for finite-life problems
  (iv) Depending upon the life \(N\) of the component, draw a vertical line passing through \(\log_{10} (N)\) on the abscissa. This line intersects AB at point F.
  (v) Draw a line FE parallel to the abscissa. The ordinate at the point E, i.e. \(\log_{10} (S_f)\) gives the fatigue strength corresponding to \(N\) cycles.
- The value of the fatigue strength \((S_f)\) obtained by the above procedure is used for the design calculations.
2.5 Design for Fluctuating Stresses

Fig 2.24  Gerber Line, Soderberg Line and Goodman Line

- When stress amplitude ($\sigma_a$) is zero, the load is purely static and the criterion of failure is $S_{ut}$, or $S_{yt}$.
- When the mean stress ($\sigma_m$) is zero, the stress is completely reversing and the criterion of failure is the endurance limit $S_e$.
- When the component is subjected to both components of stress, viz. ($\sigma_m$) and ($\sigma_a$), the actual failure occurs at different scattered points as shown in Fig. 2.24.
- There exists a border, which divides safe region from unsafe region for various combinations of ($\sigma_m$) and ($\sigma_a$).
- Different criterions are proposed to construct the border line dividing safe zone and failure zone.
- They include Gerber line, Soderberg line and Goodman line.

(i) Gerber line:
- A parabolic curve joining $S_e$ on the ordinate to $S_{ut}$ on the abscissa is called the Gerber line.
- The Gerber parabola fits the failure points of test data in the best possible way.
- According to the Gerber line, the region below this curve is considered to be safe.
- Equation for the Gerber line is:
  \[ \frac{S_a}{S_e} + \left( \frac{S_m}{S_{ut}} \right)^2 = 1 \]

(ii) Soderberg line:
- A straight line joining $S_e$ on the ordinate to $S_{yt}$ on the abscissa is called the Soderberg line.
2. DESIGN AGAINST FLUCTUATING LOADS

- The Soderberg line is more conservative failure criterion and there is no need to consider even yielding in this case.
- Equation for the Soderberg line is:
  \[ \frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1 \]

(iii) Goodman line:
- A straight line joining \( S_e \) on the ordinate to \( S_{ut} \) on the abscissa is called the Goodman line.
- Goodman line is safer from design considerations because it is completely inside the Gerber parabola and inside the failure points.
- Equation for the Goodman line is:
  \[ \frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \]

2.6 Modified Goodman Diagrams:
- The components, which are subjected to fluctuating stresses, are designed by constructing the modified Goodman diagram.
- For the purpose of design, the problems are classified into two groups:
  (i) Components subjected to fluctuating axial or bending stresses; and
  (ii) Components subjected to fluctuating torsional shear stresses
- (i) Modified Goodman Diagram for Axial and Bending Stresses:

![Modified Goodman Diagram for Axial and Bending Stresses](image)

- The Goodman line is 'modified' by combining fatigue failure with failure by yielding.
- In this diagram, the yield strength \( S_{yt} \) is plotted on both the axes and a yield line CD is constructed to join these two points to define failure by yielding.
- Obviously, the line CD is inclined at \( 45^\circ \) to the abscissa.
- A line AF is constructed to join $S_e$ on the ordinate with $S_{ut}$ on the abscissa, which is the Goodman line.
- The point of intersection of these two lines is B.
- The area OABC represents the region of safety for components subjected to fluctuating stresses.
- The region OABC is called **modified Goodman diagram**.
- All the points inside the modified Goodman diagram should cause neither fatigue failure nor yielding.
- A line OE with a slope of $\tan \theta$ is constructed in such a way that,
  \[
  \tan \theta = \frac{\sigma_a}{\sigma_m} = \frac{S_a}{S_m} = \frac{(M_b)_a}{(M_b)_m}
  \]
- The magnitudes of $(M_b)_a$ and $(M_b)_m$ can be determined from maximum and minimum bending moment acting on the component.
- The point of intersection of lines AB and OE is X.
- The point X indicates the dividing line between the safe region and the region of failure.
- The coordinates of the point X ($S_m, S_a$) represent the limiting values of stresses, which are used to calculate the dimensions of the component.
- The permissible stresses are as follows:
  \[
  \sigma_a = \frac{S_a}{f_S}, \quad \sigma_m = \frac{S_m}{f_S}
  \]

(ii) **Modified Goodman Diagram for Torsional Shear Stresses**

![Modified Goodman Diagram for Torsional Shear Stresses](image)

- In this diagram, the torsional mean stress is plotted on the abscissa while the torsional stress amplitude on the ordinate.
- The torsional yield strength $S_{sy}$ is plotted on the abscissa and the yield line is constructed, which is inclined at $45^\circ$ to the abscissa.
- It is interesting to note that up to a certain point, the torsional mean stress has no effect on the torsional endurance limit.
Therefore, a line is drawn through $S_{se}$ on the ordinate and parallel to the abscissa.

The point of intersection of this line and the yield line is B. The area OABC represents the region of safety in this case.

It is not necessary to construct a fatigue diagram for fluctuating torsional shear stresses because AB is parallel to the X-axis.

Instead, a fatigue failure is indicated if,

$$\tau_a = S_{se}$$

The permissible stresses are as follows:

$$\tau_a = \frac{S_{se}}{f_s}$$

$$\tau_{max} = \frac{S_{sy}}{f_s}$$

Example 2.1 A plate made of steel 20C8 ($S_{ut} = 440 \text{ N/mm}^2$) in hot rolled and normalized condition is shown in Fig. 2.27. It is subjected to a completely reversed load of 30 kN. The notch sensitivity factor $q$ can be taken as 0.8 and the expected reliability is 90%. The size factor is 0.85. The factor of safety is 2. Determine the plate thickness for infinite life. (V. B. Bhandari Example 5.3)

![Fig 2.27 Given Figure for Example 2.1](image-url)

**Date Given:**

Hot rolled steel

Ultimate tensile strength, $S_{ut} = 440 \text{ N/mm}^2$

Load, $P = 30 \text{ KN} = 30000 \text{ N}$

Notch sensitivity factor, $q = 0.8$

Expected reliability, $R = 90\%$

Size factor, $K_b = 0.85$

Factor of safety, $f_s = 2$

Find thickness, "t"

**Data from given figure:**

Diameter of hole in the plate, $d = 10 \text{ mm}$

Width of the plate, $w = 50 \text{ mm}$
Solution:
From Fig. 2.22, for Hot rolled steel and $S_{ut} = 440 \text{ N/mm}^2$, Surface finish factor, $K_a = 0.67$
From Table 2.2, for $R = 90\%$, Reliability factor, $K_c = 0.897$
From given figure, $d = 10 \text{ mm}$ and $w = 50 \text{ mm}$, \( \frac{d}{w} = \frac{10}{50} = 0.2 \)
From Fig. 2.7, for $\frac{d}{w} = 0.2$, theoretical stress concentration factor, $K_t = 2.51$
Fatigue stress concentration factor, $K_f = 1 + q(K_t - 1)$
\[ K_f = 1 + 0.8 (2.51 - 1) \]
\[ K_f = 2.208 \]
Modifying factor to account for stress concentration, $K_d = \frac{1}{K_f}$
\[ K_d = \frac{1}{2.208} \]
\[ K_d = 0.4529 \]
For steels, $S_e' = 0.5 S_{ut} = 0.5 (440) = 220 \text{ N/mm}^2$
Endurance limit stress of the component, $S_e = K_a K_b K_c K_d S_e'$
\[ S_e = 0.67 (0.85) (0.897) (0.4529) (220) \]
\[ S_e = 50.9 \text{ N/mm}^2 \]
Permissible stress amplitude, $\sigma_a = \frac{S_e}{f_s}$
\[ \sigma_a = \frac{50.9}{2} \]
\[ \sigma_a = 25.45 \text{ N/mm}^2 \]
Also for rectangular plate with transverse hole, $\sigma_a = \frac{P}{(w - d)t}$
\[ t = \frac{P}{(w - d)\sigma_a} = \frac{30000}{(50 - 10)25.45} \]
\[ t = 29.47 \text{ mm} \]

Example 2.2 A component machined from a plate made of steel 45C8 ($S_{ut} = 630 \text{ N/mm}^2$) is shown in Fig. 2.28. It is subjected to a completely reversed axial force of 50 kN. The expected reliability is 90\% and the factor of safety is 2. The size factor is 0.85. Determine the plate thickness $t$ for infinite life, if the notch sensitivity factor is 0.8.
(V. B. Bhandari Example 5.5)
Date Given:
Machined steel, Ultimate tensile strength, $S_{ut} = 630\text{ N/mm}^2$
Load, $P = 50\text{ KN} = 50000\text{ N (Axial)}$
Expected reliability, $R = 90\%$
Factor of safety, $f_s = 2$
Size factor, $K_b = 0.85$
Notch sensitivity factor, $q = 0.8$
Find thickness, “$t$”

Data from given figure:
Larger width of the plate, $D = 100\text{ mm}$
Smaller width of the plate, $d = 50\text{ mm}$
Fillet radius at shoulder of the plate, $r = 5\text{ mm}$

Solution:
From Fig. 2.22, for machined steel and $S_{ut} = 630\text{ N/mm}^2$, Surface finish factor, $K_a = 0.76$
From table 2.2, for $R = 90\%$, Reliability factor, $K_c = 0.897$
From given figure, $D = 100\text{ mm}$, $d = 50\text{ mm}$ and $r = 5\text{ mm}$,
\[ \frac{D}{d} = \frac{100}{50} = 2 \quad \& \quad \frac{r}{d} = \frac{5}{50} = 0.1 \]
From Fig. 2.8, for $\frac{D}{d} = 2 \quad \& \quad \frac{r}{d} = 0.1$, theoretical stress concentration factor, $K_t = 2.27$
Fatigue stress concentration factor, $K_f = 1 + q(K_t - 1)$
\[ \therefore K_f = 1 + 0.8(2.27 - 1) \]
\[ \therefore K_f = 2.016 \]
Modifying factor to account for stress concentration, $K_d = \frac{1}{K_f}$
\[ \therefore K_d = 0.496 \]
For steels, $S_e' = 0.5 S_{ut} = 0.5 \times 630 = 315\text{ N/mm}^2$
Endurance limit stress of the component, $S_e = K_a K_b K_c K_d S_e'$
\[ \therefore S_e = 0.76(0.85)(0.897)(0.496)(315) \]
\[ \therefore S_e = 90.54\text{ N/mm}^2 \]
The load acting on the plate is axial and for axial loading,
\[ (S_e)_a = 0.8 S_e = 0.8(90.54) = 72.43\text{ N/mm}^2 \]
Permissible stress amplitude, $\sigma_a = \frac{(S_e)_a}{f_s}$
\[ \therefore \sigma_a = \frac{72.43}{2} \]
\[ \therefore \sigma_a = 36.215\text{ N/mm}^2 \]
Also for plate with shoulder fillet, $\sigma_a = \frac{P}{d t}$
\[ \therefore t = \frac{P}{d \sigma_a} = \frac{50000}{50 \times 36.215} \]
\[ \therefore t = 27.61\text{ mm} \]
Example 2.3  A rotating bar made of steel 45C8 ($S_{ut} = 630 \text{ N/mm}^2$) is subjected to a completely reversed bending stress. The corrected endurance limit of the bar is 315 N/mm$^2$. Calculate the fatigue strength of the bar for a life of 90,000 cycles. (V. B. Bhandari Example 5.6)

Solution:

Date Given:

Ultimate tensile strength, $S_{ut} = 630 \text{ N/mm}^2$

Endurance limit stress, $S_e = 315 \text{ N/mm}^2$

Life of the bar, $N = 90000$ Cycles

Find fatigue strength, “$S_f$”

Solution:

Refer Fig. 2.29, for solving this example following things are required.

$\log S_e = \log 315 = 2.4983$

$\log N = \log 90000 = 4.9542$

$\log (0.9 S_{ut}) = \log (0.9 \times 630) = 2.7536$

From Fig. 2.29,

$\frac{AE}{AD} = \frac{4.9542 - 3}{2.7536 - 2.4983}$

$AE = 0.1663$

From Fig. 2.29,

$\log S_f = \log (0.9 S_{ut}) - AE$

$\log S_f = 2.7536 - 0.1663 = 2.5873$

$S_f = 386.63 \text{ N/mm}^2$

Example 2.4  A forged steel bar, 50 mm in diameter, is subjected to a reversed bending stress of 250 N/mm$^2$. The bar is made of steel 40C8 ($S_{ut} = 600 \text{ N/mm}^2$). Calculate the life of the bar for a reliability of 90%. (V. B. Bhandari Example 5.7) (GTU Example)
**Date Given:**
Forged steel bar
Diameter of bar = 50 mm
Reversed bending stress or fatigue strength, \( S_f = 250 \text{ N/mm}^2 \)
Ultimate tensile strength, \( S_{ut} = 600 \text{ N/mm}^2 \)
Expected reliability, \( R = 90\% \)
Find life of bar, “\( N \)”

**Solution:**
From Fig. 2.22, for forged bar and \( S_{ut} = 600 \text{ N/mm}^2 \), Surface finish factor, \( K_a = 0.44 \)
From table 2.1, for \( d = 50 \text{ mm} \), Size factor, \( K_b = 0.85 \)
From table 2.2, for \( R = 90\% \), Reliability factor, \( K_c = 0.897 \)
For circular bar, effect of stress concentration will be negligible, hence modifying factor to account for stress concentration, \( K_d = 1 \)
For steels, \( S_e' = 0.5 \times S_{ut} = 0.5 \times (600) = 300 \text{ N/mm}^2 \)
Endurance limit stress of the component, \( S_e = K_d K_b K_c K_d S_e' \)
\( \therefore S_e = 0.44 \times 0.85 \times 0.897 \times 300 \)
\( \therefore S_e = 100.64 \text{ N/mm}^2 \)

*Fig 2.30  S – N Diagram Drawn for Example 2.4*

Refer Fig. 2.30, for solving this example following things are required.

\[ \log S_e = \log 100.64 = 2.0028 \]
\[ \log S_f = \log 250 = 2.3979 \]
\[ \log (0.9 S_{ut}) = \log (0.9 \times 600) = 2.7324 \]

From Fig. 2.30,
\[ \frac{AE}{AD} = \frac{DB}{EF} \]
Example 2.5  A rotating shaft subjected to a non-rotating force of 5 kN and simply supported between two bearings A and E as shown in Fig. 2.31. The shaft is machined from plain carbon steel 30C8 ($S_{ut} = 500$ N/mm$^2$) and the expected reliability is 90%. The equivalent notch radius at the fillet section can be taken as 3 mm. What is the life of the shaft? Bending moment at section B is $642.9$ kN mm; surface finish factor = 0.79; size factor = 0.85; reliability factor = 0.897; stress concentration factor = 1.72; notch sensitivity = 0.78. (V. B. Bhandari Example 5.8) (GTU Example)

**Date Given:**
Material – Steel
Force, $P = 5$ KN = 5000 N
Ultimate tensile strength, $S_{ut} = 500$ N/mm$^2$
Expected reliability, $R = 90\%$
Maximum bending moment, $M_b = 642.9$ kN mm = $642.9 \times 10^3$ N mm
Surface finish factor, $K_a = 0.79$
Size factor, $K_b = 0.85$
Reliability factor, $K_c = 0.897$
Theoretical stress concentration factor, $K_t = 1.72$
Notch sensitivity factor, $q = 0.78$
Find life of the shaft, “N”

**Data from given figure:**
Smallest diameter of the shaft, $d = 30$ mm

**Solution:**
Fatigue stress concentration factor, $K_f = 1 + q(K_t - 1)$
\[ \therefore K_f = 1 + 0.78 \times (1.72 - 1) \]
\[ \therefore K_f = 1.5616 \]
Modifying factor to account for stress concentration, \( K_d = \frac{1}{K_f} \)
\[ \therefore K_d = \frac{1}{1.5616} \]
\[ \therefore K_d = 0.64 \]

For steels, \( S_e' = 0.5 \ S_{ut} = 0.5 \ (500) = 250 \ \text{N/mm}^2 \)

Endurance limit stress of the component, \( S_e = K_a \ K_b \ K_c \ K_d \ S_e' \)
\[ \therefore S_e = 0.79 \ (0.85) \ (0.897) \ (0.64) \ (250) \]
\[ \therefore S_e = 96.37 \ \text{N/mm}^2 \]

Maximum bending moment,
\[ M_b = \frac{\pi}{32} \sigma_b d^3 \]
\[ \therefore \sigma_b = \frac{32 M_b}{\pi d^3} \]

Here \( d = 30 \ \text{mm} \) because it is the weakest section of the shaft as it is having the smallest diameter of the given shaft.
\[ \therefore \sigma_b = \frac{32 \ (642.9 \times 10^3)}{\pi \ (30)^3} \]
\[ \therefore \sigma_b = 242.54 \ \text{N/mm}^2 \]

Here \( \sigma_b \) corresponds to fatigue strength, hence \( \sigma_b = S_f = 242.54 \ \text{N/mm}^2 \)

**Refer Fig. 2.32**, for solving this example following things are required.

\[ \log S_e = \log 96.37 = 1.9839 \]
\[ \log S_f = \log 242.54 = 2.3848 \]
\[ \log (0.9 \ S_{ut}) = \log (0.9 \times 500) = 2.6532 \]

From Fig. 2.32,
\[ \frac{AE}{AD} = \frac{DF}{BC} \]
2. DESIGN AGAINST FLUCTUATING LOADS

Example 2.6  A cantilever beam made of cold drawn steel 40C8 \((S_{ut} = 600 \text{ N/mm}^2\) and \(S_{yt} = 380 \text{ N/mm}^2\)) is shown in Fig. 2.33. The force \(P\) acting at the free end varies from -50 N to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter \(d\) of the beam at the fillet cross-section. (V. B. Bhandari Example 5.12)

\[
\frac{EF}{(2.6532 - 2.3848)} = \frac{(6 - 3)}{(2.6532 - 1.9839)}
\]
\[
\therefore \ EF = 1.203
\]

From Fig. 2.32,

\[
\log N = 3 + EF
\]
\[
\log N = 3 + 1.203 = 4.203
\]
\[
\therefore \ N = 15960 \text{ Cycles}
\]

Date Given:
Cold drawn steel
Ultimate tensile strength, \(S_{ut} = 600 \text{ N/mm}^2\)
Yield strength, \(S_{yt} = 380 \text{ N/mm}^2\)
Maximum load, \(P_{max} = +150 \text{ N}\)
Minimum load, \(P_{min} = -50 \text{ N}\)
Expected reliability, \(R = 90\%\)
Factor of safety, \(f_s = 2\)
Notch sensitivity factor, \(q = 0.9\)

Find diameter, “\(d\)"

Data from given figure:
\(D = 1.5 \ d\) \(\therefore \ \frac{D}{d} = 1.5\)
\(r = 0.2 \ d\) \(\therefore \ \frac{r}{d} = 0.2\)

Solution:
From Fig. 2.22, for cold drawn steel and \(S_{ut} = 600 \text{ N/mm}^2\). Surface finish factor, \(K_a = 0.77\)
Assuming \(7.5 < d < 50\), hence from table 2.1, size factor, \(K_b = 0.85\)
From table 2.2, for \(R = 90\%\), Reliability factor, \(K_c = 0.897\)
From given figure, \(\frac{D}{d} = 1.5\) & \(\frac{r}{d} = 0.2\)
From Fig. 2.10, for \(\frac{D}{d} = 1.5\) & \(\frac{r}{d} = 0.2\), theoretical stress concentration factor, \(K_t = 1.44\)
Fatigue stress concentration factor, \( K_f = 1 + q (K_t - 1) \)
\[ K_f = 1 + 0.9 (1.44 - 1) \]
\[ K_f = 1.396 \]

Modifying factor to account for stress concentration, \( K_d = \frac{1}{K_f} \)
\[ K_d = \frac{1}{1.396} \]
\[ K_d = 0.716 \]

For steels, \( S_e' = 0.5 S_{ut} = 0.5 (600) = 300 \text{ N/mm}^2 \)

Endurance limit stress of the component, \( S_e = K_a K_b K_c K_d S_e' \)
\[ S_e = 0.77 (0.85) (0.897) (0.716) (300) \]
\[ S_e = 126.11 \text{ N/mm}^2 \]

In this example, the weakest section of the beam will be fillet section. Therefore we require value of maximum and minimum bending moment at fillet section which is 100 mm away from the free end.

Maximum bending moment,
\[ (M)_{\text{max}} = P_{\text{max}} \times \text{Distance between fillet section and free end} \]
\[ (M)_{\text{max}} = + (150 \times 100) \]
\[ (M)_{\text{max}} = +15000 \text{ N mm} \]

Minimum bending moment,
\[ (M)_{\text{min}} = P_{\text{min}} \times \text{Distance between fillet section and free end} \]
\[ (M)_{\text{min}} = -(50 \times 100) \]
\[ (M)_{\text{min}} = -5000 \text{ N mm} \]

Mean bending moment,
\[ (M)_m = \frac{1}{2} [(M)_{\text{max}} + (M)_{\text{min}}] \]
\[ (M)_m = \frac{1}{2} [15000 + (-5000)] \]
\[ (M)_m = 5000 \text{ N mm} \]

Amplitude bending moment,
\[ (M)_a = \frac{1}{2} [(M)_{\text{max}} - (M)_{\text{min}}] \]
\[ (M)_a = \frac{1}{2} [15000 - (-5000)] \]
\[ (M)_a = 10000 \text{ N mm} \]

Now, \( \tan \theta = \frac{(M)_a}{(M)_m} = \frac{10000}{5000} = 2 \)

We know that, \( \frac{(M)_a}{(M)_m} = \frac{S_a}{S_m} \)
\[ \frac{S_a}{S_m} = 2 \]
\[ S_a = 2 S_m \]
2. DESIGN AGAINST FLUCTUATING LOADS

Fig 2.34 Modified Goodman Diagram Drawn for Example 2.6

Equation for the Goodman line is,

\[
\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1
\]

\[
\therefore \frac{S_a}{126.11} + \frac{S_m}{600} = 1
\]

We know that, \(S_a = 2S_m\)

\[
\therefore \frac{2S_m}{126.11} + \frac{S_m}{600} = 1
\]

\[
\therefore 2S_m(600) + S_m(126.11) = 126.11(600)
\]

\[
\therefore 1326.11S_m = 126.11(600)
\]

\[
\therefore S_m = \frac{126.11(600)}{1326.11}
\]

\[
\therefore S_m = 57.06 \text{ N/mm}^2
\]

We know that, \(S_a = 2S_m\)

\[
\therefore S_a = 114.12 \text{ N/mm}^2
\]

Permissible stress amplitude, \(\sigma_a = \frac{S_a}{fs}\)

\[
\therefore \sigma_a = \frac{114.12}{2}
\]

\[
\therefore \sigma_a = 57.06 \text{ N/mm}^2
\]

Amplitude bending moment,

\[M_a = \frac{\pi}{32} \sigma_a d^3\]

\[
\therefore d^3 = \frac{32 M_a}{\pi \sigma_a}
\]

\[
\therefore d^3 = \frac{32(10000)}{57.06 \pi} = 1785.12
\]

\[
\therefore d = 12.13 \text{ mm}
\]

Example 2.7 A cantilever beam made of cold drawn carbon steel of circular cross-section as shown in Fig. 2.35, is subjected to a load which varies from \(-F\) to \(3F\). Determine the maximum load that the member can withstand for an indefinite life using a factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9. Assume the following values: Ultimate stress = 550 MPa, Yield stress = 470 MPa, Endurance limit = 275 MPa, Size factor = 0.85, Surface finish factor= 0.89
Date Given:
Cold drawn, Carbon steel
Maximum load, $P_{max} = +3\ F$
Minimum load, $P_{min} = -\ F$
Factor of safety, $f_s = 2$
Theoretical stress concentration factor, $K_t = 1.42$
Notch sensitivity factor, $q = 0.9$
Ultimate tensile strength, $S_{ut} = 550\ \text{N/mm}^2$
Yield strength, $S_{yt} = 470\ \text{N/mm}^2$
Endurance limit, $S_e' = 275\ \text{N/mm}^2$
Size factor, $K_b = 0.85$
Surface finish factor, $K_a = 0.89$
Find, “F”

Solution:
Reliability factor is not given hence assuming $R = 50\%$, therefore from table 2.2, for $R = 50\%$, Reliability factor, $K_c = 1$
Fatigue stress concentration factor, $K_f = 1 + q(K_t - 1)$
\[ \therefore K_f = 1 + 0.9(1.42 - 1) \]
\[ \therefore K_f = 1.378 \]
Modifying factor to account for stress concentration, $K_d = \frac{1}{K_f}$
\[ \therefore K_d = \frac{1}{1.378} \]
\[ \therefore K_d = 0.7257 \]
Endurance limit stress of the component, $S_e = K_a K_b K_c K_d S_e'$
\[ \therefore S_e = 0.89(0.85)(1)(0.7257)(275) \]
\[ \therefore S_e = 151\ \text{N/mm}^2 \]
In this example, the weakest section of the beam will be fillet section. Therefore we require value of maximum and minimum bending moment at fillet section which is 125 mm away from the free end.
Maximum bending moment,
\( \left( M \right)_{\text{max}} = P_{\text{max}} \times \text{Distance between fillet section and free end} \)
\[ \therefore (M)_{\text{max}} = +(3 \times 125) \]
\[ \therefore (M)_{\text{max}} = +375F \text{ N mm} \]

Minimum bending moment,
\( \left( M \right)_{\text{min}} = P_{\text{min}} \times \text{Distance between fillet section and free end} \)
\[ \therefore (M)_{\text{min}} = -\left( F \times 125 \right) \]
\[ \therefore (M)_{\text{min}} = -125 \text{ N mm} \]

Mean bending moment,
\[ (M)_{m} = \frac{1}{2} [(M)_{\text{max}} + (M)_{\text{min}}] \]
\[ \therefore (M)_{m} = \frac{1}{2} [375F + (-125F)] \]
\[ \therefore (M)_{m} = 125F \text{ N mm} \]

Amplitude bending moment,
\( (M)_{a} = \frac{1}{2} [(M)_{\text{max}} - (M)_{\text{min}}] \)
\[ \therefore (M)_{a} = \frac{1}{2} [375F - (-125F)] \]
\[ \therefore (M)_{a} = 250F \text{ N mm} \]

Now, \( \tan \theta = \frac{(M)_{a}}{(M)_{m}} = \frac{250F}{125F} = 2 \)

We know that,
\[ \frac{S_{a}}{S_{m}} = 2 \]
\[ \therefore S_{a} = 2 S_{m} \]

Equation for the Goodman line is,
\[ \frac{S_{a}}{S_{e}} + \frac{S_{m}}{S_{ut}} = 1 \]
\[ \therefore \frac{S_{a}}{151} + \frac{S_{m}}{550} = 1 \]

We know that, \( S_{a} = 2 S_{m} \)
\[ \therefore \frac{2S_{m}}{151} + \frac{S_{m}}{550} = 1 \]
\[ \therefore 2S_{m}(550) + S_{m}(151) = 151(550) \]
\[ \therefore 1251S_{m} = 151(550) \]
\[ \therefore S_{m} = \frac{151(550)}{1251} \]
\[ \therefore S_{m} = 66.4 \text{ N/mm}^{2} \]

We know that, \( S_{a} = 2 S_{m} \)
\[ \therefore S_{a} = 132.8 \text{ N/mm}^{2} \]

Permissible stress amplitude, \( \sigma_{a} = \frac{S_{a}}{f_{s}} \)
\[ \therefore \sigma_{a} = \frac{132.8}{2} \]
\[ \sigma_a = 66.4 \text{ N/mm}^2 \]
Amplitude bending moment,
\[ M_a = \frac{\pi}{32} \sigma_a d^3 \]
Here \( d = 13 \text{ mm} \) because it is the weakest section of the shaft as it is having the smallest diameter of the given shaft.
\[ M_a = \frac{\pi}{32} (66.4)(13)^3 \]
\[ \therefore M_a = 14321.8 \]
We know that, \( (M)_a = 250F \)
\[ \therefore 250F = 14321.8 \]
\[ \therefore F = \frac{14321.8}{250} \]
\[ \therefore F = 57.28 \text{ N} \]

Example 2.8 A transmission shaft of cold drawn steel 27Mn2 (\( S_{ut} = 500 \text{ N/mm}^2 \) and \( S_{yt} = 300 \text{ N/mm}^2 \)) is subjected to a fluctuating torque which varies from -100 N.m to +400 N.m. The factor of safety is 2 and the expected reliability is 90%. Neglecting the effect of stress concentration, determine the diameter of the shaft. Assume the distortion energy theory of failure. (V. B. Bhandari Example 5.13) (GTU Example)

**Date Given:**
Cold drawn steel
Ultimate tensile strength, \( S_{ut} = 500 \text{ N/mm}^2 \)
Yield strength, \( S_{yt} = 300 \text{ N/mm}^2 \)
Maximum torque, \( T_{max} = +400 \text{ N m} = +400 \times 10^3 \text{ N mm} \)
Minimum torque, \( T_{min} = -100 \text{ N m} = -100 \times 10^3 \text{ N mm} \)
Factor of safety, \( f_s = 2 \)
Expected reliability, \( R = 90\% \)
Neglect the effect of stress concentration
Find, diameter of shaft “\( d \)”
Assume the distortion energy theory of failure

**Solution:**
From Fig. 2.22, for cold drawn steel and \( S_{ut} = 500 \text{ N/mm}^2 \), Surface finish factor, \( K_a = 0.79 \)
Assuming \( 7.5 < d < 50 \), hence from table 2.1, size factor, \( K_b = 0.85 \)
From table 2.2, for \( R = 90\% \), Reliability factor, \( K_c = 0.897 \)
It is given that neglect the effect of stress concentration, hence modifying factor to account for stress concentration, \( K_d = 1 \)
For steels, \( S_e' = 0.5 \times S_{yt} = 0.5 \times 500 = 250 \text{ N/mm}^2 \)
Endurance limit stress of the component, \( S_e = K_a K_b K_c K_d S_e' \)
\[ \therefore S_e = 0.79 \times 0.85 \times 0.897 \times 1 \times 250 \]
\[ \therefore S_e = 150.58 \text{ N/mm}^2 \]
It is given that assume the distortion energy theory of failure.
\[ \therefore S_{se} = 0.577 S_e \]
\[ \therefore S_{se} = 0.577 (150.58) \]
\[ \therefore S_{se} = 86.88 \text{ N/mm}^2 \]

For fluctuating torque,
\[ S_{se} = S_{sa} \]
\[ \therefore S_{sa} = 86.88 \text{ N/mm}^2 \]

Permissible amplitude shear stress, \( \tau_a = \frac{S_{sa}}{f_s} \)
\[ \therefore \tau_a = \frac{86.88}{2} \]
\[ \therefore \tau_a = 43.44 \text{ N/mm}^2 \]

Amplitude torque,
\[ T_a = \frac{1}{2} [T_{\text{max}} - T_{\text{min}}] \]
\[ \therefore T_a = \frac{1}{2} [400 - (-100)] = \frac{1}{2} (500) \]
\[ \therefore T_a = 250 \text{ N m} = 250 \times 10^3 \text{ N mm} \]

Amplitude torque,
\[ T_a = \frac{\pi}{16} \tau_a d^3 \]
\[ \therefore d^3 = \frac{16 T_a}{\pi \tau_a} \]
\[ \therefore d = \frac{16 (250 \times 10^3)}{43.44 \pi} = 29310 \]
\[ \therefore d = 30.8 \text{ mm} \]

**Example 2.9** A machine component is subjected to fluctuating stress that varies from 40 to 100 \text{ N/mm}^2. The corrected endurance limit stress for the machine component is 270 \text{ N/mm}^2. The ultimate tensile strength and yield strength of the material are 600 and 450 \text{ N/mm}^2 respectively. Find the factor of safety using:

(i) Gerber theory;
(ii) Soderberg line;
(iii) Goodman line; and
(iv) Static failure.

(V. B. Bhandari Example 5.17) (GTU Example)

**Date Given:**

Maximum stress, \( \sigma_{\text{max}} = 100 \text{ N/mm}^2 \)
Minimum stress, \( \sigma_{\text{min}} = 40 \text{ N/mm}^2 \)
Endurance limit, \( S_e = 270 \text{ N/mm}^2 \)
Ultimate tensile strength, \( S_{ut} = 600 \text{ N/mm}^2 \)
Yield strength, \( S_{yt} = 450 \text{ N/mm}^2 \)

Find factor of safety for different given criteria.

**Solution:**

Mean Stress, \( \sigma_m = \frac{1}{2} (\sigma_{\text{max}} + \sigma_{\text{min}}) = \frac{1}{2} (100 + 40) = 70 \text{ N/mm}^2 \)

Amplitude Stress, \( \sigma_a = \frac{1}{2} (\sigma_{\text{max}} - \sigma_{\text{min}}) = \frac{1}{2} (100 - 40) = 30 \text{ N/mm}^2 \)
Let \( n = \) Factor of safety

We know that Factor of safety, \( n = \frac{s}{\sigma} \)

\[ \therefore S = n \sigma \]

\[ \therefore S_m = n \sigma_m = 70 n \quad \& \quad S_a = n \sigma_a = 30 n \]

(i) **Gerber theory:**

Equation for the Gerber line is:

\[
\frac{S_a}{S_e} + \left( \frac{S_m}{S_{ut}} \right)^2 = 1
\]

\[ \therefore \frac{30 n}{270} + \left( \frac{70 n}{600} \right)^2 = 1 \]

\[ n \frac{49 n^2}{3600} = 1 \]

\[ n (3600) + 9 (49 n^2) = 9 (3600) \]

\[ 8.16 n + n^2 = 73.47 \]

Solving the above equation gives \( n = 5.41 \)

(ii) **Soderberg line:**

Equation for the Soderberg line is:

\[
\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1
\]

\[ \therefore \frac{30 n}{270} + \frac{70 n}{450} = 1 \]

\[ n \frac{7 n}{45} = 1 \]

\[ 45 n + 63 n = 405 \]

\[ 108 n = 405 \]

\[ \therefore n = 3.75 \]

(iii) **Goodman line; and**

Equation for the Goodman line is:

\[
\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1
\]

\[ \therefore \frac{30 n}{270} + \frac{70 n}{600} = 1 \]

\[ n \frac{7 n}{60} = 1 \]

\[ 60 n + 63 n = 540 \]

\[ 123 n = 540 \]

\[ \therefore n = 4.39 \]

(iv) **Static failure:**

Equation for static failure:

\[ n = \frac{S_{yt}}{\sigma_{max}} \]

\[ \therefore n = \frac{450}{100} \]
Example 2.10 A circular bar 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking factor of safety 1.5; size effect of 0.85; surface finish factor of 0.9; The material properties of bars are given by: ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa. (R. S. Khurmi Example 6.8) (GTU Example)

Date Given:
Length of bar, \( l = 500 \text{ mm} \) (freely supported)
Maximum load, \( P_{\text{max}} = 50 \text{ kN} = 50000 \text{ N} \)
Minimum load, \( P_{\text{min}} = 20 \text{ kN} = 20000 \text{ N} \)
Factor of safety, \( f_s = 1.5 \)
Size factor, \( K_b = 0.85 \)
Surface finish factor, \( K_a = 0.9 \)
Ultimate tensile strength, \( S_{\text{ut}} = 650 \text{ N/mm}^2 \)
Yield strength, \( S_{\text{yt}} = 500 \text{ N/mm}^2 \)
Endurance strength, \( S_{\text{e'}} = 350 \text{ N/mm}^2 \)
Find diameter, \( "d" \)

Solution:
Reliability factor is not given hence assuming \( R = 50\% \), therefore from table 2.2, for \( R = 50\% \), Reliability factor, \( K_c = 1 \)
For circular bar, effect of stress concentration will be negligible, hence modifying factor to account for stress concentration, \( K_d = 1 \)
Endurance limit stress of the component, \( S_e = K_a K_b K_c K_d S_{\text{e'}} \)
\[ S_e = 0.9 \times 0.85 \times 1 \times 1 \times 350 \]
\[ S_e = 267.75 \text{ N/mm}^2 \]
Maximum bending moment,
\[ (M)_{\text{max}} = P_{\text{max}} \times \frac{\text{Length of the bar}}{4} \]
\[ (M)_{\text{max}} = 50000 \times \frac{500}{4} \]
\[ (M)_{\text{max}} = 6250 \times 10^3 \text{ N mm} \]
Minimum bending moment,
\[ (M)_{\text{min}} = P_{\text{min}} \times \frac{\text{Length of the bar}}{4} \]
\[ (M)_{\text{min}} = 20000 \times \frac{500}{4} \]
\[ (M)_{\text{min}} = 2500 \times 10^3 \text{ N mm} \]
Mean bending moment,
\[ (M)_m = \frac{1}{2} [(M)_{\text{max}} + (M)_{\text{min}}] \]
\[ (M)_m = \frac{1}{2} [6250 \times 10^3 + 2500 \times 10^3] \]
2. DESIGN AGAINST FLUCTUATING LOADS

Design of Machine Elements (2151907)

Prepared By: Mr. SUNIL G. JANIYANI
Department of Mechanical Engineering
Darshan Institute of Engineering & Technology, Rajkot

\[ (M)_m = 4375 \times 10^3 \text{ N mm} \]

Amplitude bending moment,

\[ (M)_a = \frac{1}{2} [(M)_{\text{max}} - (M)_{\text{min}}] \]

\[ \therefore (M)_a = \frac{1}{2} [6250 \times 10^3 - 2500 \times 10^3] \]

\[ \therefore (M)_a = 1875 \times 10^3 \text{ N mm} \]

Now, \( \tan \theta = \frac{(M)_a}{(M)_m} = \frac{1875 \times 10^3}{4375 \times 10^3} = 0.4286 \)

We know that, \( \frac{(M)_a}{(M)_m} = \frac{S_a}{S_m} \)

\[ \therefore \frac{S_a}{S_m} = 0.4286 \]

\[ \therefore S_a = 0.4286 S_m \]

Equation for the Goodman line is,

\[ \frac{S_e}{S_{ut}} + \frac{S_m}{S_{ut}} = 1 \]

\[ \therefore \frac{S_a}{267.75} + \frac{S_m}{650} = 1 \]

We know that, \( S_a = 0.4286 S_m \)

\[ \therefore \frac{0.4286 S_m}{267.75} + \frac{S_m}{650} = 1 \]

\[ \therefore 0.4286 S_m (650) + S_m (267.75) = 267.75 (650) \]

\[ 546.34 S_m = 267.75 (650) \]

\[ \therefore S_m = \frac{267.75 (650)}{546.34} \]

\[ \therefore S_m = 318.55 \text{ N/mm}^2 \]

We know that, \( S_a = 0.4286 S_m \)

\[ \therefore S_a = 136.53 \text{ N/mm}^2 \]

Permissible stress amplitude, \( \sigma_a = \frac{S_a}{f_s} \)

\[ \therefore \sigma_a = \frac{136.53}{1.5} \]

\[ \therefore \sigma_a = 91.02 \text{ N/mm}^2 \]

Amplitude bending moment,

\[ M_a = \frac{\pi}{32} \sigma_a d^3 \]

\[ \therefore d^3 = \frac{32 M_a}{\pi \sigma_a} \]

\[ \therefore d^3 = \frac{32 (1875 \times 10^3)}{91.02 \pi} = 209826.64 \]

\[ \therefore d = 59.42 \text{ mm} \]
3

DESIGN OF SPRINGS

Course Contents

3.1 Introduction to Spring
3.2 Types of springs
3.3 Spring Materials
3.4 Terminology of Helical Springs
3.5 End Connections for Compression Helical Springs
3.6 End Connections for Tension Helical Springs
3.7 Buckling of Compression Springs
3.8 Surge in Springs
3.9 Concentric or Composite Springs
3.10 Springs in Series
3.11 Springs in Parallel

Examples
3. DESIGN OF SPRINGS

3.1 Introduction to Spring
- A spring is defined as an elastic machine element that deflects under the action of the load and returns to its original shape when the load is removed.
- It can take any shape and form depending upon the application.
- Functions and applications of springs are listed in the below table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>To absorb shocks and vibrations</td>
<td>Vehicle suspension springs</td>
</tr>
<tr>
<td></td>
<td>Railway buffer springs</td>
</tr>
<tr>
<td></td>
<td>Buffer springs in elevators</td>
</tr>
<tr>
<td></td>
<td>Vibration mounts for machinery</td>
</tr>
<tr>
<td>To store energy</td>
<td>Springs used in clocks, toys. Movie-cameras, circuit breakers and starters</td>
</tr>
<tr>
<td>To measure force</td>
<td>Springs used in weighing balance and engine indicators</td>
</tr>
<tr>
<td>To control Motion</td>
<td>In cam and follower mechanism, spring is used to maintain contact between two elements.</td>
</tr>
<tr>
<td>To apply force</td>
<td>In engine valve mechanism, spring is used to return the rocker arm to its normal position when the disturbing force is removed.</td>
</tr>
<tr>
<td></td>
<td>The spring used in clutch provides the required force to engage the clutch.</td>
</tr>
</tbody>
</table>

3.2 Types of springs
- Following are important types of springs according to their shape:
  1. Helical springs
  2. Conical and volute springs
  3. Torsion springs
  4. Laminated or leaf springs
  5. Disc or belleville springs

3.2.1 Helical Springs
- The helical spring is made from a wire, usually of circular cross section, that is bent in the form of a helix.
- There are two basic types of helical springs: compression spring and tension spring.
- In helical compression spring, the external force tends to shorten the spring. In other words, the spring is compressed.
- In helical tension spring, the external force tends to lengthen the spring. In other words, the spring is elongated.
3. DESIGN OF SPRINGS

3.1. Helical Springs

- It should be noted that although the spring is under compression, the wire of helical compression spring is not subjected to compressive stress.
- Also, the wire of helical tension spring is not subjected to tensile stress although the spring is under tension.
- In both cases, torsional shear stresses are induced in the spring wire.
- The words compression and tension are related to total spring and not the stresses in spring wire.

3.2.2 Conical and Volute Springs

- The conical and volute springs are used in special applications where a telescoping spring or a spring with a spring rate (load required per unit deflection) that increases with the load is desired.
- The conical spring is wound with a uniform pitch whereas the volute springs are wound in the form of paraboloid with constant pitch and lead angles.
- This characteristic is sometimes utilised in vibration problems where springs are used to support a body that has a varying mass.
3. DESIGN OF SPRINGS

3.2.3 Torsion Springs

- The construction of this spring is similar to that of compression or tension spring, except that the ends are formed in such away, that the spring is loaded by a torque, about the axis of the coils.
- Helical torsion spring is used to transmit torque to a particular component in the machine or the mechanism.
- Helical torsion spring is used in door-hinges, brush-holders, starters and door locks.

- For example, the spring transmits a torque of \( P \times r \).
- The helical torsion resists the bending moment \( P \times r \) that tends to wind up the spring.
- The bending moment induces bending stresses in the spring wire.
- The term torsion spring is somewhat misleading because wire is subjected to bending stresses, unlike torsional shear stresses induced in helical torsion or tension springs.
3.2.4 Laminated Leaf Springs

- Multi-leaf spring is widely used for the suspension of trucks and railway wagons.
- It consists of a series of flat plates, usually of semi-elliptical shape. The flat plates are called leaves of the spring.
- The leaf at the top has maximum length. The longest leaf at the top is called master leaf.
- The leaves have graduated lengths. The length gradually decreases from the top leaf to the bottom leaf.
• It is bent at both ends to form the spring eye. Two bolts are inserted through these eyes to fix the leaf spring to the automobile body.
• The leaves are held together by means of two U-bolts and a centre clip.
• Rebound clips are provided to keep the leaves in alignment and prevent lateral shifting of the leaves during operation.
• At the centre, the leaf spring is supported on the axle.
• Multi-leaf springs are provided with one or two extra full leaves in addition to master leaf.
• The extra full-length leaves are stacked between the master leaf and the graduated length leaves.
• The extra full length leaves are provided to support the transverse shear force.

3.2.5.1 Nipping of Leaf Springs

- The stresses in extra full-length leaves are more than the stresses in graduated-length leaves.
- One of the method of equalizing the stresses in different leaves is to pre-stress the spring.
- The pre-stressing is achieved by bending the leaves to different radius of curvature, before they are assembled with the clip.
- As shown in the above figure, the full length leaf is given greater radius of curvature than the adjacent leaf.
• The radius of curvature decreases with shorter leaves.
• The initial gap C between the extra full-length leaf and the graduated-length leaf before the assembly is called a nip.
• Such pre-stressing, achieved by a difference in radius of curvature, is known as nipping.

3.2.5 Disc or belleville springs

![Disc or Belleville springs](image)

• A Belleville spring consists of a coned disk, as shown in the above figure.
• It is called Belleville spring because it was invented by Julian Belleville, who patented its design in France in 1867.

![Characteristic Curves for Disc or Belleville springs](image)

• Belleville spring has typical load deflection characteristic, as shown in the above figure.
3. DESIGN OF SPRINGS

- The variation of (h/t) ratio provides a wide variety of load deflection curves.
- Belleville springs are used in plate clutches and brakes, relief valves and a wide variety of bolted connections.

![Diagram of three types of spring combinations: Series Combination, Parallel Combination, Parallel Series Combination.](image)

(a) Series Combination   (b) Parallel Combination   (c) Parallel Series Combination

---

Belleville spring offers following advantages:
1. It is simple in construction and easy to manufacture.
2. It is a compact spring unit.
3. It is especially useful where very large force is desired for small deflection of spring.
4. It provides a wide range of spring constants making it versatile.
5. It can provide any linear or non-linear load deflection characteristic.
6. The individual coned disks of a particular size and thickness can be stacked in series, parallel or series parallel combinations, as shown in Figure. These combinations provide a variety of spring constants without changing the design.
7. When two Belleville springs are arranged in series, double deflection is obtained for the same force. On the other hand, when two Belleville springs are in parallel, almost double force is obtained for a given deflection.

3.3 Spring Materials

- The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant.
- Selection of material for the spring wire depends upon following factors:
  1. The load acting on the spring
  2. The range of stress through which the spring operates
  3. The limitations on mass and volume of spring
  4. The expected fatigue life
  5. The environmental conditions in which the spring will operate such as temperature and corrosive atmosphere
  6. The severity of deformation encountered while making the spring
• The mainly used material for manufacturing the springs are as follows:
  1. Hard drawn high carbon steel
  2. Oil tempered high carbon steel
  3. Stainless steel
  4. Copper or nickel based alloys
  5. Phosphor bronze
  6. Monel
  7. Titanium
  8. Chrome vanadium
  9. Chrome silicon

• Characteristics of some typical materials are explained below
  1. **Hard-drawn wire**: This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 120° C.
  2. **Oil-tempered wire**: It is a cold drawn, quenched, tempered, and general purpose spring steel. However, it is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 180° C. When we go for highly stressed conditions then alloy steels are useful.
  3. **Chrome Vanadium**: This alloy spring steel is used for high stress conditions and at high temperature up to 220° C. It is good for fatigue resistance and long endurance for shock and impact loads.
  4. **Chrome Silicon**: This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 250° C.
  5. **Music wire**: This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. However, it cannot be used at subzero temperatures or at temperatures above 120° C. Normally when we talk about springs we will find that the music wire is a common choice for springs.
  6. **Stainless steel**: Widely used alloy spring materials.
  7. **Phosphor Bronze / Spring Brass**: It has good corrosion resistance and electrical conductivity. That is the reason it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures.

**3.4 Terminology of Helical Springs**

• The main dimensions of a helical spring subjected compressive force are as follows:
  \[ d = \text{wire diameter of spring} \]
  \[ D_i = \text{inside diameter of spring coil} \]
  \[ D_o = \text{outside diameter of spring coil} \]
  \[ D = \text{mean coil diameter} \]

Therefore,

\[ D = \frac{D_i + D_o}{2} \]
1. **Spring index (C):** The spring index is defined as the ratio of mean coil diameter to wire diameter. It is denoted by letter C.

   \[ C = \frac{D}{d} \]

   - The spring index indicates the relative sharpness of the curvature of the coil.
   - A low index means high sharpness of curvature.
   - When the spring index is low \((C < 3)\), the actual stresses in the wire are excessive due to curvature effect.
   - Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires.
   - When the spring index is high \((C > 15)\), it results in large variation in coil diameter.
   - Such a spring is prone to buckling and also tangles easily during handling.
   - Spring index from 4 to 12 is considered better from manufacturing considerations. Therefore, in practical applications, the spring index usually varies from 4 to 12.
   - However, the spring index in the range of 6 to 9 is still preferred particularly for close tolerance springs and those subjected to cyclic loading.
2. **Solid length**: When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid.
   - The solid length of a spring is the product of total number of coils and the diameter of the wire.
   - Mathematically, Solid length of the spring,
     \[ L_s = n'd \]
     where \( n' = \) Total number of coils, and \( d = \) Diameter of the wire.

3. **Free length**: The free length of a compression spring is the length of the spring in the free or unloaded condition.
   - It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).
   - Mathematically, Free length of the spring,
     \[ L_f = L_s + \text{Maximum compression} + \text{Clearance between adjacent coils} \]
     \[ L_f = n'd + \delta_{max} + 0.15\delta_{max} \]

4. **Spring Rate**: The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring.
   - Mathematically, Spring rate,
     \[ k = \frac{W}{\delta} \]
     where \( W = \) Load, and \( \delta = \) Deflection of the spring

5. **Pitch**: It is defined as the axial distance between adjacent coils in uncompressed state.
   - Mathematically,
     \[ p = \frac{L_f}{n' - 1} \]

6. **A. M. Wahl’s Stress Factor and Curvature Effect**:

   ![Curvature Effect Diagram](image)

   **Fig 3.13 Curvature Effect**
Let us look at a small section of a circular spring, as shown in the below figure.

- Suppose we hold the section b-c fixed and give a rotation to the section a-d in the anti-clockwise direction as indicated in the figure, then it is observed that line a-d rotates and it takes up another position, say a'-d'.
- The above phenomenon is termed as curvature effect.
- So more is the spring index, the lesser will be the curvature effect.
- In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl’s stress factor (K) introduced by A. M. Wahl may be used.

\[
K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}
\]

3.5 End Connections for Compression Helical Springs

- The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in above figure.
- In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring.
- Under certain conditions, especially where the number of coils is small, this effect must be taken into account.
- The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis.
- It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and hence are termed as inactive coils.
- The turns which impart spring action are known as active turns.
- As the load increases, the number of inactive coils also increases due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads.
- The following table shows the total number of turns, solid length and free length for different types of end connections.
### Table 3.2  Total number of turns, solid length and free length for various end connections

<table>
<thead>
<tr>
<th>Type of end</th>
<th>Total number of turns ($n'$)</th>
<th>Solid length</th>
<th>Free length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain ends</td>
<td>$n$</td>
<td>$(n + 1) d$</td>
<td>$p \times n + d$</td>
</tr>
<tr>
<td>Ground ends</td>
<td>$n$</td>
<td>$n \times d$</td>
<td>$p \times n$</td>
</tr>
<tr>
<td>Squared ends</td>
<td>$n + 2$</td>
<td>$(n + 3) d$</td>
<td>$p \times n + 3d$</td>
</tr>
<tr>
<td>Squared and ground ends</td>
<td>$n + 2$</td>
<td>$(n + 2) d$</td>
<td>$p \times n + 2d$</td>
</tr>
</tbody>
</table>

where  $n = \text{Number of active turns}$,

$p = \text{Pitch of the coils}$, and

$d = \text{Diameter of the spring wire}$

#### 3.6 End Connections for Tension Helical Springs

- The tensile springs are provided with hooks or loops as shown in the above figure.
- These loops may be made by turning whole coil or half of the coil.
- In a tension spring, large stress concentration is produced at the loop or other attaching device of tension spring.
- The main disadvantage of tension spring is the failure of the spring when the wire breaks.
- The total number of turns of a tension helical spring must be equal to the number of turns ($n$) between the points where the loops start plus the equivalent turns for the loops.
- It has been found experimentally that half turn should be added for each loop. Thus for a spring having loops on both ends, the total number of active turns, $n' = n + 1$.

#### 3.7 Buckling of Compression Springs

- It has been found experimentally that when the free length of the spring ($L_F$) is more than four times the mean or pitch diameter ($D$), then the spring behaves like a column and may fail by buckling at a comparatively low load as shown in the below figure.
3. DESIGN OF SPRINGS

3.8 Surge in Springs

- When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire.

- A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils.

- In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

- This phenomenon can also be observed in closed water body where a disturbance moves toward the wall and then again returns back to the starting of the disturbance.

- This wave of compression travels along the spring indefinitely.

How to prevent buckling?

- Free length ($L_F$) should be less than 4 times the coil diameter (D) to avoid buckling.

- Material should be selected having higher stiffness.

- It order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube.

- When the spring is located on a tube, the clearance between the tube walls and the spring should be kept as small as possible, but it must be sufficient to allow for increase in spring diameter during compression.
- If the applied load is of fluctuating type and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur.
- This results in very large deflections of the coils and correspondingly very high stresses.
- Under these conditions, it is just possible that the spring may fail. This phenomenon is called surge.

**How to prevent surging?**

- The surge in springs may be eliminated by using the following methods:
  1. By using friction dampers on the centre coils so that the wave propagation dies out.
  2. By using springs of high natural frequency (the operational frequency of the spring should be at least 15-20 times less than its fundamental frequency).
  3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

### 3.9 Concentric or Composite Springs

- A concentric or composite or nested spring is used for one of the following purposes:
  1. To obtain greater spring force within a given space.
  2. To insure the operation of a mechanism in the event of failure of one of the springs.
• The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in the above figure and are compressed equally.

• Such springs are used in automobile clutches; valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems.

Let \( W = \text{Axial load}, \)
\( W_1 = \text{Load shared by outer spring}, \)
\( W_2 = \text{Load shared by inner spring}, \)
\( d_1 = \text{Diameter of spring wire of outer spring}, \)
\( d_2 = \text{Diameter of spring wire of inner spring}, \)
\( D_1 = \text{Mean diameter of outer spring}, \)
\( D_2 = \text{Mean diameter of inner spring}, \)
\( \delta_1 = \text{Deflection of outer spring}, \)
\( \delta_2 = \text{Deflection of inner spring}, \)
\( n_1 = \text{Number of active turns of outer spring}, \)
\( n_2 = \text{Number of active turns of inner spring} \)

We know that,
\[
T = W \times \frac{D}{2}
\]

Also,
\[
T = \frac{\pi}{16} \times \tau \times d^3
\]

Equating the above both,
\[
W \times \frac{D}{2} = \frac{\pi}{16} \times \tau \times d^3
\]
\[
\therefore \tau = \frac{8 WD}{\pi d^3}
\]

To consider effect of curvature of wire, a Wahl’s stress factor (K) is introduced in case of helical spring.

\[
\therefore \tau = K \times \frac{8 WD}{\pi d^3}
\]

where
\[
K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}
\]

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, i.e. \( \tau_1 = \tau_2. \)

\[
\therefore K_1 \times \frac{8 W_1 D_1}{\pi (d_1)^3} = K_2 \times \frac{8 W_2 D_2}{\pi (d_2)^3}
\]

When stress factor, \( K_1 = K_2, \)
\[
\frac{W_1 D_1}{(d_1)^3} = \frac{W_2 D_2}{(d_2)^3} \quad \cdots \cdots \cdots \cdots \cdots \quad (i)
\]
Let  

\[ l = \text{Total active length of the wire} \]
\[ = \text{Length of one coil} \times \text{No. of active coils} \]
\[ = \pi D n \]
\[ \theta = \text{Angular deflection of the wire when loaded} \]

Therefore, Axial deflection of the spring,

\[ \delta = \theta \times \frac{D}{2} \]  \hspace{1cm} (ii)

We know that,

\[ \frac{T}{J} = \frac{G \theta}{l} \]
\[ \therefore \theta = \frac{T l}{G J} \]

Putting value of \( l, T \) and \( J \) in the above equation,

\[ \therefore \theta = \left( W \times \frac{D}{2} \right) \left( \pi D n \right) \]
\[ \therefore \theta = \frac{32 (W \times D) (\pi D n)}{2 G (\pi d^4)} \]
\[ \therefore \theta = \frac{16 W D^2 n}{G d^4} \]

Putting value \( \theta \) in the equation (ii),

\[ \delta = \frac{16 W D^2 n}{G d^4} \times \frac{D}{2} \]
\[ \therefore \delta = \frac{8 W D^3 n}{G d^4} \]
\[ \therefore \delta = \frac{8 W C^3 n}{G d^4} \]  \hspace{1cm} \text{where} \ C = \frac{D}{d}

If both the springs are effective throughout their working range, then their free length and deflection are equal, i.e., \( \delta_1 = \delta_2 \).

\[ \frac{8 W_1 (D_1)^3 n_1}{(d_1)^4} = \frac{8 W_2 (D_2)^3 n_2}{(d_2)^4} \]
\[ \frac{W_1 (D_1)^3 n_1}{(d_1)^4} = \frac{W_2 (D_2)^3 n_2}{(d_2)^4} \]

Multiply and divide by \( d_1 \) \& \( d_2 \) on L.H.S. and R.H.S. respectively,

\[ \frac{W_1 (D_1)^3 (n_1 d_1)}{(d_1)^5} = \frac{W_2 (D_2)^3 (n_2 d_2)}{(d_2)^5} \]

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, i.e. \( n_1 d_1 = n_2 d_2 \).

The above equation may be written as,

\[ \frac{W_1 (D_1)^3}{(d_1)^5} = \frac{W_2 (D_2)^3}{(d_2)^5} \]  \hspace{1cm} (iii)
Now dividing equation (iii) by equation (i), we have
\[
\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \\
\therefore \frac{D_1}{d_1} = \frac{D_2}{d_2} = C \text{................. (iv)}
\]
Therefore we can say that the spring index for both the springs is same.

From equations (iii) and (iv), we have
\[
\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \\
\therefore \frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2}
\]

### 3.10 Springs in Series

*Consider two springs connected in series as shown in the figure above.*

Let 
- \(W = \) Load carried by the springs,
- \(\delta_1 = \) Deflection of spring 1,
- \(\delta_2 = \) Deflection of spring 2,
- \(k_1 = \) Stiffness of spring 1 = \(W / \delta_1\)
- \(k_2 = \) Stiffness of spring 2 = \(W / \delta_2\)

* A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.
* Hence total deflection of the springs,

\[
\delta = \delta_1 + \delta_2 \text{................. (v)}
\]

* We know that

\[
k = \frac{W}{\delta} \\
\therefore \delta = \frac{W}{k}
\]
3. DESIGN OF SPRINGS

3.11 Springs in Parallel

- Putting value $\delta$ in the equation $(v)$,
  \[ \frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2} \]
  \[ \therefore \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \]
  \[ \therefore \frac{1}{k} = \frac{k_1 + k_2}{k_1 k_2} \]
  \[ \therefore k = \frac{k_1 k_2}{k_1 + k_2} \]

where $k$ = Combined stiffness of springs

- Consider two springs connected in parallel as shown in the above figure.
  Let $W =$ Load carried by the springs,
  $W_1 =$ Load shared by spring 1,
  $W_2 =$ Load shared by spring 2,
  $k_1 =$ Stiffness of spring 1
  $k_2 =$ Stiffness of spring 2

- A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

- We know that $W = W_1 + W_2$

- Also
  \[ k = \frac{W}{\delta} \]
  \[ \therefore W = k \delta \]
  \[ \therefore \delta k = \delta k_1 + \delta k_2 \]
  \[ \therefore k = \frac{k_1 + k_2}{\delta} \]

where $k$ = Combined stiffness of the springs, and
$\delta =$ Deflection produced
Table 3.3  Standard wire gauge (SWG) number and corresponding diameter of spring wire

<table>
<thead>
<tr>
<th>SWG</th>
<th>Diameter (mm)</th>
<th>SWG</th>
<th>Diameter (mm)</th>
<th>SWG</th>
<th>Diameter (mm)</th>
<th>SWG</th>
<th>Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/0</td>
<td>12.7</td>
<td>7</td>
<td>4.47</td>
<td>20</td>
<td>0.914</td>
<td>33</td>
<td>0.254</td>
</tr>
<tr>
<td>6/0</td>
<td>11.785</td>
<td>8</td>
<td>4.064</td>
<td>21</td>
<td>0.813</td>
<td>34</td>
<td>0.2337</td>
</tr>
<tr>
<td>5/0</td>
<td>10.973</td>
<td>9</td>
<td>3.658</td>
<td>22</td>
<td>0.711</td>
<td>35</td>
<td>0.2134</td>
</tr>
<tr>
<td>4/0</td>
<td>10.16</td>
<td>10</td>
<td>3.251</td>
<td>23</td>
<td>0.61</td>
<td>36</td>
<td>0.193</td>
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<tr>
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<td>9.49</td>
<td>11</td>
<td>2.946</td>
<td>24</td>
<td>0.559</td>
<td>37</td>
<td>0.1727</td>
</tr>
<tr>
<td>2/0</td>
<td>8.839</td>
<td>12</td>
<td>2.642</td>
<td>25</td>
<td>0.508</td>
<td>38</td>
<td>0.1524</td>
</tr>
<tr>
<td>0</td>
<td>8.229</td>
<td>13</td>
<td>2.337</td>
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<td>0.1321</td>
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<td>0.1118</td>
</tr>
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<td>1.626</td>
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<td>0.3454</td>
<td>42</td>
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<td>19</td>
<td>1.016</td>
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<td>0.2743</td>
<td>45</td>
<td>0.0711</td>
</tr>
</tbody>
</table>

Example 3.1  Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN / mm$^2$.

(GTU Example)

Date Given:

$W = 1000$ N,

$\delta = 25$ mm,

$C = D/d = 5$,

$\tau = 420$ MPa = 420 N/mm$^2$,

$G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm$^2$

Solution:

Let

$D =$ Mean diameter of the spring coil, and

$d =$ Diameter of the spring wire.

Wahl’s stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

$\therefore K = \frac{4(5) - 1}{4(5) - 4} + \frac{0.615}{5}$

$\therefore K = 1.31$

Maximum shear stress ($\tau$),

$$\therefore \tau = K \times \frac{8 \ W \ C}{\pi \ d^2}$$

$\therefore d^2 = K \times \frac{8 \ W \ C}{\pi \ \tau}$

$\therefore d^2 = 1.31 \times \frac{8 \ (1000) \ (5)}{420 \pi} = 39.728$
From Table 3.3, we shall take a standard wire of size SWG 3 having diameter,
\((d) = 6.401 \text{ mm}\).

\[\therefore d = 6.3 \text{ mm}\]

Mean diameter of the spring coil,
\[D = C \ d = 5 \ d = 5 \times 6.401 \quad \text{(where C = D/d)}\]
\[\therefore D = 32.005 \text{ mm}\]

Outer diameter of the spring coil,
\[D_o = D + d = 32.005 + 6.401\]
\[\therefore D_o = 38.406 \text{ mm}\]

Inner diameter of the spring coil,
\[D_i = D - d = 32.005 - 6.401\]
\[\therefore D_i = 25.604 \text{ mm}\]

Let \(n = \text{Number of active turns of the coils}\)
Deflection of the spring \((\delta)\),
\[
\delta = \frac{8W C^3 n}{G d} \delta \ G \ d
\]
\[\therefore n = \frac{25 (84 \times 10^3) (6.401)}{8 (1000) (5)^3}\]
\[\therefore n = 13.442\]

\textit{Say } n = 14

For squared and ground ends, the total number of turns,
\[n' = n + 2 = 14 + 2\]
\[\therefore n' = 16\]

Free length of the spring \((L_F)\),
\[L_F = n'd + \delta + 0.15\delta\]
\[\therefore L_F = 16 (6.401) + 25 + 0.15(25)\]
\[\therefore L_F = 131.16 \text{ mm}\]

Pitch of the coil \((p)\),
\[p = \frac{L_F}{n'-1} \quad \frac{131.16}{16-1}\]
\[\therefore p = 8.74 \text{ mm}\]

\textbf{Example 3.2} Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, \(G = 84 \text{ kN/mm}^2\). Neglect the effect of stress concentration.

(GTU Example)
\textbf{Date Given:}
\[W_s = 2250 \text{ N},\]
W₂ = 2750 N,
δ = 6 mm,
C = D/d = 5,
τ = 420 MPa = 420 N/mm²,
G = 84 kN/mm² = 84 × 10³ N/mm²

Solution:
Let
D = Mean diameter of the spring coil for a maximum load of W₂ = 2750 N, and
d = Diameter of the spring wire.

Twisting moment on the spring (T),

\[ T = W \times \frac{D}{2} \]

\[ \therefore T = 2750 \times \frac{5d}{2} \]

\[ \therefore T = 6875 d \]

Also twisting moment on the spring (T),

\[ T = \frac{\pi}{16} \times \tau \times d^3 \]

\[ \therefore 6875 d = \frac{\pi}{16} \times 420 \times d^3 \]

\[ \therefore d^2 = \frac{6875(16)}{420 \pi} \]

\[ \therefore d^2 = 83.367 \]

\[ \therefore d = 9.13 \text{ mm} \]

From Table 3.3, we shall take a standard wire of size SWG 3/0 having diameter,

(d) = 9.49 mm.

\[ \therefore \text{Mean diameter of the spring coil,} \]

\[ D = C \cdot d = 5 \times 9.49 \]

\[ \therefore D = 47.45 \text{ mm} \]

Outer diameter of the spring coil,

\[ D_o = D + d = 47.45 + 9.49 \]

\[ \therefore D_o = 56.94 \text{ mm} \]

Inner diameter of the spring coil,

\[ D_i = D - d = 47.45 - 9.49 \]

\[ \therefore D_i = 37.96 \text{ mm} \]

Let
n = Number of active turns

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (i.e. for
W = 500 N) is 6 mm.

Deflection of the spring (δ),

\[ \delta = \frac{8 W C^3 n}{G d} \]


\[ n = \frac{\delta G d}{8 W C^3} \]

\[ n = \frac{6 (84 \times 10^3) (9.49)}{8 \times (500)^3} \]

\[ n = 9.566 \]

Say \( n = 10 \)

For squared and ground ends, the total number of turns,

\[ n' = n + 2 = 10 + 2 \]

\[ n' = 12 \]

Since the deflection produced under 500 N is 6 mm, therefore maximum deflection produced under the maximum load of 2750 N is

\[ \delta_{max} = \frac{6}{500} \times 2750 \]

\[ \delta_{max} = 33 \text{ mm} \]

Free length of the spring \( (L_F) \)

\[ L_F = n'd + \delta_{max} + 0.15\delta_{max} \]

\[ L_F = 12 (9.49) + 33 + 0.15(33) \]

\[ L_F = 151.83 \text{ mm} \]

Pitch of the coil \( (p) \),

\[ p = \frac{L_F}{n' - 1} \]

\[ p = \frac{151.83}{12 - 1} \]

\[ p = 13.8 \text{ mm} \]

Example 3.3  Design a helical spring for a spring loaded safety valve (Ramsbottom safety valve) for the following conditions:

- Diameter of valve seat = 65 mm,
- Operating pressure = 0.7 N/mm²,
- Maximum pressure when the valve blows off freely = 0.75 N/mm²,
- Maximum lift of the valve when the pressure rises from 0.7 to 0.75 N/mm² = 3.5 mm,
- Maximum allowable stress = 550 MPa,
- Modulus of rigidity = 84 kN/mm²,
- Spring index = 6.

(GTU Example)

Date Given:

\( D_1 = 65 \text{ mm} \),

\( \rho_1 = 0.7 \text{ N/mm}^2 \),

\( \rho_2 = 0.75 \text{ N/mm}^2 \),

\( \delta = 3.5 \text{ mm} \),

\( \tau = 550 \text{ MPa} = 550 \text{ N/mm}^2 \),

\( G = 84 \text{ kN/mm}^2 = 84 \times 10^3 \text{ N/mm}^2 \),

\( C = D/d = 6 \)

Solution:
Let \( D \) = Mean diameter of the spring coil, and 
\( d \) = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore the spring will be under tension.

We know that initial tensile force acting on the spring (i.e. before the valve lifts),

\[
W_1 = \frac{\pi}{4} \times (D_1)^2 \times p_1
\]

\[\therefore W_1 = \frac{\pi}{4} \times (65)^2 \times 0.7\]

\[\therefore W_1 = 2323 \, N\]

Maximum tensile force acting on the spring (i.e. when the valve blows off freely),

\[
W_2 = \frac{\pi}{4} \times (D_1)^2 \times p_2
\]

\[\therefore W_2 = \frac{\pi}{4} \times (65)^2 \times 0.75\]

\[\therefore W_2 = 2489 \, N\]

\[\therefore \text{Force which produces the deflection of 3.5 mm,} \]

\[\therefore W = W_2 - W_1\]

\[\therefore W = 2489 - 2323\]

\[\therefore W = 166 \, N\]

Since the diameter of the spring wire is obtained for the maximum spring load \( W_2 \), therefore maximum twisting moment on the spring, 

\[
T = W_2 \times \frac{D}{2}
\]

\[\therefore T = 2489 \times \frac{6 \times 8.839}{2} \quad \text{where} \quad C = \frac{D}{d} = 6\]

\[\therefore T = 7467 \, d\]

Also twisting moment on the spring (\( T \)),

\[
T = \frac{\pi}{16} \times \tau \times d^3
\]

\[\therefore 7467 \times 8.839 = \frac{\pi}{16} \times 550 \times d^3\]

\[\therefore d^2 = \frac{7467 \times 16}{550 \pi}\]

\[\therefore d^2 = 69.1438\]

\[\therefore d = 8.315 \, mm\]

From Table 3.3, we shall take a standard wire of size SWG 2/0 having diameter, 

\[d = 8.839 \, mm.\]

\[\therefore \text{Mean diameter of the spring coil,} \]

\[D = C \times d = 6 \times 8.839 \quad \text{(where} \quad C = D/d)\]

\[\therefore D = 53.034 \, mm\]
Outer diameter of the spring coil,
\[ D_o = D + d = 53.034 + 8.839 \]
\[ \therefore D_o = 61.873 \text{ mm} \]

Inner diameter of the spring coil,
\[ D_i = D - d = 53.034 - 8.839 \]
\[ \therefore D_i = 44.195 \text{ mm} \]

Let \( n \) = Number of active turns of the coils

Deflection of the spring (\( \delta \)),
\[ \delta = \frac{8 W C^3 n}{G d} \]
\[ \therefore n = \frac{G d}{8 W C^3} \]
\[ \therefore n = \frac{3.5 (84 \times 10^3) (8.839)}{8 (166) (6)^3} \]
\[ \therefore n = 9.059 \]

Say \( n = 10 \)

For a spring having loop on both ends, the total number of turns,
\[ n' = n + 1 = 10 + 1 \]
\[ \therefore n' = 11 \]

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state,
the free length of the tension spring (\( L_F \)),
\[ L_F = n d + (n - 1) 1 \]
\[ \therefore L_F = 10 (8.839) + (10 - 1) 1 \]
\[ \therefore L_F = 97.39 \text{ mm} \]

Pitch of the coil (\( p \)),
\[ p = \frac{L_F}{n' - 1} \]
\[ \therefore p = \frac{97.39}{11 - 1} \]
\[ \therefore p = 9.739 \text{ mm} \]

Example 3.4 A rail wagon of mass 20 tones is moving with a velocity of 2 m/s. It is brought to rest by two buffers with springs of 300 mm diameter. The maximum deflection of springs is 250 mm. The allowable shear stress in the spring material is 600 MPa. Design the spring for the buffers.

(GTU Example)

Date Given:
\[ m = 20 \text{ tones} = 20000 \text{ kg}, \]
\[ v = 2 \text{ m/s}, \]
\[ D = 300 \text{ mm}, \]
\[ \delta = 250 \text{ mm}, \]
\[ \tau = 600 \text{ MPa} = 600 \text{ N/mm}^2 \]
3. DESIGN OF SPRINGS

Solution:
Let \( d \) = Diameter of the spring wire

We know that kinetic energy of the wagon,
\[
\frac{1}{2} \times m \times (v)^2
\]
\[
= \frac{1}{2} \times 20000 \times (2)^2
\]
\[
= 40 \times 10^3 \, N \cdot m
\]
\[
= 40 \times 10^6 \, N \cdot mm \quad \ldots \ldots \ldots \ldots \ldots (i)
\]

Let \( W \) be the equivalent load which when applied gradually on each spring causes a deflection of 250 mm.

Since there are two springs, energy stored in the springs,
\[
\frac{1}{2} \times W \times \delta \times 2
\]
\[
= W \times \delta
\]
\[
= W \times 250
\]
\[
= 250W \, N \cdot mm \quad \ldots \ldots \ldots \ldots \ldots (ii)
\]

We know that the kinetic energy of the wagon is equal to the energy stored in the springs. Hence equating (i) and (ii), we have
\[
40 \times 10^6 = 250 \times W
\]
\[
W = \frac{40 \times 10^6}{250}
\]
\[
W = 160 \times 10^3 \, N
\]

Twisting moment on the spring (T),
\[
T = W \times \frac{D}{2}
\]
\[
\therefore T = 160 \times 10^3 \times \frac{300}{2}
\]
\[
\therefore T = 24 \times 10^6 \, N \cdot mm
\]

Also twisting moment on the spring (T),
\[
T = \frac{\pi}{16} \times \tau \times d^3
\]
\[
\therefore 24 \times 10^6 = \frac{\pi}{16} \times 600 \times d^3
\]
\[
\therefore d^3 = \frac{24 \times 10^6(16)}{600 \pi}
\]
\[
\therefore d^3 = 203718
\]
\[
\therefore d = 58.84 \, mm
\]

Say \( d = 60 \, mm \)

Outer diameter of the spring coil,
\[
D_o = D + d = 300 + 60
\]
\[
\therefore D_o = 360 \, mm
\]
Inner diameter of the spring coil,
\[ D_i = D - d = 300 - 60 \]
\[ \therefore D_i = 240 \text{ mm} \]

Let \( n = \) Number of active turns of the coils

Deflection of the spring (\( \delta \)),

\[ \delta = \frac{8 W C^3 n}{G d} \quad \text{(where} \quad C = \frac{D}{d} = \frac{300}{60} = 5) \]
\[ \therefore n = \frac{\delta G d}{8 W C^3} \]
\[ \therefore n = \frac{250 (84 \times 10^3) (60)}{8 (160 \times 10^3) (5)^3} \]
\[ \therefore n = 7.875 \]

\( \text{Say} \ n = 8 \)

For squared and ground ends, the total number of turns,
\[ n' = n + 2 = 8 + 2 \]
\[ \therefore n' = 10 \]

Free length of the spring (\( L_F \))
\[ L_F = n'd + \delta + 0.15\delta \]
\[ \therefore L_F = 10 (60) + 250 + 0.15(250) \]
\[ \therefore L_F = 887.5 \text{ mm} \]

Pitch of the coil (\( p \)),
\[ p = \frac{L_F}{n' - 1} \]
\[ \therefore p = \frac{887.5}{10 - 1} \]
\[ \therefore p = 98.61 \text{ mm} \]

Example 3.5 A semi-elliptic leaf spring used for automobile suspension consists of three extra full-length leaves and 15 graduated-length leaves, including the master leaf. The center-to-center distance between two eyes of the spring is 1 m. The maximum force that can act on the spring is 75 kN. For each leaf, the ratio of width to thickness is 9:1. The modulus of elasticity of the leaf material is 207000 N/mm\(^2\). The leaves are prestressed in such a way that when the force is maximum, the stresses induced in all leaves are same and equal to 450 N/mm\(^2\). Determine:

(i) The width and thickness of the leaves;
(ii) The initial nip; and
(iii) The initial pre-load required to close the gap \( C \) between extra full-length leaves and graduated-length leaves.

\( \text{(GTU Example)} \)

Date Given:
\[ 2P = 75 \text{ kN} \]
\[ \therefore P = 37.5 \text{ kN} = 37500 \text{ N} \]
\[ 2L = 1 \text{ m} \]
3. DESIGN OF SPRINGS

\[ L = 0.5 \text{ m} = 500 \text{ mm} \]
\[ b = 9 t \]
\[ n_f = 3, \]
\[ n_g = 15, \]
\[ E = 207000 \text{ N/mm}^2, \]
\[ \sigma_b = 450 \text{ N/mm}^2 \]

**Solution:**

Total number of leaves \( n \),

\[ n = n_f + n_g = 3 + 15 \]
\[ \therefore n = 18 \]

We know that,

\[ \sigma_b = \frac{6 P L}{n b t^2} \]
\[ \therefore 450 = \frac{6 (37500)(500)}{18 (9 t) t^2} \]
\[ \therefore t^3 = \frac{6 (37500)(500)}{18 (9)(450)} \]
\[ \therefore t^3 = 1543.21 \]
\[ \therefore t = 11.56 \text{ mm} \]
\[ \therefore t = 12 \text{ mm} \]

It is given that,

\[ b = 9 t \]
\[ \therefore b = 9 (12) \]
\[ \therefore b = 108 \text{ mm} \]

Initial nip \( C \),

\[ C = \frac{2 P L^3}{E n b t^3} \]
\[ \therefore C = \frac{2 (37500)(500)^3}{207000 (18)(108)(12)^3} \]
\[ \therefore C = 13.482 \text{ mm} \]

Initial pre-load required \( P_i \),

\[ P_i = \frac{2 n_g n_f P}{n (3 n_f + 2 n_g)} \]
\[ \therefore P_i = \frac{2 (15)(3)(37500)}{18 [3(3) + 2 (15)]} \]
\[ \therefore P_i = 4807.69 \text{ N} \]

**Example 3.6** A semi-elliptic multi-leaf spring is used for the suspension of the rear axle of a truck. It consists of two extra full length leaves and ten graduated length leaves including the master leaf. The center to center distance between the two eyes of the spring is 1.2 m and the width of the leaf is 60 mm. The leaves are made of steel 55S2Mn90 \( (S_y = 1500 \text{ N/mm}^2 \text{ and } E = 207000 \text{ N/mm}^2) \) and the factor of safety is 2.5.
The spring is to be designed for a maximum force of 30 kN. The leaves are pre-stressed so as to equalize stresses in all the leaves. Determine:

(i) The thickness of the leaves; and
(ii) The deflection at the end of the spring.

(GTU Example)

Date Given:

\[ 2P = 30 \text{ kN} \]
\[ \therefore P = 15 \text{ kN} = 15000 \text{ N} \]
\[ 2L = 1.2 \text{ m} \]
\[ \therefore L = 0.6 \text{ m} = 600 \text{ mm} \]
\[ b = 60 \text{ mm} \]
\[ n_f = 2, \]
\[ n_g = 10, \]
\[ E = 207000 \text{ N/mm}^2, \]
\[ S_{yt} = 1500 \text{ N/mm}^2, \]
\[ f_s = 2.5 \]

Solution:

\[ \sigma_b = \frac{S_{yt}}{f_s} \]
\[ \therefore \sigma_b = \frac{1500}{2.5} \]
\[ \therefore \sigma_b = 600 \text{ N/mm}^2 \]

Total number of leaves (n),

\[ n = n_f + n_g = 2 + 10 \]
\[ \therefore n = 12 \]

We know that,

\[ \sigma_b = \frac{6P L}{nb t^2} \]
\[ \therefore 600 = \frac{6 \times 15000 \times 600}{12 \times (60) t^2} \]
\[ \therefore t^2 = \frac{6 \times 15000 \times 600}{12 \times (60) 	imes (600)} \]
\[ \therefore t^2 = 125 \]
\[ \therefore t = 11.18 \text{ mm} \]
\[ \therefore t = 12 \text{ mm} \]

Deflection of the spring (\( \delta \)),

\[ \delta = \frac{12PL^3}{Eb t^3(3n_f + 2n_g)} \]
\[ \therefore \delta = \frac{12 \times 15000 \times 600^3}{207000 \times 60 \times (12)^3 \{3(2) + 2(10)\}} \]
\[ \therefore \delta = 69.677 \text{ mm} \]
BELT AND CHAIN DRIVES

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4.1 Introduction
Power is transmitted from one shaft to another by means of Belt, rope, chain and gears.

Salient Features:
1. Belt, rope and chain are used where the distance between the shaft is large. For small distance gears are preferred.
2. Belt, rope and chain are flexible type of connectors, i.e., they are bent easily.
3. The flexibility of belt and rope is due to property of their materials whereas chains have a number of small rigid elements having relative motion between the two elements.
4. Belt and rope transmit power due to friction between them and the pulley. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.
5. Belts and ropes are strained during motion as tensions are developed in them.
6. Owing to slipping and straining action belts and ropes are not positive drive, i.e., velocity ratio are not constant. Chain and gears have constant velocity ratio.

4.2 Selection of Belt drive
– Speed of driving and driven shafts.
– Power to be transmitted.
– Space available.
– Service conditions.
– Centre distance between the shafts.
– Speed reduction ratio.

4.3 Types of Belt drives
1. Light drives:
   – Small power.
   – \( V \leq 10 \text{ m/s} \) Agricultural machine, Small machine.
2. Medium drives
   – Medium power
   – \( 22 < V > 10 \text{ m/s} \), Machine tool.
3. Heavy drives
   – Large power
   – \( V > 22 \text{ m/s} \) Compressor, generator.

Belt drives & its materials:

![Fig. 4.1 Types of Belts](image)
1. Flat Belt:
- Used in industry where moderate amount of power is transmitted.
- Dist. \( x \leq 8 \text{m} \) or 10m apart with 22 m/sec.
- Materials are leather, rubber, canvas, cotton & rubber Balata (higher strength than rubber belt).

2. V- Belt:
- Used in industry where moderate amount of power to be transmitted.
- Connect the shaft up to 4m.
- Speed ratio can be up to 7 to 1 and belt speed 24 m/sec.
- Made of rubber impregnated fabric with angle of V between 30° to 40°.

Note: In mutiple V – belt drive all the belt should be stratch at the same rate so that load is equally divided. When one of the self of belt break, the entire set should be replaced at the same drive. If one belt is replaced the new unwarn and unstressed will be more tightly stretched and will more with different velocity.

4.4 Types of Flat Belt Drives:

1. Open Belt Drive

An open belt drive is used when the driven pulley is desired rotate in the same direction.
- Generally the centre distance for open belt drive is 14 – 16 m. if the distance is too large, the belt whips i.e. vibrates in a direction perpendicular to the direction of motion.
- For very shorter distance, the belt slips increase.
- While transmitting power, one side of the belt is more tightened (known as tight side) as compared to other (known as slack side).
- In case of horizontal drives, it is always desired that the tight side is at the lower side of two pulleys. This is because the sag of the belt will be more on the upper side than the lower side. This is slightly increase the angle of wrap of the belts on the two pulleys than if the belt had been perfectly straight between the pulleys.
4. Belt and Chain Drives

- In case the tight side is on the upper side, the sag will be greater at the lower side, reducing the angle of wrap and slip could occur earlier. This ultimately affects the power to be transmitted.

2. Crossed Belt Drive

- A crossed belt drive is used when the driven pulley is to be rotated in the opposite direction to that of the driving pulley.
- A crossed belt drive can transmit more power than an open belt drive as the angle of wrap is more.
- However the belt has to bend in two different planes and it wears out more. To avoid this the shaft should be placed at a max dist. 20 b where b = width of belt and speed should be less than 15 m/sec.

3. Quarter Turn Belt Drive / Right Angle Belt Drive

- A guide pulley is used to connect two non-parallel shaft in such a way that they may run in either direction, and still making the pulley to deliver the belt properly in accordance with the law of belting.

- A guide pulley can also be used to connect even intersecting shaft also.
4. Belt Drive with Idler Pulley

With constant use the belt is permanently stretched in a little length. This reduces the initial tension in the belt leading to lower power transmission capacity. However the tension in the belt can be restored to the original value by using an arrangement shown in figure.

A bell–crank lever, hinged on the axis of the smaller pulley, supports adjustable weights on its one arm and the axis of a pulley on the other. The pulley is free to rotate on its axis and is known as idler pulley. Owing to weights on one arm of the lever, the pulley exerts pressure on the belt increasing the tension and the angle of contact. Thus, life of the belt is increased and the power capacity is restored to original value.

Motion of one shaft can be transmitted to two or more than two shafts by using a number of idler pulley (figure).

5. Compound Belt Drive / Intermediate Pulley

When it is required to have large velocity ratios, ordinarily the size of the larger pulley will be quite big. However, by using an intermediate (counter shaft) pulley, the size can be reduced.
6. Stepped / Cone Pulley Drive

A stepped cone pulley drive is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is done by shifting the belt from one part of the step to the other.

7. Fast and Loose Pulley drive

Many times, it is required to drive several machines from a single main shaft. In such a case, some arrangement to link or delink a machine to or from the main shaft has to be incorporated as all the machines may not be operating simultaneously. The arrangement usually provides that of using a loose pulley along with a fast pulley.

A fast pulley is keyed to the shaft and rotates with it at the same speed and thus transmits power.

A loose pulley is not keyed to the shaft and thus is unable to transmit any power.
Whenever, a machine is to be driven, the belt is mounted on the fast pulley and when it is not required to transmit any power, the belt is pushed to the loose pulley placed adjacent to the fast pulley.

4.5 Law of Belting

The law of belting states that centre line of the belt when it approaches a pulley must lie in the mid plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley.

- By following this law, non – parallel shafts may be connected by a flat belt.
- It should be observed that it is not possible to operate the belt in the reverse direction without violating the law of belting. Thus, in case of non – parallel shafts, motion is possible only in one direction. Otherwise, the belt is thrown off the pulley. However, it is possible to run a belt in either direction on the pulley of two non – parallel or intersecting shafts with the help of guide pulleys. The law of belting is satisfied.

4.6 Velocity Ratio of Belt Drive

Velocity ratio is the ratio of speed of driven pulley \((N_2)\) to that of driving pulley \((N_1)\)

Let

\[ N_1 = \text{Speed of driving pulley} \]
\[ N_2 = \text{Speed of driven pulley} \]
\[ D_1 = \text{Diameter of driving pulley} \]
\[ D_2 = \text{Diameter of the driven pulley} \]
\[ T = \text{thickness of belt} \]

Neglecting slip between belt & pulley and consider belt to be inelastic.

Let speed of belt on driving pulley = speed of belt on driven pulley

\[
\pi \frac{D_1 N_1}{60} = \pi \frac{D_2 N_2}{60}
\]

\[
\left( D_1 + 2 \frac{t}{2} \right) N_1 = \left( D_2 + 2 \frac{t}{2} \right) N_2
\]

Or Velocity Ratio (VR)

\[
(VR) = \frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}
\]

Velocity Ratio of Compound Belt Drive

Let \(D_1, D_2, D_3, D_4\) = Diameter of pulley
\(N_1, N_2, N_3, N_4\) = Speed of pulley

For pulley 1 & 2

\[
\frac{N_2}{N_1} = \frac{D_1}{D_2} \quad \text{(1)}
\]
For pulley 3 & 4

\[ \frac{N_4}{N_3} = \frac{D_3}{D_4} \]  

\[ N_2 \times N_4 = D_1 \times D_3 \]  

Multiplying (1) & (2)

\[ \frac{N_4}{N_1} = \frac{D_1}{D_2} \times \frac{D_3}{D_4} \]

### 4.7 Creep of Belt

- When the belt passes from slack side to tight side, a certain portion of the belt extends and it contracts again when the belt passes from tight side to slack side. Due to these change of length, there is a relative motion between belt & pulley surface. This relative motion is called “Creep”.
- The total effect of creep is to reduce slightly the speed of the driven pulley or follower.
- Considering the creep....

Velocity ratio \[ \frac{N_2}{N_1} = \frac{d_1}{d_2} \left( \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}} \right) \]

Where

- \( \sigma_1 \) = Stress on the tight side of belt
- \( \sigma_2 \) = Stress on the slack side of belt
- \( E \) = Young modulus for the belt material

### 4.8 Length of Open Belt Drive

Let \( r_1, r_2 \) = Radius of larger & smaller pulley.
- \( x \) = Distance between centre of two pulley.
- \( L \) = Total Length.

Let length of the belt,
L = Arc GJE + EF + Arc FKH + GH

= 2 (Arc JE + EF + Arc FK) \(\ldots\ldots\) (1)

Let

\[
\text{Arc EJG} = 2 \text{ Arc JE} = 2 \left(\frac{\pi}{2} + \alpha\right) \cdot r_1
\]

\[
\text{Arc FKH} = 2 \text{ Arc FK} = 2 \left(\frac{\pi}{2} - \alpha\right) \cdot r_2
\]

\[
\sin \alpha = \frac{r_1 - r_2}{x}
\]

\[
EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}
\]

\[
= \sqrt{(x)^2 - (r_1 - r_2)^2}
\]

\[
= x \sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}
\]

Expanding by Binomial Theorem

\[
= x \left[1 - \frac{1}{2} \left(\frac{r_1 - r_2}{x}\right)^2 + \ldots \ldots \right] = x - \frac{1}{2} \left(\frac{r_1 - r_2}{x}\right)^2
\]

Putting value in equation (1)

\[
\therefore L = 2 \left[\left(\frac{\pi}{2} + \alpha\right) r_1 + x - \frac{(r_1 - r_2)^2}{2x} + \left(\frac{\pi}{2} - \alpha\right) r_2\right]
\]

\[
= 2 \left[r_1 \cdot \frac{\pi}{2} + r_1 \cdot \alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \cdot \frac{\pi}{2} - r_2 \cdot \alpha\right]
\]

\[
= 2 \left[\frac{\pi}{2} (r_1 + r_2) + \alpha (r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]
\]

\[
= \pi (r_1 + r_2) + 2 \alpha (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}
\]

\[
\left[\begin{array}{l}
\sin \alpha = \frac{r_1 - r_2}{x} \\
\alpha \text{ is small} \\
\therefore \sin \alpha = \alpha
\end{array}\right]
\]
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\[ L = \pi (r_1 + r_2) + 2 \left( \frac{r_1 - r_2}{x} \right) (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x} \]

\[ = \pi (r_1 + r_2) + 2 \frac{(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x} \]

\[ L = \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{In terms of Radius} \]

\[ L = \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{In terms of diameter} \]

4.9 Length of Crossed Belt Drive

Fig. 4.10 Length of crossed belt drive

Let \( r_1, r_2 \) = Radius of larger & smaller pulley.
\( x \) = Distance between centre of two pulley.
\( L \) = Total Length.

\[ L = \text{Arc GJE} + \text{Arc FKH} + \text{EF} + \text{GH} \quad \text{(1)} \]

\[ \text{Arc GJE} = 2 \text{ Arc JE} = 2 \left( \frac{\pi}{2} + \alpha \right) \cdot r_1 \quad \text{(a)} \]

\[ \text{Arc FKH} = 2 \text{ Arc HK} = 2 \left( \frac{\pi}{2} + \alpha \right) \cdot r_2 \quad \text{(b)} \]

\[ \text{EF} = \text{GH} = \text{MO}_2 = \sqrt{(O_1O_2)^2 - (O_2M)^2} \]

\[ = \sqrt{x^2 - (r_1 + r_2)^2} \]
Expanding by Binomial Theorem

\[ \begin{align*}
= & \ x \left[ 1 - \frac{1}{2} \left( \frac{r_1 + r_2}{x} \right)^2 + \ldots \right] \\
= & \ x - \frac{(r_1 + r_2)^2}{2x} \ (c)
\end{align*} \]

Putting value of (a), (b) & (c) in equation (1)

\[ \begin{align*}
\therefore L &= 2 \left[ r_1 \left( \frac{\pi}{2} + \alpha \right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \left( \frac{\pi}{2} + \alpha \right) \right] \\
&= 2 \left[ \frac{\pi}{2} (r_1 + r_2) + \alpha (r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x} \right] \\
&= \pi (r_1 + r_2) + 2 \alpha (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x} \\
&= \pi (r_1 + r_2) + 2 \left( \frac{r_1 + r_2}{x} \right) (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x} \\
&= \pi (r_1 + r_2) + 2 \frac{(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x} \\
L &= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \text{ In terms of Radius}
\end{align*} \]

\[ L = \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \text{ In terms of diameter} \]
4.10 Ratio of Friction Tensions

1. Flat Belt

![Diagram of Flat Belt](image)

Fig. 4.11 Ratio of Friction Tensions for Flat Belt

$T_1 =$ Tensions on tight side.

$T_2 =$ Tensions on slack side.

$\theta =$ Angle of Lap of belt over pulley.

$\mu =$ Coefficient of friction between belt & pulley.

Consider a short length of belt PQ subtending an angle of $\delta \theta$ at the centre of pulley.

$R =$ Normal (Radial) reaction between element length of belt & pulley.

$T =$ Tension on slack side of the element.

$\delta T =$ increase in tension on tight side than that of slack side.

$T + \delta T =$ Tension on tight side of element.

- Tensions $T$ and $(T + \delta T)$ act in directions perpendicular to the radii drawn at the end of elements. The friction force $F = \mu R$ will act tangentially to the pulley rim resisting the slipping of the elementary belt on the pulley.

- Resolving the forces in tangential direction (Horizontally...),

$$\mu R + T \cos \frac{\delta \theta}{2} - (T + \delta T) \cos \frac{\delta \theta}{2} = 0$$

$$\mu R + T - (T + \delta T) = 0$$

[As $\delta \theta$ is small]

$$\frac{\delta \theta}{2} \approx 1$$

$$R = \frac{\delta T}{\mu} \quad \text{(1)}$$

- Resolving the forces in Radial Direction (Vertically...),

$$R - T \sin \frac{\delta \theta}{2} - (T + \delta T) \sin \frac{\delta \theta}{2} = 0$$

$$R - T \frac{\delta \theta}{2} - T \frac{\delta \theta}{2} - \frac{\delta T \cdot \delta \theta}{2} = 0$$
As $\delta \theta$ is small
\[
\sin \frac{\delta \theta}{2} \approx \frac{\delta \theta}{2}
\]
\[
\frac{\delta T \cdot \delta \theta}{2} \rightarrow \text{Neglect}
\]

\[
R = 2 \frac{T \delta \theta}{2} = T \delta \theta \quad \text{(2)}
\]

Compare equation (1) and (2)

\[
\frac{\delta T}{\mu} = T \delta \theta
\]

\[
\therefore \frac{\delta T}{T} = \mu \delta \theta
\]

Integrating between proper limits……..

\[
\int_{T_2}^{T_1} \frac{\delta T}{T} = \int_{0}^{\theta} \mu \delta \theta
\]

\[
\therefore \log_{e} \frac{T_1}{T_2} = \mu \theta
\]

Or taking a log to the base 10

\[
\frac{T_1}{T_2} = e^{\mu \theta} 2.3 \log \frac{T_1}{T_2} = \mu \theta
\]

Note: The above relation is valid only when the belt is on the point of slipping on the pulleys.

**2. V – Belt**

![Diagram of V-Belt](image)

*Fig. 4.12 Ratio of Friction Tensions for V-Belt*

– In case of v belt or rope, there are two normal reactions, so radial reaction is equal to $2R \sin \alpha$

Thus total friction force $= 2F = 2\mu R$

– Resolving the forces tangentially……..
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2μR + T \cos \frac{\delta \theta}{2} - (T + \delta T) \cos \frac{\delta \theta}{2} = 0

2μR + T - T - \delta T = 0

\[ \begin{align*}
\text{As } \delta \theta & \text{ is small} \\
\delta \theta & \approx 1
\end{align*} \]

\[ \begin{align*}
\delta T = 2μR \quad (1)
\end{align*} \]

– Resolving the forces radially......

\[ 2R \sin \alpha - T \sin \frac{\delta \theta}{2} - (T + \delta T) \sin \frac{\delta \theta}{2} = 0 \]

\[ 2R \sin \alpha - T \frac{\delta \theta}{2} - T \frac{\delta \theta}{2} - \delta T \cdot \delta \theta \approx 0 \]

\[ \begin{align*}
\text{As } \delta \theta & \text{ is small} \\
\sin \frac{\delta \theta}{2} & \approx \frac{\delta \theta}{2} \\
\delta T \cdot \delta \theta & \to \text{Neglect}
\end{align*} \]

\[ 2R \sin \alpha = 2 \frac{T \delta \theta}{2} \]

\[ \therefore R = \frac{T \delta \theta}{2 \sin \alpha} \quad (2) \]

From equation (1) & (2)

\[ \delta T = 2μ \frac{T \delta \theta}{2 \sin \alpha} \quad \text{or} \quad \frac{\delta T}{T} = \frac{\mu \delta \theta}{\sin \alpha} \]

Integrating between proper limits

\[ \begin{align*}
\int_{T_2}^{T_1} \frac{\delta T}{T} = \int_0^\theta \frac{\mu \delta \theta}{\sin \alpha} \\
\therefore \log_e \frac{T_1}{T_2} = \frac{\mu \theta}{\sin \alpha} \\
\frac{T_1}{T_2} = e^{\mu \theta/\sin \alpha}
\end{align*} \]

Note:-

– The expression is similar to that for a flat belt drive except that \( \mu \) is replaced by \( \mu/\sin \theta \), i.e., the coefficient of friction is increased by \( 1/\sin \theta \). Thus, the ratio \( T_1/T_2 \) is far greater in case of V – belts & ropes for same angle of lap\( \theta \) and coefficient of friction\( \mu \).

– Above expression is derived on the assumption that the belt is on the point of slipping.
4.11 Power Transmitted by a belt

Let

\[ T_1 = \text{Tensions on tight side.} \]
\[ T_2 = \text{Tensions on slack side.} \]
\[ V = \text{Linear velocity of belt} \]
\[ P = \text{Power transmitted} \]

\[ P = \text{Net Force} \times \frac{\text{Distance moved}}{\text{second}} = (T_1 - T_2) V \]

Note:- This relation gives the power transmitted irrespective of the fact whether the belt is on the point of slipping or not.

If it is the relationship between \( T_1 \) and \( T_2 \) for a flat belt is given by \( \frac{T_1}{T_2} = e^{\mu \theta} \). If it is not, no particular relation is available to calculate \( T_1 \) and \( T_2 \).

4.12 Angle of Contact or Angle of Lap

1. Open Belt Drive

\[ r_1 = \text{Radius of larger pulley.} \]
\[ r_2 = \text{Radius of smaller pulley.} \]
\[ x = \text{Centre distance between two pulley.} \]

Angle of contact (\( \theta \))

\[ \theta = (180 - 2\alpha)^\circ \]
\[ = (180 - 2\alpha) \frac{\pi}{180} \text{ Radian} \]

Where \( \sin \alpha = \left( \frac{r_1 - r_2}{x} \right) \)

Fig. 4.13 Open belt drive

2. Cross Belt Drive

Angle of contact (\( \theta \))

\[ \theta = (180 + 2\alpha)^\circ \]
\[ = (180 + 2\alpha) \frac{\pi}{180} \text{ Radian} \]

Where \( \sin \alpha = \left( \frac{r_1 + r_2}{x} \right) \)

Fig. 4.14 Crossed Belt drive
4.13 Centrifugal effect on Belt

While in motion, as a belt passes over a pulley, the centrifugal effect due to its own weight tends to lift the belt from the pulley.

The centrifugal force produces equal tensions on the two side of belt, i.e., on the tight and on the slack side.

Let

\[ m = \text{mass of belt per meter length (Kg/m).} \]
\[ v = \text{Linear velocity of belt (m/sec).} \]
\[ r = \text{Radius of pulley.} \]

\( T_C = \text{Centrifugal tension on tight side & Slack side.} \)

\( F_C = \text{Centrifugal force.} \)

\[
F_C = \text{mass of element} \times \text{Acceleration} \\
= (\text{Length of element} \times \text{mass per unit length}) \times \text{Acc.} \\
= (r \delta \theta \times m) \times \frac{v^2}{r} \quad \text{(1)}
\]

From figure resolving forces radially.....

\[
F_C = 2 T_C \sin \frac{\delta \theta}{2} \\
= 2 T_C \frac{\delta \theta}{2}
\]

\[
\left[ \frac{\delta \theta}{2} \text{ is small} \right] \\
\left[ \sin \frac{\delta \theta}{2} \approx \frac{\delta \theta}{2} \right]
\]

\[ F_C = T_C \delta \theta \quad \text{(2)} \]

From Equation (1) & (2)

\[ T_C \delta \theta = m v^2 \delta \theta \]

\[ \therefore T_C = m v^2 \]
4. Belt and Chain Drives

4.14 Maximum Power transmitted by a belt

- If it is desired that belt transmit maximum power, following two conditions must be satisfied.
  1. Larger tension must reach the maximum permissible value for the belt.
  2. The belt should be on the point of slipping i.e., maximum frictional force is developed in the belt.

Let

\[ P = (T_1 - T_2) \cdot v \]

\[ = T_1 \left(1 - \frac{T_2}{T_1}\right) \cdot v \]

\[ = T_1 \left(1 - \frac{1}{\frac{T_1}{T_2}}\right) \cdot v \]

\[
\begin{bmatrix}
\frac{T_1}{T_2} = e^{\mu \theta} \\
1 - \frac{1}{e^{\mu \theta}} = k \text{ constant}
\end{bmatrix}
\]

\[ = T_1 k \cdot v \]

\[ = (T - T_c) \cdot k \cdot v \]

\[ = kTv - T_c \cdot kv \]

\[ = kTv - (m v^2) \cdot kv \]
4. Belt and Chain Drives

\[ P = kTv - kmv^3 \]

Here maximum tension \( T \) in the belt should not exceed permissible value. Hence treating \( T \) as constant and differentiating the power with respect to \( v \) and equating the same equal to zero.

\[ \therefore \frac{dp}{dv} = kT - 3v^2(km) = 0 \]

\[ \therefore kT = 3v^2 km \]

\[ T = 3mv^2 \]

\[ T = 3T_c \quad \text{or} \quad T_c = \frac{T}{3} \]

- For maximum power transmission, the centrifugal tension in the belt is equal to 1/3 of the maximum allowable belt tension and belt should be on the point of slipping.

\[ \text{Also } T = T_1 + T_c \]

\[ \therefore T_1 = T - T_c \]

\[ = T - \frac{T}{3} = \frac{2T}{3} \]

\[ \text{Also } T = 3T_c \]

\[ = 3 \cdot mv^2 \]

\[ \therefore V_{max} = \sqrt{\frac{T}{3m}} \]

4.15 Initial Tension in the Belt

- When a belt is first fitted to a pair of pulley, an initial tension \( T_0 \) is given to the belt when the system is stationary. When transmitting power, the tension on tight increases to \( T_1 \) and that on slack side decreases to \( T_2 \).

If it is assumed that the material of the belt is perfectly elastic i.e., the strain in the belt is proportional to stress in it and the total length of the belt remains unchanged, the tension on the tight side will increase by the same amount as the tension on the slack side decreases. If the change in the tension is \( \delta T \) ... ....
Tension on tight side $T_1 = T_0 + \delta T$

Tension on slack side $T_2 = T_0 - \delta T$

∴ $T_0 = \frac{T_1 + T_2}{2} = \text{Mean of tight side & slack side tensions}$

1. Initial tension with centrifugal tensions

Total tension on tight side = $T_1 + T_C$

Total tension on slack side = $T_2 + T_C$

Let

$$T_0 = \frac{T_1 + T_2}{2} = \frac{(T_1 + T_C) + (T_2 + T_C)}{2} = \frac{T_1 + T_2 + 2T_C}{2}$$

$$T_0 = \frac{T_1 + T_2}{2} = T_C$$

OR $T_1 + T_2 = 2(T_0 - T_C)$

Let $\frac{T_1}{T_2} = e^{i\theta} = k$

$$k T_2 + T_2 = 2(T_0 - T_C)$$

$$T_2 = 2 \frac{(T_0 - T_C)}{k + 1}$$

Let

$$T_1 = T_2 \cdot k$$

$$T_1 = \frac{2k(T_0 - T_C)}{(k + 1)}$$

∴ $T_1 - T_2 = \frac{2k(T_0 - T_C)}{(k + 1)} - 2 \frac{(T_0 - T_C)}{k + 1}$

$$T_1 - T_2 = \frac{2(k - 1)(T_0 - T_C)}{(k + 1)}$$

Power transmitted $(P) = (T_1 - T_2)v$
\[ P = \frac{2 (k - 1)(T_0v - mv^3)}{k + 1} \]

To find the condition for maximum power transmission......

\[ \frac{dp}{dv} = T_0 - 3mv^2 = 0 \]

\[ \therefore T_0 = 3mv^2 \]

\[ \therefore v = \sqrt[3]{\frac{T_0}{3m}} \]

When the belt drive is started, \( v = 0 \) and thus \( T_C = 0 \) \( (T_C = mv^2) \)

\[ T_1 = \frac{2kT_0}{k + 1} \] (2)

### 4.16 Advantages & Disadvantages of V – Belt drive over Flat Belt drive

#### Advantages

1. V - Belt drive gives compactness due to small distance.
2. Drive is positive, because slip is less.
3. V – Belts are made endless, no joint so smooth drive.
4. Longer life 3 – 5 year.
5. Easily installed & removed.
6. High velocity ratio may be obtained.
7. Power transmission is more due to wedging action of the belt in groove.
8. V – Belt may be operated in either direction.

#### Disadvantages

1. V – Belt drive can’t use for large distance.
2. V – Belts are not so durable as flat belts.
3. Construction of pulley for V – Belt is more complicated than the pulleys for flat belt.
4. Since V – Belts are subjected to certain amount of creep, so they are not suitable for constant speed application such as synchronous machines, timing devices etc.

5. Belt life is effect with temperature changes, improper belt tension and mismatching of belt length.

6. The centrifugal tension prevents the use of V – Belts at speed below 5 m/s and above 50 m/sec.

### 4.17 Design of Pulleys

#### 4.17.1 Pulleys for Flat Belts

The pulleys for flat belts consist of three parts, viz. rim, hub and arms or web. The rim carries the belt. The hub connects the pulley to the shaft. T arms or web join the hub with the rim. There are two types of pulleys that are used for flat belts viz., cast iron pulleys and mild steel pulleys. The pulley diameters are calculated in belt drive design. They should comply with standard values. The minimum pulley diameter depends upon the following two factors:

(i) The number of plies in the belt
(ii) The belt speed

There is a specific term, 'crowning' of pulley in flat belt drive. The thickness of the rim is slightly increased in the centre to give it a convex or conical shape as shown in Fig. 4.16. This is called 'crown' of the pulley. The crown is provided only on one of the two pulleys. The objectives of providing crown are as follows:

(i) The crown on the pulley helps to hold the belt on the pulley in running condition.
(ii) The crown on the pulley prevents the belt from running off the Pulley.
(iii) The crown on the pulley brings the belt to running equilibrium position near the mid-plane of the Pulley.

![Crown for Pulley](image)

*Fig. 4.16 Crown for Pulley*

The crown on pulley is essential, particularly when the pulleys are mounted inaccurately or there is a possibility of slip due to non-parallelism between connected shafts.
4. Belt and Chain Drives

Fig. 4.17 Cast Iron Pulley

- Cast iron pulleys are made of grey cast iron of Grade FG 200.
- Mild steel pulleys are made of structural steels.
- The rim and the arms or web are made of low carbon steel while the hub which is subjected to crushing stress at the keyway, is made of medium carbon steel. The rim is roll-formed from a steel plate and joined either by bolts or welded.
- The arms are made of round steel bar and welded to the rim. The arms are welded to the hub, if it is made of steel. The arms are screwed to the hub, if it is made of cast iron. Mild steel pulleys have lightweight construction compared with cast iron pulleys.

4.17.2 V-Grooved Pulley

Fig. 4.18 Dimensions of V-grooved pulley

- The dimensions of V-grooved pulley for V-belts are shown in Fig. 4.18.
- Such pulleys are usually made of grey cast iron of Grade FG 250. In some applications, the pulleys are made of carbon steel casting.
The notations used in the Fig. 4.18. are as follows:

\( l_p \) = Pitch width of pulley groove or pitch width of belt. It is the width of the belt at its neutral axis. The line in the V-belt, whose length remains unchanged when the belt is deformed under tension, is called its neutral axis.

\( b \) = Minimum height of groove above the pitch line

\( h \) = Minimum depth of groove below the pitch line

\( e \) = Centre to centre distance of adjacent grooves.

\( f \) = Distance of the edge of pulley to first groove center

\( \alpha \) = Groove angle

\( d_p \) = Pitch diameter of pulley. It is diameter of the pulley measured at the pitch width of the groove

\( g \) = Minimum top width of the groove.

**Arms of Cast Iron Pulley**

There are three important things about the arms of the pulley. They are as follows:

(i) The arms of pulley have elliptical cross-section.

(ii) The major axis of elliptical cross-section is in the plane of rotation.

(iii) The major axis of elliptical cross-section is usually twice the minor axis.

Elliptical cross-section reduces aerodynamic losses during the rotation of pulley as compared with rectangular cross-section. Therefore in variably, the arms have elliptical cross-section.

The design of these arms illustrates the application of simple formula for bending stresses. It is assumed that the belt wraps around the rim of the pulley through approximately 180° and one-half of the arms carry the load at any moment. This is illustrated in Fig. 4.19.

![Fig. 4.19](image)

The torque transmitted by the pulley is given by,

\[
M_t = PR \left( \frac{n}{2} \right)
\]

or,

\[
P = \frac{2M_t}{Rn}
\]

Where, \( M_t \) = torque transmitted by the pulley (N-mm)

\( P \) = tangential force at the end of each arm (N)
R = radius of the rim (mm)

n = number of arms

As shown in Fig. 4.20, the bending moment acting on the arm is given by,

\[ M_b = PR \]  \hspace{1cm} (b)

From Eq. (a) and (b)

\[ M_b = \frac{2M_i}{n} \]  \hspace{1cm} (c)

\[ \sigma_b = \frac{M_b y}{I} = \frac{(2M_i / n)(a)}{(\pi a^4 / 8)} \]

\[ a^3 = \frac{16 M_i}{\pi n \sigma_b} \]

\[ a = 1.72 \sqrt[3]{\frac{M_i}{n \sigma_b}} \]  \hspace{1cm} (f)

Where \( \sigma_b \) is the permissible bending stress.

If we consider the minor axis in the plane of rotation as illustrated in Fig. 4.21
\[ l = \frac{\pi ba^3}{64} \]

Since the major axis is twice of the minor axis,
\[ b = 2a \]

\[ l = \frac{\pi a^4}{32} \quad \text{and} \quad y = \frac{a}{2} \]

The bending stress in the arm is given by,
\[ \sigma_b = \frac{M_y}{I} = \frac{(2M_t / n)(a/2)}{(\pi a^4 / 32)} \]

\[ a^3 = \frac{32 M_t}{\pi n \sigma_b} \]

\[ a = 2.17 \left( \frac{M_t}{n \sigma_b} \right)^{1/3} \quad \text{..................(g)} \]

It is observed from Eqs (f) and (g) that keeping the minor axis in the plane of rotation increases the cross-sectional area. It is therefore 'economical' to keep major axis of elliptical cross-section in the plane of rotation.

**Design of pulley**

Centrifugal stress induced in rim, \( \sigma_t = \rho V^2 \)

\[ \rho = \text{Density of rim material} \]

\[ V = \text{Velocity} = \frac{\pi DN}{60} \]

Width of pulley, \( B \) is 25 % higher than width of belt

\[ B = 1.25 b \]

Thickness of pulley rim

\[ t = \frac{D}{200} + 3 \quad \text{for single belt} \]

\[ t = \frac{D}{200} + 6 \quad \text{for double belt} \]

Dimensions of arms

No. of arm, \( n = 4 \) for pulley diameter 200 to 600 mm

No. of arm, \( n = 6 \) for pulley diameter 600 to 1500 mm

Dimensions of hub

Diameter of hub, \( d_1 = 1.5 d + 25 \text{ mm} \)

Length of hub, \( L_1 = \frac{\pi}{2} \times d \quad \text{d = diameter of shaft} \)

Minimum length of hub = \( 2/3 B \)

Not more than width of pulley = \( B \)
Example – 1:- Two parallel shafts connected by a crossed belt, are provided with pulleys 480 mm and 640 mm in diameters. The distance between the centre line of the shaft is 3 m. Find by how much the length of the belt should be changed if it is desired to alter the direction of rotation of the driven shaft.

Solution:

\[ R = 320 \text{ mm} \]
\[ r = 240 \text{ mm} \]
\[ X = C = 3 \text{ mtr.} \]

For cross belt drive

\[ \theta = (180 + 2\alpha) \frac{\pi}{180} \]

For cross belt drive

\[ L_{\text{cross}} = (\pi + 2\alpha)(R + r) + 2C \cos \alpha \]

\[ \begin{align*}
\sin \alpha &= \frac{R + r}{C} = \frac{320 + 240}{3} = \frac{0.320 + 0.240}{3} \\
\alpha &= 10.75^\circ = 10^\circ 45' \\
&= 0.1878 \text{ rad}
\end{align*} \]

\[ L_{\text{cross}} = (\pi + 2 \times 0.1878)(0.32 + 0.24) + 2 \times 3 \cos(10.75) = 7.865 \text{ mtr.} \]

For open belt drive

\[ L_{\text{open}} = \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x \cos \alpha \]

\[ L_{\text{open}} = \pi(R + r) + \frac{(R - r)^2}{C} + 2C \]

\[ = \pi(0.32 + 0.24) + \frac{(0.32 - 0.24)^2}{3} + 2 \times 3 = 7.761 \text{ mtr.} \]

Let length of belt reduced by

\[ L_{\text{cross}} - L_{\text{open}} = 7.865 - 7.761 = 0.104 \text{ mtr.} \]

Example – 2:- A belt runs over a pulley of 800 mm diameter at a speed of 180 rpm. The angle of lap is 165° and the maximum tension in the belt is 2kN. Determine the power transmitted if the co-efficient of friction is 0.3.

Solution:
Let

\[
V = \frac{\pi dN}{60} = \frac{\pi \times 0.8 \times 180}{60} = 7.54 \text{ m/sec}
\]

\[
2.3 \log_{10} \left( \frac{T_1}{T_2} \right) = \mu \theta
\]

\[
2.3 \log_{10} \left( \frac{T_1}{T_2} \right) = 0.3 \times 2.88
\]

\[
\Rightarrow \frac{T_1}{T_2} = 2.37
\]

\[
\Rightarrow \begin{cases} T_1 = 2000 \text{ N} \\ T_2 = \frac{T_1}{2.37} = \frac{2000}{2.37} = 843 \text{ N} \end{cases}
\]

Power = \((T_1 - T_2) V\)

\[
= (2000 - 843) \times 7.54
\]

\[
= 8724 \text{ W}
\]

\[
P = 8.724 \text{ kW}
\]

**Example – 3:** A casting weights 6 kN and is freely suspended from a rope which makes 2.5 turns round a drum of 200 mm diameter. If the drum rotates at 40 rpm, determine the force required by a man to pull the rope form the other end of the rope. Also find the power to raise the casting. The coefficient of friction is 0.25.

**Solution:**

\[
W = T_1 = 6000 \text{ N}
\]

\[
\theta = 2.5 \times 2\pi = 15.70 \text{ rad}
\]

\[
d = 0.20 \text{ mtr.}
\]

\[
N = 40 \text{ rpm}
\]

\[
\mu = 0.25
\]

\[
T_2 = ?
\]

\[
P = ?
\]

Let

\[
V = \frac{\pi dN}{60} = \frac{\pi \times 0.2 \times 40}{60} = 0.419 \text{ m/sec}
\]
4. Belt and Chain Drives

Design of Machine Elements (2151907)

2.3\log_{10} \frac{T_1}{T_2} = \mu \theta = 0.25 \times 15.70

\therefore \frac{T_1}{T_2} = 50.87 \quad \therefore T_2 = 117.93 N

\textbf{Power} = (T_1 - T_2) v

\quad = (6000 - 117.93) \times 0.419

\quad = 2464 \text{ Watt}

P = 2.46 kW

\textbf{Example - 4:} A belt drive transmits 8 kW of power from a shaft rotating at 240 rpm to another shaft rotating at 160 rpm. The belt is 8 mm thick. The diameter of smaller pulley is 600 mm and the two shafts are 5 m apart. The coefficient of friction is 0.25. If the maximum stress in the belt is limited to 3 N/mm². Find the width of the belt for (i) open belt drive and (ii) crossed belt drive.

\textbf{Solution:}

\begin{align*}
P &= 8 \times 10^3 \text{ watt} \\
N_1 &= 240 \text{ rpm, } d_1 = 0.600 \text{ m (Smaller pulley)} \\
N_2 &= 160 \text{ rpm} \\
x &= 5 \text{ m} \\
\mu &= 0.25 \\
t &= 8 \text{ mm} \\
\sigma &= 3 \text{ N/mm}^2
\end{align*}

\begin{align*}
N_1 d_1 &= N_2 d_2 \\
d_2 &= \frac{N_2 d_1}{N_2} = \frac{240 \times 0.6}{160} = 0.900 \text{ mtr. (bigger pulley)}
\end{align*}

(1) Open Belt drive

\text{Angle of contact } \theta = (180 - 2\alpha) \frac{\pi}{180}

\quad = [180 - 2(1.71)] \frac{\pi}{180}

\quad \therefore \theta = 3.08 \text{ rad}

\begin{align*}
\begin{cases}
\sin \alpha = \frac{r_1 - r_2}{x} = \frac{0.450 - 0.300}{5} \\
\alpha = 1.71^\circ
\end{cases}
\end{align*}

\text{Let}

\begin{align*}
2.3\log_{10} \frac{T_1}{T_2} &= \mu \theta = 0.25 \times 3.08 \\
\therefore \frac{T_1}{T_2} &= 2.16
\end{align*}
Let

\[
\text{Max tension } T_i = \sigma b t \\
T_1 = 1975 \sigma b t \\
T_2 = 1975 \times b \times 8 \\
b = 82.29 \text{ mm}
\]

(2) Cross belt drive

Angle of contact \( \theta = (180 + 2\alpha) \) \( \frac{\pi}{180} \)

\[
= [180 + 2(8.62)] \frac{\pi}{180} \\
\therefore \theta = 3.443 \text{ rad}
\]

\[
\sin \alpha = \frac{r_1 + r_2}{X} = \frac{0.450 + 0.300}{5} \\
\alpha = 8.62 \text{ rad}
\]

\[
2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta = 0.25 \times 3.443 \\
\therefore \frac{T_1}{T_2} = 2.36
\]

Let

Max tension \( T_i = \sigma b t \)

\[
\begin{aligned}
T_1 & = 1061 & \text{ & } T_2 = 2.36 \\
T_2 & = 1840 & \text{ & } T_1 = 780 N, T_2 = 1840 N
\end{aligned}
\]

1840 = 3 \times b \times 8

\[
b = 76.6 \text{ mm}
\]

Example – 5:- A 100 mm wide and 10 mm thick belt transmits 5 kW of power between two parallel shafts. The distance between the shaft centres is 1.5 m and the diameter of the smaller pulley is 440 mm. The driving and the driven shafts rotate at 60 rpm and 150 rpm respectively. The coefficient of friction is 0.22.

Find the stress in the belt if the two pulleys are connected by (i) an open belt drive (ii) a cross belt drive.

Solution:

\[
\begin{align*}
b & = 100 \text{ mm}, \quad t = 10 \text{ mm} \\
P & = 5 \times 10^3 \text{ watt} \quad x = 1.5 \text{ m} \\
N_1 & = 60 \text{ rpm} \quad N_2 = 150 \text{ rpm}, \\
d_1 & = 440 \text{ mm} \quad \mu = 0.22 \\
\sigma_{\text{open}} & = ? \quad \sigma_{\text{cross}} = ?
\end{align*}
\]
4. Belt and Chain Drives

\[ N_1 \frac{d_1}{d_1} = N_2 \frac{d_2}{d_2} \]
\[ d_1 = \frac{N_2 \cdot d_2}{N_1} = \frac{150 \times 440}{60} \]
\[ d_1 = 1100 \text{ mm} \]

(1) Open Belt Drive

\[ \text{Angle of contact } \theta = (180 - 2\alpha) \frac{\pi}{180} \]
\[ = \left[ 180 - 2(12.7) \right] \frac{\pi}{180} \]
\[ \therefore \theta = 2.697 \text{ rad} \]

\[ \sin \alpha = \frac{r_1 - r_2}{x} = \frac{550 - 220}{1.5} \]
\[ = \frac{0.55 - 0.22}{1.5} \]
\[ \alpha = 12.70 \]

Let
\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta = 0.22 \times 2.697 \]
\[ \therefore \frac{T_1}{T_2} = 1.81 \quad (1) \]

\[ \text{Power } = (T_1 - T_2) v \]
\[ 5 \times 10^3 = (T_1 - T_2) 3.535 \]
\[ \therefore T_1 - T_2 = 1414.5 \text{ N} \quad (2) \]

From equation (1) & (2)
\[ T_1 = 3160.8 \text{ N} \]
\[ T_2 = 1746.2 \text{ N} \]

Max tension in the belt \( T = \sigma \cdot b \cdot t \)
\[ 3160.8 = \sigma \times 100 \times 10 \]
\[ \sigma = 3.16 \text{ N/mm}^2 \]

(2) Cross belt drive

\[ \text{Angle of contact } \theta = (180 + 2\alpha) \frac{\pi}{180} \]
\[ = \left[ 180 + 2(30.88) \right] \frac{\pi}{180} \]
\[ \therefore \theta = 4.22 \]
2.3\log_{10} \frac{T_1}{T_2} = \mu \theta = 0.22 \times 4.22

\therefore \frac{T_1}{T_2} = 2.53

\text{Power} = (T_1 - T_2) \nu

5000 = (T_1 - T_2) \times 3.535

\therefore T_1 - T_2 = 1414.5 N

\text{Max tension in the belt} \ T = \sigma b t

2339 = \sigma \times 100 \times 10

\sigma = 2.339 \text{ N/mm}^2

\textbf{Example – 6:-} An open belt drive is required to transmit 10 kW of power from a motor running at 600 rpm. Diameter of the driving pulley is 250 mm. Speed of the driven pulley is 220 rpm. The belt is 12 mm thick and has a mass density of 0.001 g/mm$^3$. Safe stress in the belt is not to exceed 2.5 N/mm$^2$. Two shafts are 1.25 m apart. Take $\mu = 0.25$. Determine the width of the belt.

\textbf{Solution:}

\begin{align*}
\text{P} &= 10 \times 10^3 \text{ watt} \\
\text{N} &= 600 \text{ rpm} \\
\text{x} &= 1.5 \text{ m} \\
\text{Speed of Driving pulley} \ N_1 &= 600 \text{ rpm} \\
\text{Diameter of driving pulley} \ d_1 &= 250 \text{ mm} \\
\text{Speed of Driven pulley} \ N_2 &= 220 \text{ rpm} \\
\text{t} &= 12 \text{ mm} \\
\rho &= 0.001 \text{ g/mm}^3 = \frac{1}{10^3 \times 10^3} \text{kg} \times \frac{(10^3)^3}{\text{m}^3} = 10^3 \text{ kg/m}^3 \\
\sigma &= 2.5 \text{ N/mm}^2 = 2.5 \times 10^6 \text{ N/m}^2 \\
\text{x} &= 1.25 \text{ m} \\
\mu &= 0.25 \\
b &= ?
\end{align*}

\begin{align*}
N_1 \ d_1 &= N_2 \ d_2 \\
d_2 &= \frac{N_1 \ d_1}{N_2} = \frac{600 \times 250}{220} \\
d_2 &= 681.81 \text{ mm}
\end{align*}

\begin{align*}
\nu &= \left( \frac{r + \frac{t}{2}}{2} \right) \\
\omega &= \left( \frac{r + \frac{t}{2}}{2} \right) \times \frac{2 \pi N}{60} \\
&= \left( 125 + \frac{12}{2} \right) \times \frac{2 \pi \times 600}{60}
\end{align*}
Let

\[ P = (T_1 - T_2) v \]

\[ 10 \times 10^3 = (T_1 - T_2) \times 8.23 \]

\[ T_1 - T_2 = 1215 \quad \text{(1)} \]

Let

\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta \]

\[ 0.25 \times 2.79 \]

\[ \Rightarrow \frac{T_1}{T_2} = 2.01 \quad \text{(2)} \]

\[
\begin{cases}
\text{open belt drive} \\
\theta = (180 - 2\alpha) \frac{\pi}{180} \ \text{rad} \\
\theta = (180 - 2(9.94)) \frac{\pi}{180} \\
\theta = 2.79
\end{cases}
\]

\[ \text{where } \sin \alpha = \frac{r_1 - r_2}{X} = \frac{340.90 - 125}{1.25} \]

\[ \alpha = 9.94 \text{ rad} \]

From equation (1) & (2)

\[ T_2 = 1203 \text{ N} \]
\[ T_1 = 2418 \text{ N} \]

**Centrifugal Tension \( T_c \)**

\[ T_c = m v^2 \]
\[ = (12b) (8.23)^2 \]
\[ T_c = 812.8b \text{ N} \]

\[ m = \text{Mass of belt / unit length} \]
\[ = \text{volume per unit length} \times \rho \]
\[ = x \text{ sectional area} \times \text{length} \times \rho \]
\[ = \text{width} \times \text{thickness} \times \text{length} \times \rho \]
\[ = b \times 0.012 \times 1 \times 10^3 \]
\[ = 12b \]

Max tension in the belt \[ T = \sigma b t \]

\[ = 2.5 \times 10^6 \times b \times 0.012 \]
\[ = 30,000b \]

Let

\[ T = T_1 + T_c \]

\[ 30,000b = 2418 + 812.8b \]

\[ \therefore b = 0.0828 \text{ m} \]
\[ b = 82.8 \text{ mm} \]
Example – 7: - Two parallel shafts that are 3.5 m apart are connected by two pulley of 1 m and 400 mm diameter. The larger pulley being the driver runs at 220 rpm. The belt weight 1.2 kg/meter length. The maximum tension in the belt is not to exceed 1.8 kN. The coefficient of friction is 0.28. Owing to slip on one of the pulleys, the velocity of driven shaft is 520 rpm only. Determine
1. Torque on each shaft
2. Power transmitted
3. Power lost in friction
4. Efficiency of the drive.

Solution:

\[ x = 3.5 \text{ m} \]
\[ d_1 = 1 \text{ m}, \; N_1 = 220 \text{ rpm} \]
\[ d_2 = 0.400 \text{ m}, \; N_2 = 520 \text{ rpm} \]
\[ m = 1.2 \text{ kg/m} \]
\[ t = 1.8 \times 10^3 \text{ N} \]
\[ \mu = 0.28 \]

\[ v = \frac{\pi d_1 N_1}{60} \]
\[ = \frac{\pi \times 1 \times 220}{60} \]
\[ v = 11.52 \text{ m/sec} > 10 \]

\[ T_c = m v^2 \]
\[ = 1.2 (11.52)^2 \]
\[ T_c = 159 \text{ N} \]

For open belt drive

\[ \theta = (180 - 2\alpha) \frac{\pi}{180} \text{ rad} \]
\[ \text{where } \sin \alpha = \frac{r_1 - r_2}{X} \]
\[ = [180 - 2 (4.91)] \frac{\pi}{180} \]
\[ \therefore \theta = 2.97 \text{ rad} \]

\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta = 0.28 \times 2.97 \]
\[ \therefore \frac{T_1}{T_2} = 2.299 \]

Max Tension
\[ T = T_1 + T_c \]
\[ 1.8 \times 10^3 = T_1 + 159 \]
\[ \therefore T_1 = 1641 \text{ N} \]
\[ \therefore T_2 = 714 \text{ N} \]
1. Torque on larger pulley \( T_L \) = \( (1641 - 714) \times 0.5 \) = 463.5 N·m
   Torque on smaller pulley \( T_S \) = \( (1641 - 714) \times 0.200 \) = 185.4 N·m

2. Power transmitted \( P \) = \( (T_1 - T_2) \times v \) = \( (1641 - 714) \times 11.52 \) = 10679 watt
   \( P \) = 10.679 kw

3. Power lost in friction
   Input Power \( P_{in} \) = \( \frac{2\pi N_1 T_1}{60} \) = \( \frac{2\pi \times 220 \times 463.5}{60} \) = 10678 watt
   Output Power \( P_{out} \) = \( \frac{2\pi N_2 T_2}{60} \) = \( \frac{2\pi \times 920 \times 185.4}{60} \) = 10096 watt
   Power Loss = 10678 - 10096 = 582 watt

4. Efficiency of the drive \( \eta \)
   \( \eta = \frac{Output Power}{Input Power} = \frac{10096}{10678} = 0.945 = 94.5\% \)

**Example – 8:** A V- belt drive with the following data transmits power from an electric motor to compressor:

- Power transmitted = 100 kW
- Speed of the electric motor = 750 rpm
- Speed of compressor = 300 rpm
- Diameter of compressor pulley = 800 mm
- Centre distance between pulleys = 1.5 m
- Max speed of the belt = 30 m/sec
- Mass of density = 900 kg / m³
- C/s area of belt = 350 mm²
- Allowable stress in the belt = 2.2 N / mm²
- Groove angle of pulley = 38° = 2β
- Coefficient of friction = 0.28

Determine the number of belts required and length of each belt.

**Solution:**
\[
N_1 d_1 = N_2 d_2 \\
\therefore d_1 = \frac{N_2 d_2}{N_1} = \frac{300 \times 800}{750} = 320 \text{ mm}
\]

Let centrifugal tension \( T_c \)
\[
T_c = m v^2 \\
= 0.315 (30)^2 = 283.5 \text{ N}
\]

Let Maximum Tension in belt
\[
T = \sigma b t \\
= 2.2 \times 350 \\
T = 770 \text{ N}
\]

Let
\[
T = T_1 + T_c \\
\therefore T_1 = T - T_c \\
= 770 - 283.5 \\
T_1 = 486.5 \text{ N}
\]

Let
\[
2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta \ \cot \beta \\
= 0.28 \times 2.82 \\
\\frac{\sin 19^\circ}{\sin 19^\circ}
\]

\[
\therefore \frac{T_1}{T_2} = 11.3 \\
\therefore T_2 = 43.1 \text{ N}
\]

\[
\therefore \text{Power} = (T_1 - T_2) v \\
= (486.5 - 43.1) \times 30 \\
= 13302 \text{ Watt} \\
P = 13.3 \text{ kW}
\]

\[
\text{No. of belt} = \frac{\text{Total power transmitted}}{\text{Power transmitted / belt}} = \frac{100}{13.3} = 7.51 \approx 8 \text{ belts}
\]
Length of belt drive (using approximate relation)

\[ L_o = \pi \left( r_1 + r_2 \right) + 2x + \frac{(r_1 - r_2)^2}{x} \]

\[ = \pi (0.4 + 0.16) + 2(1.5) + \frac{(0.4 - 0.16)^2}{3} \]

\[ L_o = 4.79 \text{ m} \]

**Example – 9:** Determine the maximum power transmitted by a V–belt drive having the included v–groove angle of 35°. The belt used is 18 mm deep with 18 mm maximum width and weight 300 g per metre length. The angle of lap is 145° and the maximum permissible stress is 1.5 N/mm². \( \mu = 0.2 \)

**Solution:**

V Belt Drive

\[ 2\beta = 35^\circ \]

\[ t = 18 \text{ mm}, \ w = 18 \text{ mm} \]

\[ m = 0.3 \text{ kg/m} \]

\[ \theta = 145^\circ \times \frac{\pi}{180} \text{ rad} = 2.53 \text{ rad} \]

\[ \sigma = 1.5 \times 10^6 \text{ N/m}^2 \]

\[ \mu = 0.2 \]

\[ P_{\text{max}} = ? \]

Let

Max tension in the belt \( T = \sigma b t \)

\[ = 1.5 \times 18 \times 18 \]

\[ = 486 \text{ N} \]

Velocity for Max Power \( v = \sqrt{\frac{T}{3m}} \)

\[ = \sqrt{\frac{486}{3 \times 0.3}} \]

\[ v = 23.23 \text{ m/sec} \]

Now

\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta \cos \beta \]

\[ = \frac{0.2 \times 2.53}{\sin (17.5^\circ)} \]

\[ \therefore \frac{T_1}{T_2} = 5.39 \]
Let

Centrifugal Tension \( T_c = m v^2 \)
\[ = 0.3 (23.23)^2 = 162 \text{ N} \]

\begin{align*}
\text{OR} & \\
\text{Max Tension condi.} & \\
T_c = \frac{T}{3} = \frac{486}{3} = 162 \text{ N}
\end{align*}

Let

\[ T = T_1 + T_c \]
\[ \therefore T_1 = T - T_c = 486 - 162 = 324 \text{ N} \]

\[ \therefore \frac{T_1}{T_2} = 5.39 \quad \therefore T_2 = 60.2 \text{ N} \]

Max Power \( P = (T_1 - T_2) v \)
\[
= (324 - 60.2) 23.23 \\
= 6128.074 \text{ Watt} \\
P = 6.12 \text{ kW}
\]

**Example – 10:** 2.5 kW power is transmitted by an open belt drive. The linear velocity of the belt is 2.5 m/sec. The angle of lap on the smaller pulley is 165°. The coefficient of friction is 0.3.

Determine the effect on power transmission in the following cases.
1. Initial tension in the belt is increased by 8%.
2. Initial tension in the belt is decreased by 8%.
3. Angle of lap is increased by 8% by the use of an idler pulley, for the same speed and the tension on the tight side.
4. Coefficient of friction is increased by 8% by suitable dressing to the friction surface of the belt.

**Solution:**

Let

Power \( P = (T_1 - T_2) v \)
\[ 2500 = (T_1 - T_2) 2.5 \quad \therefore T_1 - T_2 = 1000 \text{ N} \quad (1) \]

Also

\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta = 0.3 \times 165^\circ \times \frac{\pi}{180} \]
\[ \therefore \frac{T_1}{T_2} = 2.37 \quad (2) \]

By Solving
4. Belt and Chain Drives

Design of Machine Elements (2151907)

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\[ T_2 = 729.9 \text{ N} \]
\[ T_1 = 1729.92 \text{ N} \]

Initial Tension \( T_0 = \frac{T_1 + T_2}{2} = 1229.9 \text{ N} \)

1. **Initial Tension increased by 8%**

\[ T_0 = 1229.9 \times 1.08 = 1328.30 \text{ N} \]

Or \[ T_0 = \frac{T_1 + T_2}{2} \]

\[ 1328.30 = \frac{T_1 + T_2}{2} \rightarrow T_1 + T_2 = 2656.60 \text{ N} \] (3)

As \( \mu \) and \( \theta \) remain unchanged......

\[ \therefore \frac{T_1}{T_2} = 2.37 \] (4)

By solving equation (3) & (4)

\[ T_1 = 1868.3 \text{ N} \]
\[ T_2 = 788.30 \text{ N} \]

Let

\[ \text{Power } P = (T_1 - T_2) v \]

\[ = (1868.3 - 788.30)2.5 \]
\[ P = 2.7 \text{ kW} \]

\[ \therefore \text{ increase in power} = \frac{2.7 - 2.5}{2.5} = 0.08 = 8\% \]

2. **Initial Tension decreased by 8%**

\[ T_0 = 1229.9 \times (1 - 0.08) = 1131.50 \text{ N} \]

Or \[ T_0 = \frac{T_1 + T_2}{2} \]

\[ 1131.50 = \frac{T_1 + T_2}{2} \rightarrow T_1 + T_2 = 2263 \text{ N} \] (5)

As \( \mu \) and \( \theta \) remain unchanged......

\[ \therefore \frac{T_1}{T_2} = 2.37 \] (6)

By solving equation (5) & (6)

\[ T_1 = 1591.5 \text{ N} \]
\[ T_2 = 671.5 \text{ N} \]

Let

\[ \text{Power } P = (T_1 - T_2) v \]

\[ = (1591.5 - 671.5)2.5 \]
\[ P = 2.3 \text{ kW} \]


\[ \therefore \text{decrease in power} = \frac{2.5 - 2.3}{2.5} = 0.08 = 8\% \]

3. **Angle of Lap** \((\theta)\) **is increased by 8%** with same speed and \(T_1\)

Let

\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta = 0.3 \times \left(165^\circ \times \frac{\pi}{180} \times 1.08\right) \]

\[ \therefore \frac{T_1}{T_2} = 2.54 \]

As \(T_1\) is same

\[ T_1 = 1729.92\text{ N} \]
\[ T_2 = 681.07\text{ N} \]

Power \(P = (T_1 - T_2)v\)

\[ = (1729.92 - 681.07)2.5 \]
\[ P = 2.62\text{ kW} \]

\[ \therefore \text{increase in power} = \frac{2.62 - 2.5}{2.5} = 0.048 = 4.8\% \]

4. **Coefficient of friction is increased by 8%**

Let

\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta \]

\[ = (0.3 \times 1.08) \times 165 \times \frac{\pi}{180} \]

\[ \therefore \frac{T_1}{T_2} = 2.54 \quad (7) \]

Let

\[ T_0 = \frac{T_1 + T_2}{2} = 1229.9 \]

\[ \therefore T_1 + T_2 = 2459.8\text{ N} \quad (8) \]

By solving equation (7) & (8)

\[ T_2 = 694.9\text{ N} \]
\[ T_1 = 1764.9\text{ N} \]

Let

Power \(P = (T_1 - T_2)v\)

\[ = (1764.9 - 694.9)2.5 \]
\[ P = 2.67\text{ kW} \]

\[ \therefore \text{increase in power} = \frac{2.67 - 2.5}{2.5} = 0.07 = 7\% \]
Example – 11:- In a belt drive the mass of belt is 1 kg/m length and its speed is 6 m/sec. The drive transmits 9.6 kW of power. Determine the initial tension in the belt and strength of belt. The coefficient of friction is 0.25 and angle of lap is 220°.

Solution:
\[ m = 1 \text{ kg/m} \quad v = 6 \text{ m/sec} \]
\[ P = 9.6 \times 10^3 \text{ Watt} \]
\[ T_0 = \ ? \]
Strength = ?
\[ \mu = 0.25 \]
\[ \theta = 220^\circ \times \frac{\pi}{180} \text{ rad} \]

Let
\[ \text{Power } P = (T_1 - T_2)v \]
\[ 9600 = (T_1 - T_2)6 \quad \therefore T_1 - T_2 = 1600 \text{ N } \text{(1)} \]

Let
\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta \]
\[ = 0.25 \times \left( 220 \times \frac{\pi}{180} \right) \]
\[ \therefore \frac{T_1}{T_2} = 2.31 \text{ } \text{(2)} \]

Solving equation (1) & (2)
\[ T_1 = 2594 \text{ N} \]
\[ T_2 = 994 \text{ N} \]

Let
\[ \text{Centrifugal Tension } T_c = m v^2 \]
\[ = 1(6)^2 = 36 \text{ N} \]
\[ \text{Initial Tension } T_0 = \frac{T_1 + T_2}{2} + T_c = \frac{2594 + 994}{2} + 36 = 1830 \text{ N} \]

Strength of the belt = Total Tension on tight side
\[ = T_1 + T_c \]
\[ = 2594 + 36 = 2630 \text{ N} \]

Example – 12:- In an open belt drive, the diameters of larger and smaller pulley are 1.2 m & 0.8 m respectively. The smaller pulley rotates at 320 rpm. The centre distance between the shafts is 4 m. When stationary, the initial tension in the belt is 2.8 kN. The mass of the belt is 1.8 kg/m and \( \mu = 0.25 \). Determine the power transmitted.

Solution:
\[ d_1 = 1.2 \text{ m}, \quad N_1 = ? \]
\[ d_2 = 0.8 \text{ m}, \quad N_2 = 320 \text{ rpm} \]
\[ x = 4 \text{ m} \quad T_0 = 2800 \text{ N} \]
\[ m = 1.8 \text{ kg/m} \quad \mu = 0.25 \]
Power = ?
Velocity of Belt \( = \frac{\pi d N}{60} = \frac{\pi \times 0.8 \times 320}{60} = 13.40 \text{ m/sec} \)

Centrifugal Tension \( T_c = m v^2 \)
\[ \quad = 1.8 (13.4)^2 = 323.4 \text{ N} \]

Initial Tension \( T_0 = \frac{T_1 + T_2}{2} + T_c \)
\[ 2800 = \frac{T_1 + T_2}{2} + 323.4 \]
\[ \therefore T_1 + T_2 = 4953 \text{ N} \quad (1) \]

For open belt drive

Angle of contact \( \theta = (180 - 2\alpha) \frac{\pi}{180} \) \[ \begin{cases} \sin\alpha = \frac{r_1 - r_2}{x} = \frac{0.6 - 0.4}{4} \\ \alpha = 2.86 \end{cases} \]
\[ = [180 - 2(2.86)] \frac{\pi}{180} \]
\[ \therefore \theta = 3.042 \text{ rad} \]

Let
\[ 2.3 \log_{10} \frac{T_1}{T_2} = \mu \theta \]
\[ = \left( 0.25 \times 3.042 \right) \]
\[ \therefore \frac{T_1}{T_2} = 2.14 \quad (2) \]

By solving equation (1) & (2)
\[ T_1 = 3376 \text{ N} \]
\[ T_2 = 1577 \text{ N} \]

Power \( P = (T_1 - T_2) v \)
\[ = (3376 - 1577) 13.4 = 24106 \text{ Watt = 24.10 kW} \]

Example – 13:- The initial tension in a belt drive is found to be 600 N and the ratio of friction tension is 1.8. The mass of the belt is 0.8 kg/m length. Determine the
1. Velocity of the belt for maximum power transmission
2. Tension on the tight side of the belt when it is started
3. Tension on the tight side of the belt when running at maximum speed.

Solution:
1. Velocity of the belt \( (v) \)
   Let max power condition for initial tension
   \[ v = \sqrt{\frac{T_0}{3m}} = \sqrt{\frac{600}{3 \times 0.8}} = 15.81 \text{ m/sec} \]
2. Tension on tight side when belt is started \( v=0, T_c = 0 \)
3. Tension on tight side when belt is running at max speed

\[ T_1 = \frac{2k(T_o - T_c)}{k + 1} = \frac{2 \times 1.8 \times 600}{1.8 + 1} = 771.4 \text{ N} \]

\[ T_1 = 514.6 \text{ N} \]

Example – 14:- The driving pulley of an open belt drive is 800 mm diameter and rotates at 320 rpm while transmitting power to a driven pulley of 250 mm diameter. The Young’s modulus of elasticity of the belt material is 110 N/mm². Determine the speed lost by the driven pulley due to creep if the stresses in the tight and slack sides of belt are found to be 0.8 N/mm² and 0.32 N/mm² respectively.

Solution:
Open Belt Drive
\[ D_1 = 0.8 \text{ m}, \quad N_1 = 320 \text{ rpm} \]
\[ D_2 = 0.250 \text{ m} \]
\[ E = 110 \text{ N/mm}^2 \]
\[ \sigma_1 = 0.8 \text{ N/mm}^2 \]
\[ \sigma_2 = 0.32 \text{ N/mm}^2 \]

\[ \frac{N_2}{N_1} = \frac{D_1}{D_2} \left[ \frac{E + \sqrt{\sigma_1}}{E + \sqrt{\sigma_2}} \right] \quad (\text{Velocity Ratio with creep}) \]

\[ N_2 = 320 \times \frac{800}{250} \left[ \frac{110 + \sqrt{0.32}}{110 + \sqrt{0.8}} \right] = 1021 \text{ rpm} \]

Let velocity ratio without creep......

\[ \frac{N_2}{N_1} = \frac{D_1}{D_2} \]

\[ N_2 = 320 \times \frac{800}{250} = 1024 \text{ rpm} \]

Speed lost due to creep = 1024 – 1021 = 3 rpm
Example – 15:- The following data refers to flat belt drive

- Power to be transmitted = 15 KW
- Motor speed = 1440 r.p.m
- Speed of driven pulley = 480 r.p.m.
- Density of belt material = 950 Kg/m³
- Centre distance between two pulleys = 810 mm
- Belt speed = 20 m/sec
- Modulus of elasticity for belt material = 100 MPa
- Coefficient of friction = 0.35
- Permissible stress for belt material = 2.25 MPa
- Belt thickness = 5 mm

Calculate (i) pulley diameters (ii) length and width of the belt

Solution:

\[ P = 15 \text{ KW} \]
\[ V = 20 \text{ m/sec} \]
\[ N_1 = 1440 \text{ rpm} \]
\[ E_b = 100 \text{ MPa} \]
\[ N_2 = 480 \text{ rpm} \]
\[ \mu = 0.35 \]
\[ \rho = 950 \text{ Kg/m}^3 \]
\[ \sigma = 100 \text{ MPa} \]
\[ c = 810 \text{ mm} \]
\[ t = 5 \text{ mm} \]

Velocity,
\[ V = \frac{\pi d_1 N_1}{60} \]
\[ 20 = \frac{\pi d_1 (1440)}{60} \]
\[ d_1 = 0.265 \text{ m} \]
\[ d_2 N_1 = d_2 N_2 \]
\[ 0.265 \times 1440 = d_2 (1440) \]
\[ d_2 = 0.795 \text{ m} \]

Length of belt,
\[ L = \frac{\pi}{2} (d_1 + d_2) + 2c + \frac{(d_1 - d_2)^2}{4c} \]
\[ L = \frac{\pi}{2} (0.795 + 0.265) + 2(0.81) + \frac{(0.795 - 0.265)^2}{4 \times 0.81} \]
\[ L = 1.66 + 1.62 + 0.086 \]
\[ L = 3.36 \text{ m} \]

\[ \sin \alpha = \frac{r_2 - r_1}{c} = \frac{0.3975 - 0.1325}{0.81} \]
\[ \alpha = 19.09^\circ \]
\[ \theta = 180 - 2\alpha \]
\[ = 180 - 2(19.09) \]
\[ = 141.82^\circ \]
\[ = 141.82 \times \pi/180 = 2.47 \text{ rad} \]
Mass of belt per meter length, \( m = \text{Area} \times l \times \text{density} \)

\[
= \text{b.t.l.\( \rho \)}
\]

\[
= \text{b} \times 0.005 \times 1 \times 950
\]

\[
= 4.75\text{ b kg/m}
\]

Centrifugal tension, \( T_C = mv^2 \)

\[
= 4.75\text{ b (20)}^2
\]

\[
= 1900\text{ b}
\]

Maximum tension in belt, \( T = T_1 + T_C \)

\[
\sigma\text{.b.t} = 1297.44 + 1900\text{ b}
\]

\[
2.25 \times 10^6 \times \text{b} \times 0.005 = 1297.44 + 1900\text{ b}
\]

\[
9350\text{ b} = 1297.44
\]

\[
\text{b} = 0.138\text{ m}
\]

\[
\text{b} = 140\text{ mm}
\]

**Example – 16:-** Determine the percentage increase in power capacity made possible in changing over from a flat pulley to a V belt drive. The diameter of the flat pulleys is the same as the pitch circle diameter of the V belt grooved pulleys. Pulley rotates at the same speed as the grooved pulley. The belt materials are the same and they have the same cross sectional area, with coefficient of friction for both as 0.3. The groove angle of the V belt pulley is 60\(^0\) and the angle of contact for both the cases is 150\(^0\).

**Solution:** \( d, N, \text{material, area are same} \)

\[
\mu = 0.3
\]

\[
2\beta = 60^{\circ}
\]

\[
\theta = 150 \times \pi/180 = 2.6\text{ rad}
\]

**Flat belt**

\[
\frac{T_1}{T_2} = e^{\mu \theta}
\]

\[
\frac{T_1}{T_2} = e^{0.35 \times 2.47}
\]

\[
\frac{T_1}{T_2} = 2.37
\]

Power, \( P = (T_1 - T_2) V \)

\[
15 \times 10^3 = (T_1 - T_2) 20
\]

\[
T_1 - T_2 = 750
\]

\[
2.37 T_2 - T_2 = 750
\]

\[
T_2 = 547.44\text{ N}
\]

\[
T_1 = 1297.44\text{ N}
\]
\[
\frac{T_1}{T_2} = e^{0.3 \times 2.6}
\]
\[
\frac{T_1}{T_2} = 2.18
\]
\[
T_1 = 2.18T_2
\]

Power transmitted by Flat belt, \( P_F = (T_1 - T_2) \cdot V \)
\[
= (2.18T_2 - T_2) \cdot V
\]
\[
= 1.18T_2 V
\]

V-Belt
\[
\frac{T_1}{T_2} = \frac{\mu \theta}{\sin \beta}
\]
\[
\frac{T_1}{T_2} = e^{0.3 \times 2.6} \quad \text{in} \sin 30^\circ
\]
\[
\frac{T_1}{T_2} = 4.76
\]
\[
T_1 = 4.76T_2
\]

Power transmitted by V-belt, \( P_V = (T_1 - T_2) \cdot V \)
\[
= (4.76T_2 - T_2) \cdot V
\]
\[
= 3.76T_2 V
\]

Percentage increase in power capacity = \( \frac{P_V - P_F}{P_F} \times 100 \)
\[
= \frac{3.76 - 1.18}{1.18} \times 100
\]
\[
= 218.6 \%
\]

**Example – 17:** Following data given for open V-belt drive
- Diameter of driving pulley = 100 mm
- Diameter of driven pulley = 200 mm
- Center distance = 1000 mm
- Groove angle = 40º
- Mass of belt = 0.2 kg/m
- Maximum permissible tension = 700 N
- Coefficient of friction = 0.2

Plot a graph of maximum tension and power transmitted against belt velocity. Calculate maximum power transmitted by belt and corresponding belt velocity. Neglect power losses.

**Solution:**

\[
d_1 = 100 \text{ mm}
\]
\[
d_2 = 200 \text{ mm}
\]
c = 1 m  
\(2\beta = 40^\circ\)  
m = 0.2 kg/m  
\(T_1 = 700\) N  
\(\mu = 0.2\)

Arc of contact, \(\theta = 180 - 2\sin^{-1}\left(\frac{d_2 - d_1}{2c}\right)\)

\[
\theta = 180 - 2\sin^{-1}\left(\frac{200 - 100}{2 \times 1000}\right) 
= 174.268^\circ
= 174.268^\circ \times \pi/180 = 3.04\text{ rad}
\]

\[
\frac{T_1 - mV^2}{T_2 - mV^2} = e^{\frac{\mu\theta}{\sin\theta}}
\]

\[
\frac{T_1 - mV^2}{T_2 - mV^2} = e^{\frac{0.2 \times 3.04}{\sin 20^\circ}}
\]

\[
\frac{T_1 - 0.2 \times V^2}{T_2 - 0.2 \times V^2} = 5.916
\]

\[\text{.........(a)}\]

The belt tension is maximum when \(V = 0\). For this condition,

\[
\frac{T_1}{T_2} = 5.916
\]

The maximum permissible tension is 700 N.

\(T_1 = 700\) N  
\(T_2 = 118.32\) N

Initial tension, \(T_0 = \frac{T_1 + T_2}{2} = \frac{700 + 118.32}{2}\)

\[= 409.16\text{ N}\]

For maximum power transmission,

\[
V = \sqrt{\frac{T_0}{3m}} = \sqrt{\frac{409.16}{3 \times 0.2}}
\]

\[= 26.11\text{ m/s}\]

\[T_1 + T_2 = 2T_0\]

\[T_1 + T_2 = 818.32\text{ ..........(b)}\]

Substituting \((V = 26.11\text{ m/s})\) in Eq. (a),

\[
\frac{T_1 - 0.2 \times (26.11)^2}{T_2 - 0.2 \times (26.11)^2} = 5.916
\]

\[\text{.........(c)}\]

Solving Eq. (b) and (c)
\[ T_2 = 215.24 \text{ N} \]
\[ T_1 = 603.08 \text{ N} \]

Power transmitted by belt, \( P = (T_1 - T_2) V \)
\[ = (603.08 - 215.24) \times 26.11 \]
\[ = 10.126 \text{ kW} \]

Variation of tension and power against belt velocity

For an intermediate velocity of \( V = 10 \text{ m/s} \),
\[ \frac{T_1 - 0.2 \times (10)^2}{T_2 - 0.2 \times (10)^2} = 5.916 \]

\[ \text{...(d)} \]

Solving Eq. (b) and (d),
\[ T_2 = 132.54 \text{ N} \]
\[ T_1 = 685.78 \text{ N} \]

Power transmitted by belt, \( P = (T_1 - T_2) V \)
\[ = (685.78 - 132.54) \times 10 \]
\[ = 5.532 \text{ kW} \]

For an intermediate velocity of \( V = 40 \text{ m/s} \),
\[ \frac{T_1 - 0.2 \times (40)^2}{T_2 - 0.2 \times (40)^2} = 5.916 \]

\[ \text{...........}(d) \]

Solving Eq. (b) and (d),
\[ T_2 = 345.78 \text{ N} \]
\[ T_1 = 472.53 \text{ N} \]

Power transmitted by belt, \( P = (T_1 - T_2) V \)
\[ = (472.53 - 345.78) \times 40 \]
\[ = 5.07 \text{ kW} \]

Similar calculations can be made for other values of the belt velocity and plot a graph of maximum tension and power transmitted against belt velocity.
Example – 18:– 11 KW, 1440 rpm motor is used to transmit power through V belt drive having following details.

- Each belt has area of cross section = 140 mm$^2$
- Groove angle for pulley = 38°
- Density of belt material = 1350 Kg/m$^3$
- Diameter of pulley on motor shaft = 140 mm
- Speed ratio = 2:1
- Centre distance = 400 mm
- Maximum permissible stress for belt = 2.5 MPa
- Coefficient of friction between belt and pulley = 0.25

Find the number of belts required.

Solution:

\[
P = 11 \text{ KW} \quad \quad d_1 = 140 \text{mm}
\]
\[
N_1 = 1440 \text{ rpm} \quad \quad \text{V.R.} = 2:1
\]
\[
A = 140 \text{ mm}^2 \quad \quad c = 400 \text{ mm}
\]
\[
2\beta = 38^\circ \quad \quad \sigma = 2.5 \text{MPa}
\]
\[
\rho = 1350 \text{ Kg/m}^3 \quad \quad \mu = 0.25
\]

Velocity, \(V= \frac{\pi d_1 N_1}{60}\)

\[
V = \frac{\pi \times 0.140 \times 1440}{60}
\]

\[
V = 10.55 \text{ m/sec}
\]
Design of Machine Elements (2151907) 4. Belt and Chain Drives

\[ N_1 \frac{d_1}{d_2} = N_2 \]

\[ 0.140 \times 2 = d_2 \]

\[ d_2 = 0.28 \text{ m} \]

\[ \sin \alpha = \frac{r_2 - r_1}{c} = \frac{0.28 - 0.14}{0.4} \]

\[ \alpha = 20.487^\circ \]

\[ \theta = 180 - 2\alpha \]

\[ = 180 - 2(20.487) \]

\[ = 139.025^\circ \]

\[ = 139.025 \times \pi/180 = 2.426 \text{ rad} \]

Mass of belt per meter length, \( m = \text{Area} \times l \times \text{density} \)

\[ = 140 \times 10^{-6} \times 1 \times 1350 \]

\[ = 0.189 \text{ kg/m} \]

Maximum tension in belt, \( T = T_1 + T_2 \)

\[ \sigma \Delta = T_1 + mv^2 \]

\[ 2.5 \times 140 = T_1 + 0.189(10.55)^2 \]

\[ T_1 = 328.96 \text{ N} \]

\[ \frac{T_1}{T_2} = e^{\frac{\mu_1}{\sin \beta}} \]

\[ \frac{T_1}{T_2} = e^{\frac{0.25 + 2.426}{\sin 19^\circ}} \]

\[ = 6.44 \]

\[ T_2 = 51.06 \text{ N} \]

Power transmitted by belt, \( P = (T_1 - T_2) V \)

\[ = (328.96 - 51.06) \times 10.55 \]

\[ = 2.93 \text{ KW} \]

No. of belts = \( \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} \)

\[ \frac{11}{2.93} = 3.75 \]

\[ \approx 4 \text{ belts} \]

**Example-19:-** Select a HI-SPEED belt for a light machine tool form following given data:

Power = 14.5 KW  Motor speed = 1440 r.p.m.

Machine pulley speed = 360 r.p.m.  Load factor = 1.2
Centre distance between pulley = 1000 mm  

Load rating for belt = 0.023 KW per mm per width per ply at 180° arc of contact at 10 m/sec belt speed

**Preferable pulley sizes:** 200, 224, 250, 280, 315, 400, 450, 500, 560, 630, 710, 800, 900, 1000 mm

<table>
<thead>
<tr>
<th>Arc of contact (degree)</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>150</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc of contact factor</td>
<td>1.33</td>
<td>1.26</td>
<td>1.19</td>
<td>1.13</td>
<td>1.08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of ply</th>
<th>Standard width of belt in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-ply</td>
<td>40</td>
</tr>
<tr>
<td>5-ply</td>
<td>76</td>
</tr>
<tr>
<td>6-ply</td>
<td>112</td>
</tr>
</tbody>
</table>

**Solution:**

\[ P = 14.5 \text{ KW} \]
\[ N_1 = 1440 \text{ rpm} \]
\[ N_2 = 360 \text{ rpm} \]
\[ F_a = 1.2 \]
\[ c = 1 \text{ m} \]

Load rating for belt = 0.023 KW

\[ V = 10 \text{ m/sec} \]

**Diameter of smaller and bigger pulleys**

\[ V = \frac{\pi d \cdot N}{60} \]

\[ 10 = \frac{\pi d_1 (1440)}{60} \]

\[ d_1 = 0.133 \text{ m} = 133 \text{ mm} \]

Selecting the preferred pulley diameter of 200 mm.

\[ d_1 N_1 = d_2 N_2 \]
\[ 200 \times 1440 = d_2 (360) \]
\[ d_2 = 800 \text{ mm} \]

Maximum power, \( P_{\text{max}} = F_a \cdot P \)

\[ = 1.2 \times 14.5 \]
\[ = 17.4 \text{ KW} \]

Arc of contact,

\[ \theta = 180 - 2\sin^{-1} \left( \frac{d_2 - d_1}{2c} \right) \]
\[ \theta = 180 - 2\sin^{-1} \left( \frac{800 - 200}{2 \times 1000} \right) \]
= 145.084

Related Arc of contact factor, \( F_d = 1.16 \)

Corrected Power, \( P_{co} = F_d \times P_{max} \)

\[
= 1.16 \times 17.4 \\
= 20.184 \text{ KW}
\]

\[
\text{Width} \times \text{No. of plies} = \frac{\text{Corrected power}}{\text{Load rating for belt}}
\]

\[
= \frac{20.184}{0.023}
\]

= 877.56

Belt widths,

4 plies \( b = \frac{877.56}{4} = 219.39 \text{ mm} \)

5 plies \( b = \frac{877.56}{5} = 175.5 \text{ mm} \)

6 plies \( b = \frac{877.56}{6} = 146.26 \text{ mm} \)

We should select belt of 152 mm preferred width and 6 plies.

Example – 20:- Design a V-belt drive from the following data:

- Power to be transmitted = 7.5 kW
- Motor speed = 1440 r.p.m.
- Speed of driven pulley = 480 r.p.m
- Centre distance between two pulleys = 1000 mm
- Service factor = 1.3
- Driver pulley diameter = 200 mm.

<table>
<thead>
<tr>
<th>Arc of contact (degree)</th>
<th>151</th>
<th>154</th>
<th>157</th>
<th>160</th>
<th>163</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arc of contact factor</td>
<td>0.93</td>
<td>0.93</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correction factor for belt length</th>
<th>Belt Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>1.09</td>
<td>2480</td>
</tr>
<tr>
<td>1.10</td>
<td>2570</td>
</tr>
<tr>
<td>1.11</td>
<td>2700</td>
</tr>
</tbody>
</table>

Solution: \( P = 7.5 \text{ kW} \)

\( N_1 = 1440 \text{ rpm} \)

\( N_2 = 480 \text{ rpm} \)

\( c = 1 \text{ m} \)
4. Belt and Chain Drives

Design of Machine Elements (2151907)

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Darshan Institute of Engineering & Technology, Rajkot

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$F_a = 1.3$

$d_1 = 200 \text{ mm}$

Design power $= F_a \times P = 1.3 \times 7.5$

$= 9.75 \text{ kW}$

$d_1N_1 = d_2N_2$

$200 \times 1440 = d_2(480)$

$d_2 = 600 \text{ mm}$

Length of belt,

$L = \frac{\pi}{2} (d_1 + d_2) + 2c + \frac{(d_1 - d_2)^2}{4c}$

$L = \frac{\pi}{2} (0.2 + 0.6) + 2(1) + \frac{(0.6 - 0.2)^2}{4 \times 1}$

$L = 3.296 \text{ m}$

Select length of belt as 3200 mm

Related correction factor for belt length, $F_C = 1.09$

Arc of contact,

$\theta = 180 - 2\sin^{-1}\left(\frac{d_2 - d_1}{2c}\right)$

$\theta = 180 - 2\sin^{-1}\left(\frac{600 - 200}{2 \times 1000}\right)$

$= 156^\circ$

Approximately Arc of contact factor, $F_d = 0.96$

Assuming, Power rating of belt = 6.36 kW

No. of belts $= \frac{\text{Power to be transmitted} \times F_a}{\text{Power Rating} \times F_C \times F_d}$

No. of belts $= \frac{7.5 \times 1.3}{6.36 \times 1.09 \times 0.96}$

$= 1.51$

$\approx 2.$

Example – 21:- A pulley made of gray C.I. transmits 10 kW power at 720 rpm. The diameter of pulley is 500 mm. The pulley has four arms of elliptical cross-section, in which the major axis is twice of the minor axis. Determine the dimensions of cross-section of arm, if factor of safety is 5.

Solution: $P = 10 \text{ kW}$

$N = 720 \text{ rpm}$

$D = 500 \text{ mm}$

$n = 4$

For gray C.I., $\sigma_{ut} = 150 \text{ N/mm}^2$
Power, \( P = \frac{2\pi NM_t}{60} \)

\[ 10 \times 10^4 = \frac{2 \times \pi \times 720 \times M_t}{60} \]

\[ M_t = 132.63 \times 10^3 \text{ N.mm} \]

One half of arms carry the load at any instance, tangential force at end of each arm,

\[ \text{Tangential force, } P = \frac{M_b}{R \left( \frac{n}{2} \right)} = \frac{132.63 \times 10^3}{250 \left( \frac{4}{2} \right)} = 265.26 \text{ N} \]

Bending moment, \( M_b = P \times R \)

\[ = 265.26 \times 250 \]

\[ = 66315 \text{ N.mm} \]

For an elliptical cross-section with \( a \) & \( b \) as minor and major axis

\[ l = \frac{\pi}{64} a b^3 , \quad b = 2a \]

\[ l = \frac{\pi}{64} a (2a)^3 = \frac{\pi a^4}{8} \quad y = b/2 = a \]

\[ \sigma_b = \frac{M_b}{l} \cdot y \]

\[ \frac{30}{\pi} = \frac{66315}{8} \cdot a \]

\[ a = 17.78 \text{ mm} \approx 20 \text{ mm} \]

\[ b = 40 \text{ mm} \]

**Example – 22:** An overhung cast iron pulley transmits 7.5 kW at 400 r.p.m. The belt drive is vertical and angle of wrap may be taken as 180°. Find:

(i) Diameter of the pulley. The density of CI is 7200 kg/m³

(ii) Width of the belt, if the co-efficient of friction between the belt and the pulley is 0.25 assuming thickness \( t = 10 \text{ mm} \).

(iii) Diameter of the shaft, if the distance of the pulley center line from the nearest bearing is 300 mm.

(iv) Dimensions of the key for securing the pulley on to the shaft.

(v) Size of the arms six in number.

The section of the arms may be taken as elliptical, the major axis being twice the minor axis. The following stresses may be taken for design purpose: Shaft and key: 80 MPa(Tension), 50 MPa (Shear); Belt: 2.5 MPa (Tension); Pulley rim: 4.5 MPa (Tension); Pulley arms: 15 MPa (Tension).

**Solution:**

\[ P = 7.5 \text{ kW} \quad N = 400 \text{ rpm} \]

\[ \theta = 180^\circ \quad \rho_{CI} = 7200 \text{ kg/m}^3 \]
μ = 0.25  t = 10 mm
L = 300 mm  n = 6
Shaft and Key – σ_t = 80 MPa,
τ = 50 MPa
Belt – σ_t = 2.5 MPa
Pulley rim – σ_t = 4.5 MPa
Pulley arms – σ_t = 15 MPa

Diameter of pulley
Centrifugal stress or tensile stress induced in pulley rim
\[ \sigma_t = \rho V^2 \]
\[ 4.5 \times 10^6 = 7200 V^2 \]
V = 25 m/s
Velocity, \( V = \frac{\pi d N}{60} \)
\[ 25 = \frac{\pi d(400)}{60} \]
d = 1.19 m

Width of belt
\[ \frac{T_1}{T_2} = e^{\mu \theta} \]
\[ \frac{T_1}{T_2} = e^{0.25 \times 3.14} \]
\[ \frac{T_1}{T_2} = 2.19 \]
Power, \( P = (T_1 - T_2) V \)
\[ 7.5 \times 10^3 = (T_1 - T_2) 25 \]
\[ T_1 - T_2 = 300 \]
\[ 2.19 T_2 - T_2 = 300 \]
\[ T_2 = 252.1 N \]
\[ T_1 = 552.1 N \]

Assuming the leather belt for which density of belt may be taken as 1000 kg/m^3
Mass of belt per meter length, \( m = b \times t \times l \times \text{density} \)
\[ = b \times t \times l \times 1000 \]
\[ = 10 b \text{ kg/m} \]
Centrifugal tension, \( T_C = mv^2 \)
\[ = 10 b (25)^2 \]
\[ = 6250 b \]

Maximum tension in belt, \( T = T_1 + T_C \)
\[ \sigma.b.t = 552.1 + 6250 b \]
\[ 2.5 \times 10^6 \times b \times 0.01 = 552.1 + 6250 b \]
\[ 18750 b = 552.1 \]
\[ b = 0.029 \text{ m} \]
\[ b ≈ 30 \text{ mm} \]

**Diameter of the shaft**

Power, \( P = \frac{2\pi NM_1}{60} \)
\[ 7.5 \times 10^3 = \frac{2 \times \pi \times 400 \times M_1}{60} \]

Torque, \( M_1 = 179 \text{ N.m} \)

Bending moment on shaft due to tensions of belt,
\[ M = (T_1 + T_2 + 2T_C) L \]
\[ = [552.1 + 252.1 + 2 \times 10 \times 0.3 \times (25)^2] \times 0.3 \]
\[ = 353.76 \text{ N.m} \]

Equivalent twisting moment, \( T_e = \sqrt{T^2 + M^2} \)
\[ T_e = \sqrt{(179)^2 + (353.76)^2} \]
\[ T_e = 396.47 \text{ N.m} \]
\[ T_e = \frac{\pi}{16} d^3 \tau \]
\[ 396.47 \times 10^3 = \frac{\pi}{16} d^3 \times 50 \]
\[ d = 34.31 \text{ mm} \]
\[ d ≈ 38 \text{ mm} \]

**Dimension of the key**

The standard dimensions of the key for 38 mm shaft diameter

Width of key, \( w = 12 \text{ mm} \)

Thickness of key, \( t = 8 \text{ mm} \)
\[ M_t = l \times w \times t \times d/2 \]
\[ 179 \times 10^3 = l \times 12 \times 50 \times 38/2 \]
\[ l = 15.7 \text{ mm} \]
Length of the key should be equal to hub length
   Length of key, \( l = d \frac{\pi}{2} = 38.\frac{\pi}{2} \)
   \( = 59.7 \text{ mm} \)

**Size of arms**

Minor axis of elliptical cross-section of arms

\[
a = 1.72 \sqrt{\frac{M_t}{n \sigma_b}} = 1.72 \sqrt{\frac{179 \times 10^3}{6 \times 15}}
\]

\( = 21.63 \text{ mm} \)
\( = 22 \text{ mm} \)

Major axis, \( b = 2a = 2 \times 22 \)
\( = 44 \text{ mm} \)

**Example – 23:** A pulley of 0.9m diameter transmits 7.5 kW power at 200 rpm. Find the width of a leather belt if maximum tension is not to exceed 14.5 N per mm width. The tension in the tight side is twice that in the slack side. Also determine the dimensions of the various parts of the flat belt pulley, assuming it to have six arms. The arms are of C.I. for which tensile stress may be taken as 15 N/mm\(^2\). The diameter of the shaft is 35 mm.

**Solution:**

\( D = 0.9 \text{ m} \)
\( P = 7.5 \text{ kW} \)
\( N = 200 \text{ rpm} \)
\( T = 14.5 \text{ N per mm width} \)
\( T_1 = 2T_2 \)
\( n = 6 \)
\( \sigma_t = 15 \text{ MPa} \)
\( d = 35 \text{ mm} \)

Velocity,

\[
V = \frac{\pi DN}{60}
\]

\( V = \frac{\pi \times 0.9 \times 200}{60} \)

\( V = 9.42 \text{ m/s} \)

Power, \( P = (T_1 - T_2) V \)

\( 7.5 \times 10^3 = (T_1 - T_2) \times 9.42 \)

\( T_1 - T_2 = 796.18 \)

\( 2T_2 - T_2 = 796.18 \)

\( T_2 = 796.18 \text{ N} \)

\( T_1 = 1592.3 \text{ N} \)

Maximum tension is 14.5 N per mm width

\( b = \frac{T_1}{14.5} = \frac{1592.3}{14.5} \)

\( = 109.8 \text{ mm} \)

\( = 110 \text{ mm} \)
Width of Pulley, \( B = 1.25 \, b \)
\[ = 137.5 \, \text{mm} \]

Thickness of pulley rim for single belt
\[ t = \frac{D}{200} + 3 = \frac{900}{200} + 3 \]
\[ = 7.5 \, \text{mm} \approx 8 \, \text{mm} \]

Power, \( P = \frac{2\pi N M_t}{60} \)
\[ 7.5 \times 10^3 = \frac{2\pi \times 200 \times M_t}{60} \]
Torque, \( M_t = 358.098 \, \text{N.m} \)

Assuming the cross-section of arms as elliptical with major axis equal to twice minor axis
\[ b_1 = 2a_1 \]
\[ a_1 = 1.72 \sqrt[3]{\frac{M_t}{n \sigma_b}} = 1.72 \sqrt[3]{\frac{358.098 \times 10^3}{6 \times 15}} \]
\[ a_1 = 27.25 \, \text{mm} \approx 30 \, \text{mm} \]
\[ b_1 = 60 \, \text{mm} \]

Diameter of hub, \( d_1 = 1.5 \, d + 25 = (1.5 \times 35) + 25 \)
\[ = 77.5 \, \text{mm} \]

Length of hub, \( L = d \cdot \frac{\pi}{2} = 35 \cdot \frac{\pi}{2} \)
\[ = 54.97 \, \text{mm} \approx 55 \, \text{mm} \]

Since the length of hub should not less than \( 2/3B \)
\[ L = \frac{2}{3} \times B = \frac{2}{3} \times 137.5 \]
\[ = 92 \, \text{mm} \]

4.18 Design of chain drives

- A chain drive consists of an endless chain wrapped around two sprockets.
- A chain can be defined as a series of links connected by a pin joints.
- The sprocket is a toothed wheel with a special profile for the teeth.
- The chain drive is intermediate between belt and gear drives. It has some features of belt drives and some of gear drives.
- **Pitch of chain**: It is distance between the hinge centre of a link and the corresponding hinge centre of adjacent link.
- **Pitch circle diameter of sprocket chain**: It is the diameter of an imaginary circle that passes through the centre of link pins as the chain is wrapped on sprocket.
Advantages
1. Constant velocity due to no slip, so it is positive drive.
2. No effect on overload on the velocity ratio.
3. Oil or grease on the surface does not affect the velocity ratio.
4. Chains occupy less space as they made of metals.
5. Lesser loads are put on the shaft.
6. High transmission efficiency due to “No slip”.
7. Through one chain only motion can be transmitted to several shaft.

Disadvantages
1. It is heavier as compared to the belt.
2. There is gradual stretching and increase in length of chains. From time to time some of its links have to be removed.
3. Lubrication of its parts is required.
4. Chains are costlier compared to belts.

Classification of Chains
1. Hoisting Chain
Hoisting chain includes oval link and stud link chains. An oval link is a common form of hoisting chain. It consists of an oval link and is also known as coil chain. Such chains are used for lower speed only.

![Fig. 4.23 Hoisting Chain](image)

2. Conveyor Chain
Conveyor chain may be detachable or hook type or closed joint type. The sprocket teeth are so shaped that the chain should run onto and off the sprocket smoothly without interference.
Such chains are used for low speed agricultural machinery. The material of the link is usually malleable cast iron. The motion of the chain is not very smooth.
3. Power transmission Chain

These chains are made of steel in which the wearing parts are hardened. They are accurately machined and run on carefully designed sprockets.

1. Block Chain: - This type of chain is mainly used for transmission of power at low speeds. Sometimes they are also used as conveyor chains in place of malleable conveyor chains.

2. Roller Chain: - A common form of a roller chain is shown in figure. A bushing is fixed to a inner link whereas the outer link has a pin fixed to it. There is only sliding motion between pin and bushing. The roller is made of a hardened material and is free to turn on the bushing.

3. Silent Chain (Inverted tooth chain):- Though roller chain can run quietly at fairly high speed. The silent chains are used where maximum quietness is desired. Silent chains do not have rollers. The links are so shaped as to engage directly with the sprocket teeth. The included angle is either 60° or 75°.

Geometric Relationships (Relation between Pitch and Pitch circle diameter for Chain)

The engagement of chain on sprocket wheel is shown in Fig. 4.8. D is the pitch circle diameter of the sprocket and α is called the pitch angle. The pitch circle diameter of the
sprocket is defined as the diameter of an imaginary circle that passes through the centres of link pins as the chain is wrapped on the sprocket.

![Diagram](image)

\[ \alpha = \frac{360}{z} \]  \hspace{1cm} \text{..........(a)}

where \( z \) is the number of teeth on the sprocket. From the figure, it can be proved that

\[ \sin \left( \frac{\alpha}{2} \right) = \frac{p}{2} \frac{D}{2} \]

\[ D = \frac{p}{\sin \left( \frac{\alpha}{2} \right)} \]  \hspace{1cm} \text{..........(b)}

The velocity ratio \( i \) of the chain drives is given by

\[ i = \frac{n_1}{n_2} = \frac{z_2}{z_1} \]

Where, \( n_1, n_2 \) = speeds of rotation of driving and driven shafts (rpm)

\( z_1, z_2 \) = number of teeth on driving and driven sprockets.

The average velocity of the chain is given by,

\[ V = \frac{\pi D n}{60} = \frac{z p n}{60} \]  \hspace{1cm} \text{..........(c)}

Where, \( V \) is the average velocity in m/s.

The length of the chain is always expressed in terms of the number of links, or

\[ L = L_n \times p \]  \hspace{1cm} \text{..........(d)}

Where, \( L \) = length of the chain (mm)

\( L_n \) = number of links in the chain

The number of links in the chain is by the following approximate relationships:
The above formula is derived by analogy with the length of the belt. The first two terms represent the number of links when \( z_1 = z_2 \) and the sides of the chain are parallel. The third term takes into consideration the inclination of the sides. It is obvious that the chain should contain a whole number of links. Therefore, the number of links \( L_n \) is adjusted to the previous or next digit so as to get an even number. It is always preferred to have an 'even' number of links, since the chain consists of alternate pairs of inner and outer link plates.

When the chain has an odd number of links, an additional link, called 'offset' link, is provided. The offset link is, however, weaker than the main links. After selecting the exact number of links, the centre to centre distance between the axes of the two sprockets is calculated by the following formula:

\[
a = \frac{p}{4} \left\{ L_n - \left( \frac{z_1 + z_2}{2} \right) \right\} + \sqrt{L_n^2 - \left( \frac{z_1 + z_2}{2} \right)^2} - 8 \left( \frac{z_1 - z_2}{2\pi} \right)^2 \quad \text{.........(f)}
\]

The above equation can be easily derived from Eq. (e). The centre distance calculated by the formula does not provide any sag. In practice, a small amount of sag is essential for the links to take the best position on the sprocket wheel. The centre distance is, therefore, reduced by a margin of \((0.002a \text{ to } 0.004a)\) to account for the sag.

**Polygonal Effect**

The chain passes around the sprocket as a series of chordal links. This action is similar to that of a non-slipping belt wrapped around a rotating polygon. The chordal action is illustrated in Figure 4.8, where the sprocket has only four teeth. It is assumed that the sprocket is rotating at a constant speed of \( n \) rpm. In Fig. 4.9, the chain link AB is at a distance of \((D/2)\) from the centre of the sprocket wheel and its linear velocity is given by,

\[
V_{\text{max}} = \frac{\pi D n}{60} \quad \text{m/s}
\]

As the sprocket rotates through an angle \((\alpha/2)\), the position of the chain link AB is shown in Figure 4.8. In this case, the link is at a distance of \((D/2) \times \cos(\alpha/2)\) from the centre of the sprocket and its linear velocity is given by,

\[
V_{\text{min}} = \frac{\pi D n \cos \left( \frac{\alpha}{2} \right)}{60} \quad \text{m/s}
\]
It is evident that the linear speed of the chain is not uniform but varies from $V_{\text{max}}$ to $V_{\text{min}}$ during every cycle of tooth engagement. This results in a pulsating and jerky motion. The variation in velocity is given by

$$(V_{\text{max}} - V_{\text{min}}) \propto \left[ 1 - \cos \left( \frac{\alpha}{2} \right) \right]$$

$$(V_{\text{max}} - V_{\text{min}}) \propto \left[ 1 - \cos \left( \frac{180}{z} \right) \right]$$

As the number of teeth ($z$) increases to $\infty$, $\cos \left( \frac{180}{z} \right)$ or $\cos \left( \frac{180}{\infty} \right)$, i.e., $\cos (0^\circ)$ will approach unity and $(V_{\text{max}} - V_{\text{min}})$ will become zero. Therefore, the variation will be zero. In order to reduce the variation in chain speed, the number of teeth on the sprocket should be increased.

It has been observed that the speed variation is 4% for a sprocket with 11 teeth, 1.6% for a sprocket with 17 teeth, and less than 1% for a sprocket with 24 teeth.

For smooth operation at moderate and high speeds, it is considered a good practice to use a driving sprocket with at least 17 teeth.

From durability and noise considerations, the minimum number of teeth on the driving sprocket should be 19 or 21.

**Power Rating of Roller Chains**

The power transmitted by the roller chain can be expressed by the elementary equation

$$P = T_1 \cdot V$$

Where, $T_1$ = allowable tension in the chain (N)

$V$ = average velocity of chain (m/s),

The Power rating of the chain is determined by the following relationship:
Power rating of chain = \frac{\text{Power to be transmitted} \times K_S}{K_1 \times K_2}

Where, 
- \(K_S\) = service factor
- \(K_1\) = multiple strand factor
- \(K_2\) = tooth correction factor

- The service factor takes into consideration the effect of shocks and vibrations on the power to be transmitted.
- The tooth correction factor \(K_2\) accounts for the variation in teeth.
- There are duplex and triplex chains too. The variation in the number strands is taken into accounts by the multiple strand factor \(K_1\).

**Modes of chain failure**

There are four modes of chain failures

(i) **Wear**: The wear of the chain is caused by the articulation of pins in the bushings. The wear results in elongation of the chain, or in other words, the chain pitch is increased. This makes the chain 'ride out' on the sprocket teeth, resulting in a faulty engagement. When the elongation is excessive, it becomes necessary to replace the chain. The permissible elongation for the chain is 1.5 to 2.5%. When the chain is properly lubricated, a layer of oil film between the contacting surfaces of the pin and the bushing reduces wear.

(ii) **Fatigue**: As the chain passes around the sprocket wheel, it is subjected to a tensile force, which varies from a maximum on the tight side to a minimum on the loose side. The chain link is, therefore, subjected to one complete cycle of fluctuating stresses during every revolution of the sprocket wheel. This results in a fatigue failure of side link plates. For infinite life, the tensile stress should be lower than the endurance limit of the link plates.

(iii) **Impact**: The engagement of rollers with the teeth of the sprocket results in impact. When excessive, this may lead to the breakage of roller or bushing. Increasing the number of teeth on the sprocket or reducing chain tension and speed reduces the magnitude of the impact force.

(iv) **Galling**: Galling is a stick-slip phenomenon between the pin and the bushing. When the chain tension is high, welds are formed at the high spots of the contacting area. Such microscopic welds are immediately broken due to relative motion of contacting surfaces and leads to excessive wear, even in the presence of the lubricant.
Example – 24:- The center to Centre distance between two sprockets of a chain drive is 600 mm. The chain drive is used to reduce the speed from 180 rpm to 90 rpm on the driving sprocket has 18 teeth and a pitch circle diameter of 480 mm. Determine
1. No. of teeth on the driven sprocket
2. Pitch and the length of chain.

**Solution:**

1. \[ \frac{N_2}{N_1} = \frac{Z_1}{Z_2} \quad \therefore \quad Z_2 = Z_1 \times \frac{N_1}{N_2} \]
   
   \[ = 18 \times \frac{180}{90} = 36 \text{ Teeth} \]

2. Pitch of chain (p)

   \[ p = 2R \sin \left( \frac{180^\circ}{Z_2} \right) = 2 \times 0.240 \times \sin \left( \frac{180^\circ}{36} \right) = 0.04183 \text{ m} = 41.83 \text{ mm} \]

3. Length of chain (L)

   No. of links in chain, \( L_n = \frac{2a}{p} + \frac{z_1 + z_2}{2} + \left( \frac{z_2 - z_1}{2\pi} \right)^2 \frac{p}{a} \)

   \[ L_n = \frac{2 \times 600}{41.83} + \frac{18 + 36}{2} + \left( \frac{36 - 18}{2\pi} \right)^2 \frac{41.83}{600} \]

   \[ L_n = 56.26 \]

   \[ = 56 \text{ links} \]

   Length of chain, \( L = L_n \times \pi \)

   \[ L = 56 \times 41.83 \]

   \[ = 2342.48 \text{ mm} \]

Example – 25:- A simple chain No. 10B is used to transmit power from a 1400 rpm electric motor to a line shaft running at 350 rpm. The number of teeth on the driving sprocket wheel is 19. The operation is smooth without any shocks. Calculate: (i) The rated power for which the chain drive is recommended. (ii) The tension in the chain for this rated power; and (iii) The factor of safety for the chain based on the breaking load.

Use following data:

At 1400 rpm, for chain 10B, power rating is 11.67 kW,

Service factor \( K_S \): 1.3, Multiple strand factor \( K_s \): 1.0, Breaking Load: 22200 N,
Tooth correction factor \( K_2 \): 1.11, Pitch: 15.875 mm.

**Solution:**

\[ N = 1400 \text{ rpm} \]

\[ z = 19 \]

Power to be transmitted = \( \frac{\text{Power rating} \times K_1 \times K_2}{K_3} \)

\[ P = \frac{11.67 \times 1 \times 1.11}{1.3} \]

\[ P = 9.96 \text{ kW} \]

Velocity, \( V = \frac{zpN}{60} \)

\[ V = \frac{19 \times 15.875 \times 10^{-3} \times 1400}{60} \]

\[ V = 7.038 \text{ m/s} \]

The tension in the chain, \( T_1 = \frac{P}{V} = \frac{9.96 \times 10^3}{7.038} \]

\[ = 1415.17 \text{ N} \]

F.O.S. = \( \frac{\text{Breaking Load}}{\text{Chain Tension}} \)

F.O.S. = \( \frac{22200}{1415.17} \)

\[ = 15.68. \]

**Example – 26:** A chain drive with double strands of 16B type has a pitch of 25.4 mm. It is used to transmit power between a 15 tooth driving sprocket rotating at 700 rpm and a 60 tooth driven sprocket. For the drive conditions, a service factor of 1.3 can be used. Find

(i) The power that can be transmitted by the drive.

(ii) Length of the chain, if the centre distance between the sprockets is 475 mm.

Use the following data:

At 700 rpm, for chain 16B, power rating is 27.73 kW.

<table>
<thead>
<tr>
<th>No. of strands</th>
<th>The multiple strand factor , ( K_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>1.7</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>3.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of teeth</th>
<th>Tooth correction factor , ( K_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.85</td>
</tr>
<tr>
<td>60</td>
<td>2.80</td>
</tr>
</tbody>
</table>

**Solution:**

\[ p = 25.4 \text{ mm} \]

\[ z_1 = 15 \]
Example – 27:- It is required to design a chain drive to connect a 12 kW, 1400 rpm electric motor to a centrifugal pump running at 700 rpm. The service condition involves moderate shock. Pitch is 19.05 mm

(i) Determine the P.C.D. of driving and driven sprocket.
(ii) Determine no. of chain links.
(iii) Specify the correct center distance between the axes of sprockets.

Solution:  

\[ P = 12 \text{ kW} \]

\[ N_1 = 1400 \text{ rpm} \]

\[ N_2 = 700 \text{ rpm} \]

\[ p = 19.05 \text{ mm} \]

P.C.D. for driving sprocket,
\[ D_1 = \frac{p}{\sin\left(\frac{180}{Z_1}\right)} = \frac{19.05}{\sin\left(\frac{180}{17}\right)} \]

\[ = 103.67 \text{ mm} \]

\[ Z_1N_1 = Z_2N_2 \]

\[ Z_2 = Z_1\left(\frac{N_1}{N_2}\right) \]
Design of Machine Elements (2151907)  4. Belt and Chain Drives

\[ Z_2 = 17 \left( \frac{1400}{700} \right) \]

\[ = 34 \]

For driven sprocket, \( D_2 = \frac{p}{\sin \left( \frac{180}{Z_2} \right)} = \frac{19.05}{\sin \left( \frac{180}{34} \right)} \)

\[ = 206.463 \text{ mm} \]

The centre distance between sprocket wheel should be between (30p) to (50p). Taking a mean value (40p).

The approximate centre distance, \( a = 40 \cdot p = 40 \times 19.05 \)

\[ = 762 \text{ mm} \]

No. of links in chain, \( L_n = \frac{2a}{p} + \frac{Z_1 + Z_2}{2} + \left( \frac{Z_2 - Z_1}{2\pi} \right)^2 \cdot \frac{p}{a} \)

\[ L_n = \frac{2 \times 762}{19.05} + \frac{17 + 34}{2} + \left( \frac{34 - 17}{2\pi} \right)^2 \cdot \frac{19.05}{762} \]

\[ L_n = 106 \text{ links} \]

Correct centre distance, \( a = \frac{p}{4} \left\{ \left[ L_n - \left( \frac{Z_1 + Z_2}{2} \right) \right] + \sqrt{\left[ L_n - \left( \frac{Z_1 + Z_2}{2} \right) \right]^2 - 8 \left( \frac{Z_2 - Z_1}{2\pi} \right)^2} \right\} \)

\[ a = \frac{19.05}{4} \left\{ 106 - \left( \frac{17 + 34}{2} \right) + \sqrt{\left[ 106 - \left( \frac{17 + 34}{2} \right) \right]^2 - 8 \left( \frac{34 - 17}{2\pi} \right)^2} \right\} \]

\[ = 765.026 \text{ mm} \]

To provide small slag, for allowing chain links to take the best position on the sprocket teeth, the centre distance is reduced by (0.002a).

Correct centre distance is given by

\[ a = 0.998 \times 765.026 \]

\[ = 763.49 \text{ mm} \]

Example – 28:- Select a simple roller chain drive to transmit 5 kW power 1400 r.p.m. from an electric motor to a drilling machine.

Speed reduction = 3:1
Approximate centre distance = 500 mm
Service factor = 1.3
Assume moderate shock conditions.
Number of teeth on pinion = 21
Also find no. of chain links and correct centre distance.

<table>
<thead>
<tr>
<th>Power rating (kW) of a simple roller chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinion Speed (r.p.m.)</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>1000</td>
</tr>
<tr>
<td>1400</td>
</tr>
</tbody>
</table>

**Solution:**

\[ P = 5 \text{ kW} \]
\[ N_1 = 1400 \text{ rpm} \]
\[ a = 500 \text{ mm} \]
\[ i = 3 \]
\[ K_S = 1.3 \]
\[ z = 21 \]

For single strand chain, \( K_1 = 1 \)

For 21 teeth, \( K_2 = 1.26 \)

Power to be transmitted = \[ \frac{\text{Power rating} \times K_1 \times K_2}{K_S} \]

\[ 5 = \frac{\text{Power rating} \times 1 \times 1.26}{1.3} \]

Power rating = 5.16 kW

Referring to Table, the power rating of the chain 8A at 1400 rpm is 5.28 kW. Therefore the chain number 8A is selected.

Assume pitch, \( p = 12.70 \text{ mm} \)

\[ z_2 = i z_1 = 3 \times 21 = 63 \]

No. of links in chain, \( L_n = \frac{2a + \frac{z_1 + z_2}{2} + \left( \frac{z_2 - z_1}{2\pi} \right)^2}{p} \)

\[ L_n = \frac{2 \times 500}{12.70} + \frac{21 + 63}{2} + \left( \frac{63 - 21}{2\pi} \right)^2 \]

\[ L_n = 121.87 = 122 \text{ links} \]

Correct centre distance,

\[ a = \frac{p}{4} \left\{ L_n - \left( \frac{z_1 + z_2}{2} \right) + \sqrt{L_n - \left( \frac{z_1 + z_2}{2} \right)^2 - 8 \left( \frac{z_2 - z_1}{2\pi} \right)^2} \right\} \]

\[ a = \frac{12.70}{4} \left\{ 122 - \left( \frac{21 + 63}{2} \right) + \sqrt{122 - \left( \frac{21 + 63}{2} \right)^2 - 8 \left( \frac{63 - 21}{2\pi} \right)^2} \right\} \]

\[ a = 500.81 \text{ mm} \]
5

PRESSURE VESSELS

Course Contents

5.1 Introduction to Pressure Vessels
5.2 General Materials for Pressure Vessels
5.3 Classification of Pressure Vessels
5.4 Stresses in a Thin Cylindrical Shell Due to an Internal Pressure
5.5 Thin Spherical Shells Subjected to an Internal Pressure
5.6 Thick Cylindrical Shells Subjected to an Internal Pressure
5.7 Autofrettage or Pre-stressing
5.8 Types of End Closures
5.9 Gaskets and Gasketed Joint
5.10 Openings in Pressure Vessel Examples
5.1 Introduction to Pressure Vessels

- Vessels, tanks, and pipelines that carry, store, or receive fluids are called pressure vessels.

- A pressure vessel is defined as a container with a pressure differential between inside and outside. The inside pressure is usually higher than the outside, except for some isolated situations.

![Typical Components of Pressure Vessels](image)

- The fluid inside the vessel may undergo a change in state as in the case of steam boilers, or may combine with other reagents as in the case of a chemical reactor.

- Pressure vessels often have a combination of high pressures together with high temperatures, and in some cases flammable fluids or highly radioactive materials. Because of such hazards it is imperative that the design be such that no leakage can occur.

- In addition these vessels have to be designed carefully to cope with the operating temperature and pressure.

- It should be borne in mind that the rupture of a pressure vessel has a potential to cause extensive physical injury and property damage. Plant safety and integrity are of fundamental concern in pressure vessel design.

5.1.1 Applications of Pressure Vessels

- Industrial compressed air receivers
- Domestic hot water storage tanks
- Diving cylinders (Scuba diving)
- Recompression chambers
- Distillation towers
• Autoclaves (In medical industry to sterilize)
• Oil refineries and petrochemical plants
• Nuclear reactor vessels
• Pneumatic and Hydraulic Reservoirs
• Storage vessels for liquefied gases such as ammonia, chlorine, propane, butane and LPG.

5.1.2 ASME Codes for Pressure Vessels
• It is a standard that provides rules for the design, fabrication, and inspection of boilers and pressure.
• This establishes and maintains design, construction and inspection standards providing for maximum protection of life and property.
• ASME Section VIII : Boiler and Pressure Vessel Code (BPVC)
  – Division 1- Rules for Construction of Pressure Vessels
  – Division 2 - Alternative Rules
  – Division 3 - Alternative Rules for Construction of High Pressure Vessels

5.2 General Materials for Pressure Vessels
• The materials that are used in pressure vessel construction are:
  – Steels
  – Nonferrous materials such as aluminum and copper
  – Metals such as titanium and zirconium
  – Nonmetallic materials, such as, plastic, composites and concrete
  – Metallic and nonmetallic protective coatings
• Various materials has some typical characteristics as below:
  – Carbon steel: strength & moderate corrosion resistance
  – Low-alloy steels: strength at high temperatures
  – Stainless steels: corrosion resistance
  – Nickel alloys: corrosion resistance
  – Copper alloys: sea water resistance
  – Aluminum: light, low temperature toughness
  – Titanium: sea water, chemical resistance
  – Refractories: very high temperatures
  – Non-metallic: corrosion & chemicals

5.2.1 Factors Affecting Selection of Material:
• Factors Affecting Selection of Material are as following:
  – Process fluids (i.e. a plastic might be perfect for the fluid corrosiveness, but will melt when the operators ‘steam’ the equipment during cleaning)
  – Operating temperature
  – Operating pressure
5. PRESSURE VESSELS

- Fluid Velocity
- Contamination of product
- Required life of the equipment (May choose to incur shorter life and replace more often)
- Cost of the materials of construction (base material + fabrication costs)

5.3 Classification of Pressure Vessels

- Based on Wall Thickness:
  1) Thin Wall Vessel
  2) Thick Wall Vessel

- Based on Geometric Shapes:
  1) Cylindrical Vessels
  2) Spherical Vessels
  3) Rectangular Vessels
  4) Combined Vessels

- Based on Installation Methods:
  1) Vertical Vessels
  2) Horizontal Vessels

- Based on Operating Temperature:
  1) Low Temperature Vessels (less than or equal to -20°C)
  2) Normal Temperature Vessels (Between -20°C to 150°C)
  3) Medium Temperature Vessels (Between 150°C to 450°C)
  4) High Temperature Vessels (more than or equal to 450°C)

- Based on Design Pressure:
  1) Low Pressure Vessels (0.1 MPa to 1.6 MPa)
  2) Medium Pressure Vessels (1.6 MPa to 10 MPa)
  3) High Pressure Vessels (10 MPa to 100 MPa)
  4) Ultra High Pressure Vessels (More than 100 MPa)

- Based on Technological Processes:
  1) Reaction Vessel
  2) Heat Exchanger Vessel
  3) Separation Vessel
  4) Storage Container Vessel

5.3.1 Difference Between Thin Shell and Thick shell Pressure Vessels

- The pressure vessels, according to their dimensions, may be classified as thin shell or thick shell.

- If the wall thickness of the shell (t) is less than 1/10 to 1/15 of the diameter of the shell (d), then it is called a thin shell. On the other hand, if the wall thickness of the shell is greater than 1/10 to 1/15 of the diameter of the shell, then it is said to be a thick shell.
• Thin shells are used in boilers, tanks and pipes, whereas thick shells are used in high pressure cylinders, tanks, gun barrels etc.
• Another criterion to classify the pressure vessels as thin shell or thick shell is the internal fluid pressure (p) and the allowable stress ($\sigma_t$).
• If the internal fluid pressure (p) is less than $1/6$ of the allowable stress, then it is called a thin shell. On the other hand, if the internal fluid pressure is greater than $1/6$ of the allowable stress, then it is said to be a thick shell.

5.4 Stresses in a Thin Cylindrical Shell due to an Internal Pressure

• The analysis of stresses induced in a thin cylindrical shell are made on the following assumptions:
  1) The effect of curvature of the cylinder wall is neglected.
  2) The tensile stresses are uniformly distributed over the section of the walls.
  3) The effect of the restraining action of the heads at the end of the pressure vessel is neglected.
• When a thin cylindrical shell is subjected to an internal pressure, it is likely to fail in the following two ways:
  1) It may fail along the longitudinal section (i.e. circumferentially) splitting the cylinder into two troughs, as shown in Fig 5.2.
  2) It may fail across the transverse section (i.e. longitudinally) splitting the cylinder into two cylindrical shells, as shown in Fig 5.3.
• Thus the wall of a cylindrical shell subjected to an internal pressure has to withstand tensile stresses of the following two types: (a) Circumferential or hoop stress, and (b) Longitudinal stress.

5.4.1 Circumferential or Hoop Stress

• Consider a thin cylindrical shell subjected to an internal pressure as shown in fig. 5.2.
• Tensile stress acting in a direction tangential to the circumference is called circumferential or hoop stress.
• In other words, it is a tensile stress on longitudinal section (or on the cylindrical walls).
• Let $p = \text{Intensity of internal pressure},$

  $d = \text{Internal diameter of the cylindrical shell},$
5. PRESSURE VESSELS

We know that the total force acting on a longitudinal section (i.e. along the diameter X-X) of the shell

\[ P \times d \times l = \sigma_t \times 2 \times t \times l \] (i)

Also the total resisting force acting on the cylinder walls for two sections
\[ \sigma_t \times 2 \times t \times l \] (ii)

Equating equations (i) and (ii), we have
\[ P \times d \times l = \sigma_t \times 2 \times t \times l \]
\[ \therefore \sigma_t = \frac{P \times d}{2 \times t} \]

5.4.2 Longitudinal Stress

Consider a closed thin cylindrical shell subjected to an internal pressure as shown in the above figure.

Tensile stress acting in the direction of the axis is called longitudinal stress.

In other words, it is tensile stress acting on the transverse or circumferential section Y-Y (or on the ends of the vessel).

Let \( \sigma_l = \) Longitudinal stress.

In this case, the total force acting on the transverse section (i.e. along Y-Y)

\[ P \times \frac{\pi}{4} d^2 = \sigma_l \times \pi \times d \times t \] (i)

Also the total resisting force
\[ \sigma_l \times \pi \times d \times t \] (ii)

Equating equations (i) and (ii), we have
\[ P \times \frac{\pi}{4} d^2 = \sigma_l \times \pi \times d \times t \]
\[ \therefore \sigma_l = \frac{P \times d}{4 \times t} \]
5.5 Thin Spherical Shells Subjected to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure as shown in fig. 5.4.

Let
- \( V = \) Storage capacity of the shell,
- \( p = \) Intensity of internal pressure,
- \( d = \) Diameter of the shell,
- \( t = \) Thickness of the shell,
- \( \sigma_t = \) Permissible tensile stress for the shell material.

In designing thin spherical shells, we have to determine the diameter of the shell and the thickness of the shell.

We know that the storage capacity of the shell,

\[
V = \frac{4}{3} \times \pi \times r^3
\]

\[
\therefore V = \frac{4}{3} \times \pi \times \left(\frac{d}{2}\right)^3
\]

\[
\therefore V = \frac{\pi}{6} \times d^3
\]

\[
\therefore d = \left(\frac{6V}{\pi}\right)^{\frac{1}{3}}
\]

As a result of the internal pressure, the shell is likely to rupture along the centre of the sphere.

Therefore force tending to rupture the shell along the centre of the sphere or bursting force

\[
= \text{Pressure} \times \text{Area}
= p \times \frac{\pi}{4} d^2 \text{ } \ldots (i)
\]

Also resisting force of the shell

\[
= \text{Stress} \times \text{Resisting area}
= \sigma_t \times \pi \times d \times t \text{ } \ldots (ii)
\]

Equating equations (i) and (ii), we have

\[
p \times \frac{\pi}{4} d^2 = \sigma_t \times \pi \times d \times t
\]

\[
\therefore t = \frac{pd}{4\sigma_t}
\]
5.6 Thick Cylindrical Shells Subjected to an Internal Pressure

- When the ratio of the inner diameter \( d \) of the cylinder to the wall thickness \( t \) is less than 10 to 15, the cylinder is called as thick cylinder.
- Hydraulic cylinders, high pressure pipes and gun barrels are examples of thick cylinders.
- The radial stress \( \sigma_r \) is neglected in thin cylinders, while it is of significant magnitude in case of thick cylinders.
- There are number of equations for the design of thick cylinders. The choice of equation depends upon two parameters: Cylinder material (whether brittle or ductile) and Condition of cylinder ends (open or closed).
- In the design of thick cylindrical shells, the following equations are mostly used:
  1. Lame’s equation,
  2. Birnie’s equation,
  3. Clavarino’s equation and
  4. Barlow’s equation.

5.6.1 Lame’s Equation

- When the material of the cylinder is brittle, such as cast iron or cast steel, Lame’s equation is used to determine the wall thickness.
- It is based on the maximum principal stress theory of failure, where maximum principal stress is equated to permissible stress for the material.
- Three principal stresses at the inner surface of the cylinder are as follows:
  \[ \sigma_r = -P_i \] .............................. (i)
  \[ \sigma_t = + \frac{P_i(D_o^2 + D_i^2)}{(D_o^2 - D_i^2)} \] .............................. (ii)
  \[ \sigma_l = + \frac{P_iD_i^2}{(D_o^2 - D_i^2)} \] .............................. (iii)

- Therefore,
  \[ \sigma_t > \sigma_l > \sigma_r \]
- Hence \( \sigma_l \) is the criterion of design. From equation (ii)
  \[ \frac{\sigma_t}{P_i} = \frac{D_o^2 + D_i^2}{D_o^2 - D_i^2} \]
  \[ \therefore \sigma_t(D_o^2 - D_i^2) = P_i(D_o^2 + D_i^2) \]
  \[ \therefore \sigma_tD_o^2 - \sigma_lD_i^2 = P_iD_o^2 + P_iD_i^2 \]
  \[ \therefore \sigma_tD_o^2 - P_iD_o^2 = \sigma_lD_i^2 + P_iD_i^2 \]
  \[ \therefore D_o^2(\sigma_t - P_i) = D_i^2(\sigma_l + P_i) \]
  \[ \therefore \frac{D_o^2}{D_i^2} = \frac{\sigma_t + P_i}{\sigma_l - P_i} \]
\[ \therefore \left( \frac{D_o}{D_i} \right)^2 = \frac{\sigma_t + P_i}{\sigma_t - P_i} \]

\[ \therefore \frac{D_o}{D_i} = \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}} \]

Substituting \( D_o = D_i + 2\, t \) in the above equation

\[ \therefore \frac{D_i + 2\, t}{D_i} = \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}} \]

\[ \therefore 1 + \frac{2\, t}{D_i} = \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}} \]

\[ \therefore \frac{2\, t}{D_i} = \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i}} - 1 \]

\[ \therefore t = \frac{D_i}{2} \sqrt{\frac{\sigma_t + P_i}{\sigma_t - P_i} - 1} \]

where

\[ \sigma_t = \frac{S_{ut}}{(f_s)} \]

### 5.6.2 Birnie’s equation

- In case of open-end cylinders (such as pump cylinders, rams, gun barrels etc.) made of ductile material (i.e. low carbon steel, brass, bronze, and aluminium alloys), the allowable stresses cannot be determined by means of maximum-stress theory of failure.
- In such cases, the maximum-strain theory is used. According to this theory, the failure occurs when the strain reaches a limiting value.
- Birnie’s equation for the wall thickness of a cylinder,

\[ t = \frac{D_i}{2} \left[ \frac{\sigma + (1 - \mu)P_i}{\sigma - (1 + \mu)P_i} - 1 \right] \]

### 5.6.3 Clavarino’s equation

- This equation is also based on the maximum-strain theory of failure, but it is applied to closed-end cylinders (or cylinders fitted with heads) made of ductile material.
- According to this equation, the thickness of a cylinder,

\[ t = \frac{D_i}{2} \left[ \frac{\sigma + (1 - 2\mu)P_i}{\sigma - (1 + \mu)P_i} - 1 \right] \]

where

\[ \sigma = \frac{S_{yt}}{(f_s)} \]
5.6.4 Barlow’s equation

- This equation is generally used for high pressure oil and gas pipes.
- According to this equation, the thickness of a cylinder,

\[ t = \frac{p_1 D_o}{2 \sigma_t} \]

where, \( D_o \) is outer diameter of cylinder

5.7 Autofrettage or Pre-stressing

- Autofrettage is a process of pre-stressing the cylinder before using it in service.
- It is used in case of high-pressure cylinders and gun barrels.
- When the cylinder is subjected to internal pressure, the circumferential stress \( \sigma_t \) at the inner surface limits the pressure capacity of the cylinder.
- In the pre-stressing process, residual compressive stresses are developed at the inner surface.
- When the cylinder is loaded in service, the residual compressive stresses at the inner surface begin to decrease, become zero and finally become tensile as the pressure is gradually increased.
- Autofrettage increases the pressure capacity of the cylinder.
- It has another advantage that the residual compressive stresses close the cracks within the cylinder resulting in increased endurance strength.
- There are three methods of pre-stressing the cylinder. They are as follows:

1. **Compound cylinder**: It consists of two concentric cylinders with outer cylinder shrunk onto the inner one. This induces compressive stresses in the inner cylinder. Compound cylinder is extensively used in practice.

A compound cylinder, consisting of a cylinder and a jacket is shown in figure (a). The inner diameter of the jacket is slightly smaller than the outer diameter of the cylinder.
• When the jacket is heated, it expands sufficiently to move over the cylinder.
• As the jacket cools, it tends to contract onto the inner cylinder, which induces residual compressive stresses.
• There is a shrinkage pressure $P$ between the cylinder and the jacket.
• The pressure $P$ tends to contract the cylinder and expand the jacket, as shown in figures (b) and (c).
• The shrinkage pressure $P$ can be evaluated from the below equation for a given amount of interference ($\delta$).

$$\delta = \frac{P D_2}{E} \left( \frac{2 D_2^2 (D_3^2 - D_1^2)}{(D_3^2 - D_2^2)(D_2^2 - D_1^2)} \right)$$

• The resultant stresses in a compound cylinder are found by superimposing the two stresses—stresses due to shrink fit and those due to internal pressure.

2. **Overloading the cylinder:** The second method consists of overloading the cylinder before it is put into service.
• The overloading pressure is adjusted in such a way that a portion of the cylinder near the inner diameter is subjected to stresses in the plastic range, while the outer portion is still in the elastic range.
• When the pressure is released, the outer portion contracts exerting pressure on the inner portion which has undergone permanent deformation.
• This induces residual compressive stresses at the inner surface.

3. **Wire method:** In the third method, a wire under tension is closely wound around the cylinder, which results in residual compressive stresses.

### 5.8 Types of End Closures

- Formed heads are used as end closures for cylindrical pressure vessels.
- There are two types of end closures:
  1. Domed heads:
     - a) Hemispherical
     - b) Semi-ellipsoidal
     - c) Torispherical
  2. Conical heads
    1. **Hemispherical heads:**
       - Hemispherical heads have minimum plate thickness, minimum weight and consequently lowest material cost.
       - However, the amount of forming required to produce the hemispherical shape is more, resulting in increased forming cost.
       - The thickness of the hemispherical head is given by,

$$t = \frac{P_i R_i}{2 \sigma_t \eta - 0.2 P_i} + CA$$
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where $R_i$ is the inner radius of the cylindrical shell,
CA is Corrosion allowance
$\eta$ is weld joint efficiency

![Hemispherical heads](image)

**Fig 5.6** Hemispherical heads

2. **Semi-ellipsoidal Heads:**
   - In a semi-ellipsoidal head, the ratio of the major axis to the minor axis is taken as 2:1.

![Semi-ellipsoidal Heads](image)

**Fig 5.7** Semi-ellipsoidal Heads

- The thickness of a semi-ellipsoidal head is given by,
  
  $$t = \frac{P_i D_i}{2 \sigma_t \eta - 0.2 P_i} + CA$$

- From the above equations, it is observed that the thickness of the semi-elliptical head is more (almost twice) than the corresponding hemispherical head, and to that extent, the material cost is more.
- However due to the shallow dished shape, the forming cost is reduced.
- The length of the straight portion ($S_f$) is given by,
  
  $$S_f = 3t \text{ or } 20 \text{ mm}$$
  
  (whichever is more)

3. **Torispherical Heads:**
   - Torispherical heads are extensively used as end closures for a large variety of cylindrical pressure vessels.
   - They are shaped by using two radius – crown radius $L$ and knuckle radius $r_i$.
- The crown radius \( L \) is the radius of the dish that constitutes the major portion of the head.
- The knuckle radius \( r_i \) is the corner radius joining the spherical crown with the cylindrical shell.
- Torispherical heads require less forming than semi-ellipsoidal heads.
- Their main drawback is the local stresses at the two discontinuities namely the junction between “the crown head and the knuckle radius” and the junction between “the knuckle radius and the cylindrical shell”.
- The localised stresses may lead to failure due to brittle fracture.
- The thickness of a torispherical head is given by,
  \[
  t = \frac{0.885 P_i L}{\sigma_t \eta - 0.1 P_i} + CA
  \]
- The knuckle radius \( r_i \) is taken as 6% of the crown radius, \( r_i = 0.06 L \)

4. **Conical Heads:**

- The thickness of a conical head is given by,
  \[
  t = \frac{P_i D_i}{2 \cos \alpha (\sigma_t \eta - 0.6 P_i)} + CA
  \]
  where, \( \alpha \) is half the apex angle.
- The half apex angle is usually less than 30°.
5.9 Gaskets and Gasketed Joints

- A gasket is a device used to create and maintain a barrier against the transfer of fluid across the mating surfaces of a mechanical assembly. It is used in static joints, such as cylinder block and cylinder head.
- There are two types of gaskets—metallic and non-metallic.
- Metallic gaskets consist of sheets of lead, copper or aluminium.
- Non-metallic gaskets are made of asbestos, cork, rubber or plastics.

![Fig. 5.10 Shapes of Gasket](image)

- Different shapes of gasket for cylinder head are illustrated in the above figure.
- Metallic gaskets are used for high temperature and high pressure applications. They can have corrugated construction or they can be made in the form of plain sheets.
- The limiting temperatures of metallic gaskets are as follows:
  - Lead: 90°C
  - Copper/brass: 250°C
  - Aluminium: 400°C
- The metallic gasket takes a permanent set when compressed in assembly and there is no recovery to compensate for separation of contact faces.
- They are also susceptible to corrosion and chemical atmosphere. Their performance also depends upon surface finish of the contacting surfaces.
- Asbestos gaskets have excellent resistance to be crushing loads and cutting action due to sharp edges of the flanges. Dimensional stability is another advantage.
- They are used in cylinder head, water and steam pipe fittings and manifold connections.
- Vulcanized compounds of rubber and cork are employed as gaskets in steam lines, combustion chambers and chemical environment.
• They are used for applications involving irregular surfaces. They are of low cost, but are affected by fungus and alkalis.
• Rubber compounds have excellent impermeability and ability to flow into joint imperfections when compressed.
• Asbestos gaskets can be used up to 250° C, while other non-metallic gaskets have a limiting temperature of 70° C.

5.10 Openings in Pressure Vessel

• Openings are provided in the pressure vessel; these could be an inlet and outlet pipe connections, manhole or hand hole, connections for pressure gauges, temperature gauges and safety valves.
• The openings are circular, elliptical or obround. The inner diameter of a manhole is generally 380 mm. Such openings are designed by the area compensation method.
• The basic principle of the area compensation method is illustrated in Fig. 5.11. When the opening is cut in the pressure vessel, an area is removed from the shell. It must be reinforced by an equal amount of area near the opening.
• The area 'removed' should be equal to the area 'added'. The area is added by providing a reinforcing pad in the form of annular circular plate around the opening.
• It should be noted that in this method, we are considering cross-sectional area in the form of a rectangular strip. It is not the compensation of volume of metal that has been cut due to the opening by means of the reinforcing pad.

![Fig. 5.11 Principle of Area Compensation](image)

• It is not always necessary to replace the actually removed area of the metal. The plate of the shell and nozzle are usually thicker than would be required to withstand pressure. This partially compensates for loss of area in the opening. As shown in Fig. 5.12,

\[ A = d t_r \]  \hspace{1cm} (i)

where,

\[ A = \text{area of metal removed in corroded condition (mm}^2) \]
\[ d = \text{inner diameter of opening in corroded condition} \]
\[ = (d_i + 2CA) \text{ mm} \]
\[ d_i = \text{inner diameter (nominal) of nozzle (mm)} \]
\[ t_r = \text{required thickness of cylindrical shell (mm)} \]

The required thickness \( t_r \) is given by

\[ t_r = \frac{P_D}{2\sigma\eta - P_i} \]
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Fig. 5.12 Area Compensation for Nozzle

- The metal used for reinforcement should be located in the vicinity of the opening. The limiting dimension $X$ parallel to the wall of the cylindrical shell is given by
  
  \[ X = d \quad \text{or} \quad X = \left[ \frac{d}{2} + t + t_n - 3CA \right] \quad \text{(Whichever is maximum)} \]

- The limiting dimensions $h_1$ and $h_2$ parallel to the nozzle wall are given by,
  
  $h_1$ or $h_2 = 2.5 \, (t - CA)$

  $h_1$ or $h_2 = 2.5 \, (t_n - CA)$ (whichever is minimum)

  where,
  
  $t = \text{total thickness of the wall of cylinder shell (mm)}$

  $t_n = \text{total thickness of nozzle wall (mm)}$

- The area $A_1$ of excess thickness in the vessel wall, which is available for reinforcement, is given by
  
  \[ A_1 = (2X - d) \, (t - t_n - CA) \] ..........................(ii)

- The area $A_2$ of excess thickness in the nozzle wall is given by,
  
  \[ A_2 = 2h_1 \, (t_n - t_n - CA) \] ..........................(iii)

  where $(t_n)$ is the thickness required for the nozzle wall to be able to withstand the pressure, or

  \[ t_n = \frac{PD_i}{2\sigma_i - P_i} \]

- The area $A_3$ of the inside extension of the nozzle is given by,
  
  \[ A_3 = 2h_2 \, (t_n - 2CA) \] ..........................(iv)

- The total area available for reinforcement is $(A_1 + A_2 + A_3)$.
  
  When, $(A_1 + A_2 + A_3) \geq A$

  the opening is adequately reinforced and no reinforcing pad is required. When this condition is not fulfilled, a reinforcing pad of area $A_4$ is required.

  \[ A_4 = A - (A_1 + A_2 + A_3) \]

- Sometimes a reinforcing pad of area equal to $A$ is used for the opening to avoid detailed calculations. This results in oversized reinforcement.
Example – 1:- A hydraulic press has a maximum capacity of 1000 kN. The piston diameter is 250 mm. Calculate the wall thickness if the cylinder is made of material for which the permissible strength may be taken as 80 MPa. This material may be assumed as a brittle material.

Solution:

\[ W = 1000 \text{ kN} \quad D_i = 250 \text{ mm} \]
\[ \sigma_t = 80 \text{ MPa} \]

Load on the hydraulic press,

\[ W = \frac{\pi D_i^2}{4} \times p_i \]
\[ 1000 \times 10^3 = \frac{\pi (250)^2}{4} \times p_i \]
\[ p_i = 20.37 \text{ N/mm}^2 \]

Wall thickness of the cylinder,

\[ t = \frac{D_i}{2} \left( \sqrt{\frac{\sigma_t + p_i}{\sigma_t - p_i}} - 1 \right) \]
\[ t = 250 \left( \sqrt{\frac{80 + 20.37}{80 - 20.37}} - 1 \right) \]
\[ t = 37 \text{ mm} \]

Example – 2:- The piston rod of a hydraulic cylinder exerts an operating force of 10 kN. The friction due to piston packing and stuffing box is equivalent to 10% of the operating force. The pressure in the cylinder is 10 MPa. The cylinder is made of cast iron FG 200 and the factor of safety is 5. Determine the diameter and the thickness of the cylinder.

Solution:

\[ p_i = 10 \text{ MPa} \quad \sigma_{ut} = 200 \text{ MPa} \]
\[ \text{Operating force} = 10 \text{ kN} \quad \text{F.S.} = 5 \]
\[ \text{Frictional force} = 10\% \text{ of Operating force} \]

Total force on the piston, \( W = \text{Operating force} + \text{Frictional force} \)
\[ = 10 + 10\% \text{ of 10} \]
\[ = 11 \text{ kN} \]
\[ W = \frac{\pi D_i^2}{4} \times p_i \]
\[ 11 \times 10^3 = \frac{\pi (D_i)^2}{4} \times 10 \]
\[ D_i = 37.4 \text{ mm} \]
\[ D_i \approx 40 \text{ mm} \]
\[ \sigma_t = \frac{\sigma_{ut}}{\text{F.S.}} = \frac{200}{5} \]
\[ \sigma_t = 40 \text{ MPa} \]
The cylinder material is brittle and Lame’s equation is applicable.

Using Lame’s equation,

\[ t = \frac{D_i}{2} \left[ \frac{\sigma_t + p_i}{\sigma_t - p_i} - 1 \right] \]

\[ t = \frac{40}{2} \left[ \frac{40 + 10}{40 - 10} - 1 \right] \]

\[ t = 5.82 \text{ mm} \]

\[ t \approx 6 \text{ mm} \]

**Example – 3:-** The inner diameter of a cylindrical tank for liquefied gas is 250 mm. The gas pressure is limited to 15 MPa. The tank is made of plain carbon steel 10C4 (\(\sigma_{ut} = 340 \text{ N/mm}^2\) and \(\mu = 0.27\)) and the factor of safety is 5. Calculate the cylinder wall thickness.

**Solution:**

\( D_i = 250 \text{ mm} \)

\( P_i = 15 \text{ MPa} \)

\( \sigma_{ut} = 340 \text{ MPa} \)

\( \mu = 0.27 \)

\( F.S. = 5 \)

\[ \sigma_t = \frac{\sigma_{ut}}{F.S.} = \frac{340}{5} \]

\[ \sigma_t = 68 \text{ MPa} \]

The cylindrical tank is made of ductile material and ends are closed. Therefore, Clavarino’s equation is applicable.

Using Lame’s equation,

\[ t = \frac{D_i}{2} \left[ \frac{\sigma_t + (1 - 2\mu)p_i}{\sigma_t - (1 + \mu)p_i} - 1 \right] \]

\[ t = \frac{250}{2} \left[ \frac{68 + [1 - 2(0.27)]15}{68 - [1 + 0.27]15} - 1 \right] \]

\[ t = 29.62 \text{ mm} \]

\[ t \approx 30 \text{ mm} \]

**Example – 4:-** A seamless steel pipe of 100 mm internal diameter is subjected to internal pressure of 12 MPa. It is made of steel (\(\sigma_{yt} = 230 \text{ N/mm}^2\) and \(\mu = 0.27\)) and the factor of safety is 2.5. Determine the thickness of the pipe.

**Solution:**

\( D_i = 100 \text{ mm} \)

\( P_i = 12 \text{ MPa} \)

\( \sigma_{yt} = 230 \text{ MPa} \)

\( \mu = 0.27 \)

\( F.S. = 2.5 \)

\[ \sigma_t = \frac{\sigma_{yt}}{F.S.} = \frac{230}{2.5} \]

\[ \sigma_t = 92 \text{ MPa} \]
The pipe has open ends and it is made of ductile material. Therefore, Bernie’s equation is applicable.

Using Bernie’s equation,
\[
t = \frac{D_i}{2} \left[ \frac{\sigma_t + (1 - \mu) \rho_i}{\sigma_t - (1 + \mu) \rho_i} - 1 \right]
\]
\[
t = \frac{100}{2} \left[ \frac{92 + (1 - 0.27)12}{92 - (1 + 0.27)12} - 1 \right]
\]
\[
t = 7.29 \text{ mm}
\]
\[
t \approx 8 \text{ mm}
\]

**Example – 5:** The cylinder of a portable hydraulic riveter is 220 mm in diameter. The pressure of the fluid is 14 N/mm² by gauge. Determine suitable thickness of the cylinder wall assuming that the maximum permissible tensile stress is not to exceed 105 MPa.

**Solution:**

\[
D_i = 220 \text{ mm}
\]
\[
\rho_i = 14 \text{ N/mm}^2
\]
\[
\sigma_t = 105 \text{ MPa}
\]

Since the pressure of the fluid is high, therefore thick cylinder equation is used. Assuming the material of the cylinder as steel, taking Poisson’s ratio for steel, \( \mu = 0.3 \). The thickness of the cylinder wall (\( t \)) may be obtained by using Birnie’s equation

\[
t = \frac{D_i}{2} \left[ \frac{\sigma_t + (1 - \mu) \rho_i}{\sigma_t - (1 + \mu) \rho_i} - 1 \right]
\]
\[
t = \frac{220}{2} \left[ \frac{105 + (1 - 0.3)14}{105 - (1 + 0.3)14} - 1 \right]
\]
\[
t = 16.5 \text{ mm}
\]

**Example – 6:** A high pressure cylinder consists of a steel tube with inner and outer diameters of 20 mm and 40 mm respectively. It is jacketed by an outer steel tube, having an outer diameter of 60 mm. The tubes are assembled by a shrinking process in such a way that maximum principal stress induced in any tube is limited to 100 MPa. Calculate the shrinkage pressure and original dimensions of the tubes (\( E = 207 \text{ kN/mm}^2 \)).

**Solution:**

\[
D_1 = 20 \text{ mm}
\]
\[
D_2 = 40 \text{ mm}
\]
\[
D_3 = 60 \text{ mm}
\]
\[
\sigma_{\text{max}} = 100 \text{ MPa}
\]
\[
E = 207 \text{ kN/mm}^2
\]

The maximum principal stress is the tangential stress at the inner surface of the jacket.

\[
\sigma_t = \frac{P(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)}
\]
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\[
100 = \frac{P(60^2 + 40^2)}{(60^2 - 40^2)}
\]

Shrinkage Pressure, \( P = 38.46 \text{ N/mm}^2 \)

Amount of Interference,

\[
\delta = \frac{PD_2}{E} \left[ \frac{2D_2^2(D_3^2 - D_1^2)}{(D_3^2 - D_2^2)(D_2^2 - D_1^2)} \right]
\]

\[
\delta = \frac{38.46 \times 40}{207 \times 10^3} \left[ \frac{2(40)^2(60^2 - 20^2)}{(60^2 - 40^2)(40^2 - 20^2)} \right]
\]

\[
\delta = 0.0317 \text{ mm}
\]

Inner diameter of jacket = Outer diameter of inner tube – Amount of Interference

\[
= D_2 - \delta
\]

\[
= 40 - 0.0317
\]

\[
= 39.9683 \text{ mm}
\]

**Example – 7:** A steel tube 240 mm external diameter is shrunk on another steel tube of 80 mm internal diameter. After shrinking, the diameter at the junction is 160 mm. Before shrinking, the difference of diameters at the junction was 0.08 mm. If the Young’s modulus for steel is 200 GPa, find: 1. tangential stress at the outer surface of the inner tube; 2. tangential stress at the inner surface of the outer tube; and 3. radial stress at the junction.

**Solution:**

\[
D_3 = 240 \text{ mm} \quad D_1 = 80 \text{ mm}
\]

\[
D_2 = 160 \text{ mm} \quad \delta = 0.08 \text{ mm}
\]

\[
E = 200 \text{ GPa}
\]

\[
\delta = \frac{PD_2}{E} \left[ \frac{2D_2^2(D_3^2 - D_1^2)}{(D_3^2 - D_2^2)(D_2^2 - D_1^2)} \right]
\]

\[
P = \frac{\delta E}{D_2} \left[ \frac{(D_3^2 - D_2^2)(D_2^2 - D_1^2)}{2D_2^2(D_3^2 - D_1^2)} \right]
\]

\[
P = \frac{0.08 \times 200 \times 10^3}{160} \left[ \frac{(240^2 - 160^2)(160^2 - 80^2)}{2 \times 160^2(240^2 - 80^2)} \right]
\]

\[
P = 23.4 \text{ N/mm}^2
\]

The tangential stress at the outer surface of the inner tube,

\[
\sigma_u = \frac{P(D_2^2 + D_1^2)}{(D_2^2 - D_1^2)}
\]

\[
\sigma_u = \frac{23.4 \times (160^2 + 80^2)}{(160^2 - 80^2)}
\]

\[
= 39 \text{ MPa (Compressive)}
\]

The tangential stress at the inner surface of the outer tube,
\[
\sigma_{to} = \frac{P(D_3^2 + D_2^2)}{(D_3^2 - D_2^2)}
\]
\[
\sigma_{to} = \frac{23.4 \times (240^2 + 160^2)}{(240^2 - 160^2)}
\]
\[
= 60.84 \text{ MPa (Compressive)}
\]
The radial stress at the junction, (i.e. at the inner radius of the outer tube),
\[
\sigma_{ro} = -p = -23.4 \text{ N/mm}^2
\]
\[
= 23.4 \text{ MPa (compressive)}
\]

Example – 8:- A hydraulic press has the following specifications:
- Capacity = 80 kN
- Fluid pressure = 16 MPa
- Stroke = 80 mm
- Permissible tensile stress for pillar and ram = 75 MPa
- Permissible stress for C.I. cylinder = 30 MPa
- Distance between the center line of pillars = 800 mm
- Distance between top supporting platform and bottom of top plate when the ram is in the down most position = 800mm
- Design the ram, cylinder and pillars.

Solution:
- \( p = 16 \text{ N/mm}^2 \)
- \( W = 80 \text{ kN} \)
  - For ram and pillar, \( \sigma_t = 75 \text{ N/mm}^2 \)
  - For C.I. cylinder, \( \sigma_t = 30 \text{ N/mm}^2 \)

Design of Ram
In case of solid ram,
- Maximum force to be exerted by ram,
  - \( W = \frac{\pi}{4} d_r^2 \times p \)
  - \( 80 \times 10^3 = \frac{\pi}{4} (d_r)^2 \times 16 \)
  - \( d_r = 79.79 \text{ mm} \)
- Diameter of ram, \( d_r \approx 80 \text{ mm} \)

In case the ram is made hollow in order to reduce its weight, then it can be designed as a thick cylinder subjected to external pressure.
- \( d_{ro} \) = Outer diameter of ram = \( d_r = 80 \text{ mm} \)
- \( d_{ri} \) = Inner diameter of ram, and
- Permissible stress for ram,
  - \( \sigma_c = 2P \left( \frac{(d_{ri})^2}{(d_{ro})^2 - (d_{ri})^2} \right) \)
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\[
75 = 2 \times 16 \left[ \frac{(d_i)^2}{(80)^2 - (d_n)^2} \right]
\]

\[
\frac{75}{2 \times 16} = \left[ \frac{(d_n)^2}{6400 - (d_n)^2} \right]
\]

\[
2.3437 = \frac{(d_n)^2}{6400 - (d_n)^2}
\]

\[
2.3437 \times [6400 - (d_n)^2] = (d_n)^2
\]

\[
15000 = 3.3437(d_n)^2
\]

\[
d_n = 66.97 \text{ mm}
\]

\[
d_n \approx 67 \text{ mm}
\]

\[
d_{co} = 80 \text{ mm}
\]

**Design of Cylinder**

\(d_{ci} = \) Inner diameter of cylinder, and \(d_{co} = \) Outer diameter of cylinder.

Assuming a clearance between the ram and the cylinder bore = 15 mm,

\[d_{ci} = d_{co} + \text{Clearance} = 80 + 15 = 95 \text{ mm}\]

The cylinder can be considered as thick cylinder, material of cylinder is cast iron. According to Lame’s equation,

\[
t = \frac{d_{ci}}{2} \left[ \sqrt{\frac{\sigma_t + p}{\sigma_t - p}} - 1 \right]
\]

\[
t = \frac{95}{2} \left[ \sqrt{\frac{30 + 16}{30 - 16}} - 1 \right]
\]

\[
t = 38.6 \text{ mm}
\]

\[
t \approx 40 \text{ mm}
\]

Outside diameter of the cylinder,

\[d_{co} = d_{ci} + 2 \times t = 95 + 2 \times 40 = 175 \text{ mm}\]

**Design of Pillars**

\(d_p = \) Diameter of the pillar.

It is assumed that four pillars are equally sharing the load.

Load on each pillar = \[\frac{80 \times 10^3}{4} = 20 \times 10^3 \text{ N}\]

Load on each pillar = \[\frac{\pi}{4} d_p^2 \times \sigma_t\]


\[ 20 \times 10^3 = \frac{\pi}{4} d_p^2 \times 75 \]

\[ d_p = 18.43 \text{ mm} \]

From fine series of metric threads, let us adopt the threads on pillars as M 20 \times 1.5 having major diameter as 20 mm and core diameter as 18.16 mm.

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**Example – 9:** A pressure vessel consists of a cylindrical shell with an inner diameter of 1500 mm, and thickness of 20 mm. It is provided with a nozzle with an inner diameter of 250 mm and thickness of 15 mm. The yield strength of the material for the shell and nozzle is 200 N/mm\(^2\) and the design pressure is 2.5 MPa. The extension of the nozzle inside the vessel is 15 mm. The corrosion allowance is 2 mm, while the weld joint efficiency is 0.85. Neglecting the area of welds, determine whether or not a reinforcing pad is required for the opening. If so, determine the dimensions of pad made from a plate of 15 mm thickness.

**Solution:**

For cylindrical shell, \( D_i = 1500 \text{ mm} \quad \text{t} = 20 \text{ mm} \)

For nozzle, \( d_i = 250 \text{ mm} \quad t_n = 15 \text{ mm} \)

\[ h_2 = 15 \text{ mm} \]

\[ P_i = 2.5 \text{ MPa} \]

\[ S_{yt} = 200 \text{ N/mm}^2 \]

\[ CA = 2 \text{ mm} \]

\[ \eta = 0.85 \]

For pad, \( t = 15 \text{ mm} \)
Area of removed metal

\[ \sigma_t = \frac{S_{nt}}{F.S.} = \frac{200}{2.5} = 133.33 \text{ N/mm}^2 \]

\[ t_r = \frac{P_i D_i}{2\sigma_t \eta - P_i} = \frac{2.5 \times 1500}{2 \times 133.33 \times 0.85 - 2.5} \]

\[ = 16.73 \text{ mm} \]

\[ d = d_i + 2(CA) = 250 + 2(2) = 254 \text{ mm} \]

\[ A = d t_r = 254 \times (16.73) = 4249.42 \text{ mm}^2 \]

Area available for reinforcement

\[ t_m = \frac{P_i D_i}{2\sigma_t \eta - P_i} = \frac{2.5 \times 250}{2 \times 133.33 \times 0.85 - 2.5} \]

\[ = 2.79 \text{ mm} \]

The limiting dimension \( X \) is the higher of the following two values:

\[ X = d = 254 \text{ mm} \]

\[ X = \left[ \frac{d}{2} + t + t_m - 3CA \right] \]

\[ = (125 + 20 + 15 - 6) \]

\[ = 154 \text{ mm} \]

Therefore,

\[ X = 254 \text{ mm} \]

The limiting dimension \( h_1 \) is the lower of the following two values:

\[ h_1 = 2.5 \times (t - CA) = 2.5 \times (20 - 2) = 45 \text{ mm} \]

\[ h_1 = 2.5 \times (t_m - CA) = 2.5 \times (15 - 2) = 32.5 \text{ mm} \]

Therefore,

\[ h_1 = 32.5 \text{ mm} \text{ and } h_2 = 15 \text{ mm} \]

The areas available for reinforcement within the above limits are as follows:

\[ A_1 = (2X - d) \times (t - t_r - CA) \]

\[ = \left[ 2(254) - 254 \right] \times (20 - 16.73 - 2) \]

\[ = 322.58 \text{ mm}^2 \]

\[ A_2 = 2h_1 \times (t_m - t_m - CA) \]

\[ = 2 \times 32.5 \times (15 - 2.79 - 2) \]

\[ = 663.65 \text{ mm}^2 \]

\[ A_3 = 2h_2 \times (t_m - 2CA) \]

\[ = 2 \times 15 \times (15 - 4) \]

\[ = 330 \text{ mm}^2 \]

\[ (A_1 + A_2 + A_3) = 1316.23 \text{ mm}^2 \]

Area of pad

From (a) and (b),

\[ A > (A_1 + A_2 + A_3) \]
Therefore, a reinforcing pad is necessary. The area of the reinforcing pad $A_4$ is given by

$$A_4 = A - (A_1 + A_2 + A_3)$$

$$= 4249.42 - 1316.23$$

$$= 2933.19 \text{ mm}^2$$

**Dimensions of pad**

The thickness of the reinforcing pad is 15 mm.

Therefore, the width of the pad is given by,

$$w = \frac{2933.19}{15} = 195.55 \text{ or } 200 \text{ mm}$$

The inner diameter of the pad is equal to the outer diameter of the nozzle, i.e., $(250 + 30) = 280 \text{ mm}.$

Outer diameter of pad = $280 + 200 = 480 \text{ mm}$