3. Design of DC Machines

3.1. Output equation of DC machines

- Output equation of a DC machine is the mathematical expression that relates the main dimension, specific magnetic loading, specific electric loading and speed.

\[ P_a = \text{Power developed by armature (kW)} \]
\[ P = \text{Rating of machine (kW)} \]
\[ E = \text{Generated emf (V)} \]
\[ I_a = \text{Armature current (A)} \]
\[ I_z = \text{Current in each conductor (A)} \]
\[ n_s = \text{Synchronous speed (rps)} \]
\[ n = \text{Machine speed (rps)} \]
\[ p = \text{Number of pole} \]
\[ a = \text{Number of parallel path} \]
\[ Z = \text{Total number of armature conductor} \]
\[ S = \text{Number of armature slots} \]
\[ Z_s = \text{Conductor per slot} \]
\[ Y_s = \text{Slot pitch (m)} \]
\[ \phi = \text{Total flux around the airgap (Wb)} \]
\[ B_{av} = \text{Specific magnetic loading (Wb/m}^2) \]
\[ ac = \text{Specific electric loading (AT/m)} \]
\[ D = \text{Armature diameter (m)} \]
\[ L = \text{Armature core length (m)} \]
\[ C_o = \text{Output co-efficient} \]

- Power developed by armature in kW

\[ P_a = E \times I_a \times 10^{-3} \]
\[ = \frac{\phi Znp}{a} \times I_a \times 10^{-3} \]
\[ = \left( \phi p \right) \left( \frac{I_z Z}{a} \right) \times n \times 10^{-3} \]
\[ = (\pi DLB_{av}) (\pi Dac) \times n \times 10^{-3} \]
\[ = (\pi^2 B_{av} ac \times 10^{-3}) D^2 Ln \]
\[ = C_o D^2 Ln \]

- In case of DC generator and motor amount of armature power \( P_a \) and rated power \( P \) govern by armature copper loss, field copper loss, friction and windage loss and iron loss.
3. Design of DC Machines

(a) Power developed by DC generator

- In case of DC generator, prime mover supplies friction, windage and iron loss. Armature supplies its own copper loss and field copper loss. Amount of friction, windage and iron loss may be taken as one third of total loss.

\[ P_a = \text{Output power} + \text{Armature copper loss} + \text{Field copper loss} \]

\[ = \text{Input power} - \left(\text{Friction, windage and iron loss}\right) \]

\[ = \frac{P}{\eta} - \frac{1}{3}(\text{Total loss}) \]

\[ = \frac{P}{\eta} - \frac{1}{3}(\text{Input power - Output power}) \]

\[ = \frac{P}{\eta} - \frac{1}{3}\left(\frac{P}{\eta} - P\right) \]

\[ = \frac{P}{\eta} - \frac{P}{3}\left(\frac{1}{\eta} - 1\right) \]

\[ = \frac{P}{\eta} - \frac{P}{3}\left(\frac{1 - \eta}{\eta}\right) \]

\[ = P\left(\frac{1}{\eta} - \left(\frac{1 - \eta}{3\eta}\right)\right) \]

\[ = P\left(\frac{2 + \eta}{3\eta}\right) \]

(b) Power developed by DC motor

- In case of DC motor, armature copper loss and field copper loss are taken from supply and friction, windage and iron loss are supplied by armature. Amount of friction, windage and iron loss may be taken as one third of total loss.

\[ P_a = \text{Output power} + \text{Friction, windage and iron loss} \]

\[ P_a = P + \frac{1}{3}(\text{Total loss}) \]

\[ = P + \frac{1}{3}(\text{Input power - Output power}) \]

\[ = P + \frac{1}{3}\left(\frac{P}{\eta} - P\right) \]

\[ = P + \frac{P}{3}\left(\frac{1 - \eta}{\eta}\right) \]

\[ = P\left(1 + \frac{1 - \eta}{3\eta}\right) \]

\[ = P\left(\frac{1 + 2\eta}{3\eta}\right) \]
3. Design of DC Machines

3.2. Factors affecting size of rotating machines

- Output equation of DC machine reflects that the product $D^2L$ will decrease with the increase of speed and/or output coefficient.

$$D^2L = \frac{P_o}{C_o n}$$

- Product $D^2L$ reflects the volume of active parts of rotating machine, hence decrease in its value ultimately reduces the size and cost.

(a) Speed

- Volume of active part of rotating machine varies inversely proportional to the speed.
- For the same output a machine designed with greater speed will have smaller size and hence lesser cost.
- When the speed is not specific criteria, highest practical speed should be selected.

(b) Output coefficient

$$C_o = \frac{P_o}{D^2Ln} = \pi^2 B_{av} ac \times 10^{-3}$$

- Volume of active part of rotating machine varies inversely proportional to the output coefficient.
- Economic point of view, output coefficient should be high as possible to reduce size and cost.
- As output coefficient is product of specific magnetic loading and specific electrical loading, its value depends on selection of these loadings.
- Higher value of specific magnetic loading and electrical loading gives much higher value of output coefficient which further helps to reduce volume of machine and consecutively size and cost.
- But beyond some extent higher value of specific magnetic loading and electrical loading adversely affect the performance characteristics such as temperature rise, efficiency, power factor, commutation etc.

3.3. Factors affecting selection of specific magnetic loading for DC machines

- The total flux around the armature or stator periphery at the air gap is called the total magnetic loading.
- The average flux density over the air gap of rotating machine is called specific magnetic loading.

$$B_{av} = \frac{\text{Total flux around the airgap}}{\text{Area of flux path around the airgap}} = \frac{p \phi}{\pi DL}$$

- In some of the cases, maximum gap density is selected in design instead of specific magnetic loading.

$$B_g = \frac{\text{Average gap density}}{\text{Ratio of pole arc to pole pitch}} = \frac{B_{av}}{\psi}$$
3. Design of DC Machines

- The choice of specific magnetic loading is subjective to types of machines.

(a) **Maximum flux density in tooth**

- Maximum flux density in iron part of machine is directly proportional to average flux density of air gap.

![Diagram of Flux over one slot pitch](image)

Flux over one slot pitch = \[ \frac{p\phi}{s} = \frac{B_{av}\pi DL}{s} = \frac{\pi D}{s}L = B_{av}Y_sL \]

Flux density in tooth, \( B_t \) = \[ \frac{\text{Flux in each tooth}}{\text{Area of each tooth}} = \frac{B_{av}Y_sL}{W_tL} = B_{av}\frac{Y_s}{W_t} \]

- In salient pole machine flux is concentrated over the pole arc, hence teeth under that pole arc will carry almost flux. It is clear from above equation, that ratio \( B_t/B_{av} \) is constant, hence value of \( B_{av} \) must be selected such a way that value of \( B_t \) does not exceed the specific limit.

- Small machines are designed with parallel sided slot with tapered tooth i.e. width of tooth is not same over the height. Tooth flux density will vary from minimum to maximum over tooth height from large to smaller tooth width. Thus, low value of specific magnetic loading is selected for small machine to compensate flux variation.

- When higher value of average gap density is selected, mmf required for iron part of machine will be very high, which further results in large field ampere-turn, field copper loss and cost of copper.

(b) **Frequency**

- Due to rotation of armature, teeth and armature core comes under the influence of field pole alternately. This reversal of frequency at armature will yield hysteresis and eddy current loss in it.

Frequency of reversal, \( f = \frac{np}{2} \)

Also,

\( P_h \propto f \left( B_{av} \right)^2 \) and \( P_e \propto f^2 \left( B_{av} \right)^2 \)

- To keep total armature hysteresis and eddy current loss within limit, DC machine with high frequency of reversal should not be designed for large value of specific magnetic loading.
3. Design of DC Machines

(c) Size of machine

- Large machine has huge diameter and hence large tooth width. Increased tooth width permits higher value of gap flux density without causing saturation.

  The usual value of gap flux density are;
  - Maximum gap density \((B_g)\): 0.55 to 1.15 Wb/m²
  - Average gap density \((B_{av})\): 0.4 to 0.8 Wb/m²

3.4. Factors affecting selection of specific electric loading for DC machines

- The total number of ampere conductor around the armature periphery is called the total electric loading.

- The number of armature ampere conductor per meter of armature periphery at the air gap is called specific electric loading.

\[
ac = \frac{\text{Total armature ampere conductor}}{\pi D} = \frac{I_z Z}{\pi D}
\]

(a) Temperature rise

- The value of specific electric loading is governed by allowable temperature rise of machine.

Specific electric loading,

\[
ac = \frac{I_z Z}{\pi D} = \frac{I_z Z}{\pi D} = \frac{I_z Z}{Y_s}
\]

\[
I^2R \text{ loss in each slot} = Z_s (I_z)^2 \left( \frac{L}{a_z} \right)
\]

Heat dissipation surface of slot = \(Y_s L\)

Loss dissipated/unit area of armature,

\[
q = \frac{Z_s (I_z)^2 \left( \frac{L}{a_z} \right)}{Y_s L} = \frac{Z_s (I_z)^2 \rho L}{Y_s a_z} = \left( \frac{I_z Z_s}{Y_s} \right) \left( \frac{I_z}{a_z} \right) \rho = (ac) (\delta) \rho
\]

Temperature rise,

\[
\theta = qc = (ac)(\delta)c
\]

\[
ac = \frac{\theta}{\delta c}
\]

Where, \(c\) is cooling co-efficient

- Higher value of specific electric loading can be selected where higher temperature rise is allowed i.e. machine with the insulation that can withstand higher temperature rise. It also depends on the type of enclosure and cooling techniques employed.

(b) Machine speed

- For high speed machine, higher value of \(ac\) can be selected because ventilation is better which will dissipate heat at faster rate i.e. high permissible temperature rise.
3. Design of DC Machines

(c) Voltage

- For high voltage machine, insulation occupies more space hence space available for conductor is less. So, for this type of machine if high value of \( ac \) is selected, it will result in more number of conductor and further it will be difficult to accommodate conductor in slot.

Let,

\[ W_s = \text{Width of slot (m)} \]
\[ d_s = \text{Depth of slot (m)} \]

Total area of slot \( = S \left( W_s d_s \right) = \left( \frac{\pi D}{Y_s} \right) \left( W_s d_s \right) \)

Total area of conductor in slot \( = Z a_z = \left( \frac{\pi D}{I_z} \right) a_z = \left( \frac{\pi D}{\delta} \right) (ac) \)

Space factor for slot, \( S_f = \frac{\delta}{\left( \frac{\pi D}{Y_s} \right) \left( W_s d_s \right)} = \frac{Y_s}{\delta} \left( \frac{ac}{W_s d_s} \right) \)

\[ \therefore \ ac = \left( \frac{W_s d_s}{Y_s} \right) \delta S_f \]

- For fixed value of slot dimension and number, specific electric loading is directly proportional to space factor of slot. For high voltage machine due to thick insulation over the conductor, value of space factor is small. Hence, it is advisable to select lower value of \( ac \).

- Consequently, space factor depends on shape of conductor. For high voltage machine, current rating is small which allows to choose conductor cross section as circular. The space factor for circular conductor is small compared to rectangular.

(d) Machine size

- In large size machine, more space is available for accommodation of conductor in slot which permits to select higher \( ac \).

(e) Armature reaction

- When higher value of \( ac \) is selected, higher value of armature mmf makes armature magnetically very strong. So, under load condition large air gap flux distortion weakens the main flux.

- In order to reduce above said armature reaction, field mmf needs to be very strong. It will be achieved by adding more field conductor. This means that increased cost of conductor and large size of machine.

(f) Commutation

- Machine designed with high value of \( ac \) has large armature conductor. It will result in large number of coil i.e. more number of turns and large coil inductance.
3. Design of DC Machines

- Hence, high reactance voltage in coil delays the commutation and creates worse commutation condition.

The usual value of specific electric loading is 15,000 to 50,000 AT/m for the machine using class A insulation.

3.5. Factors affecting the separation of the main dimensions D and L for DC machines

- Output equation of DC machine in terms of product $D^2L$ represents the size or volume of active iron parts of machine.

- Good design is that maximum output with less $D^2L$ product i.e. small in size. On other side certain parameter that influence the performance of machine and doesn’t allow to go beyond specific value of D and L.

(a) Peripheral speed

- Peripheral speed influence the value of machine diameter i.e. $V_a = \pi Dn$

- Usual value ranges between 15 to 50 m/s. Design limit of peripheral speed is 30 m/s. For the exceeded value of the peripheral speed, rotor assembly and the overhang bands fly out due to excessive centrifugal force.

(b) $L/\tau$ ratio

- Ratio of $L/\tau$ is the prime factor to decide length of core and proportion of pole.

- Pole cross section i.e. circular or rectangular is the guiding factor to control mean turn length and weight/cost of copper in case of field winding.

- Circular pole cross section gives smallest length of mean turn length and weight of copper. At the same time it necessitates the solid structure.

- Rectangular pole cross section contributes in larger mean turn length and weight of copper. At the same time gives facility to use laminated structure which reduces the production cost.

- For given flux and cross section area, the mean turn length of field winding is minimum when periphery is square i.e. length of pole is approximately equal to pole arc.
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\[ b_p = \text{Width of pole body (m)} \]
\[ b = \text{Pole arc (m)} \]
\[ \tau = \text{Pole pitch (m)} \]
\[ \psi = \frac{b}{\tau} = 0.64 \text{ to } 0.72 \]

For,
\[ L = b = \psi \tau \]

Ratio \( \frac{L}{\tau} \) also lies between 0.64 to 0.72

Practically length of core is taken slightly higher, so \( \frac{L}{\tau} \) lies between 0.7 to 0.9

(c) Voltage between adjacent commutator segments

- The value of voltage between adjacent commutator segments is limited to a value where there is no possibility of flash over. Usually machine is designed with the voltage between commutator segments not more than 30 V.

\[ Z_c = \text{Number of conductors between adjacent commutator segments} \]
\[ T_c = \text{Turns per coil} \]
\[ e_z = \text{Average voltage per conductor at no load (V)} \]
\[ E_c = \text{Average voltage between adjacent commutator segments at no load} \]
\[ \quad = Z_c e_z \]
\[ \quad = (2T_c) \left( B_{av} L V_a \right) \]
\[ E_{cm} = \text{Maximum voltage between adjacent commutator segments at no load} \]
\[ \quad = (2T_c) \left( B_{gm} L V_a \right) \]

- For large machine, typical values of \( B_{gm} = 1.2 \text{ Wb/m}^2, V_a = 30 \text{ m/s}, E_{cm} = 30 \text{ V} \) and \( T_c = 1 \), provides \( L = 0.4166 \text{ m} \).

- It indicates that due to limiting value of \( L \), DC machines are designed for large diameter D.

(d) Moment of inertia

- Moment of inertia for cylindrical mass is proportional to the product \( D^4 L \).

- For impact load application, moment of inertia required is very much high and hence DC machine is designed for large diameter.

- For control system application, small moment of inertia is essential, hence DC machine is designed for small diameter.

3.6. Factors affecting selection of number of poles for DC machines

- Appropriate value of magnetic loading and electric loading is required in order to get right design of a machine. So far magnetic loading is concerned, suitable number of pole is must.

- In case of AC machine supply frequency and speed of machine fixes the number of pole. While in case of DC machine any number of pole can be selected. However a small range of number of pole gives good commercial design.
3. Design of DC Machines

(a) Field mmf per pole

- In practice, total field mmf is taken in proportion to total armature mmf. Irrespective of number of pole, total armature mmf is constant.

\[
\text{Field mmf per pole} = k \left( \frac{\text{Total armature mmf}}{2p} \right)
\]

where,

\[
k \text{ is constant}
\]

- It is clear that field mmf developed by each coil is inversely proportional to number of pole. So, for same depth of field winding, height of field winding increases with less number of pole. Which further increases height of pole and overall diameter of machine.

- So, it is good to have more number of pole in order to reduce field mmf, weight of iron and overall diameter.

(b) Weight of iron parts

\[
\phi = \text{Total flux around the airgap}
\]

\[
\frac{\phi}{4} = \text{Flux in yoke}
\]

\[
\frac{\phi}{8} = \text{Flux in armature core}
\]

- Flux carried by yoke is inversely proportional to number of pole, so for more number of pole, yoke cross section area required is less. As yoke carries almost steady flux, iron loss is negligible and hence large number of pole can be good choice to reduce weight of yoke.

- Flux per pole divides itself in two parts in armature core, so for more number of pole, armature core cross section area required is less. But increased number of pole increases the iron loss in armature core due to increased frequency of reversal.

- Eddy current loss in armature core

\[\text{Figure 3.3 Two pole machine}\]

\[\text{Figure 3.4 Four pole machine}\]
3. Design of DC Machines

\[ P_e \propto B_c^2 f^2 \]
\[ P_e \propto \left( \frac{\phi}{A_c} \right)^2 \left( \frac{pn}{2} \right)^2 \]
\[ P_e \propto \left( \frac{\phi}{4A_c} \right)^2 \left( \frac{2n}{2} \right)^2 \]
\[ P_e \propto \frac{\phi^2 n^2}{16A_c^2} \]

\[ P_e \propto B_c^2 f^2 \]
\[ P_e \propto \left( \frac{\phi}{A_c} \right)^2 \left( \frac{pn}{2} \right)^2 \]
\[ P_e \propto \left( \frac{\phi}{8A_c} \right)^2 \left( \frac{4n}{2} \right)^2 \]
\[ P_e \propto \frac{\phi^2 n^2}{32A_c^2} \]

- To obtain same eddy current loss for all value of number of pole, area of armature core has to be same. Armature core area decreases for large number of pole, eddy current loss increases.

- Hysteresis loss in armature core

\[ P_h \propto B_c^2 f \]
\[ P_h \propto \left( \frac{\phi}{A_c} \right)^2 \left( \frac{pn}{2} \right) \]
\[ P_h \propto \left( \frac{\phi}{4A_c} \right)^2 \left( \frac{2n}{2} \right) \]
\[ P_h \propto \frac{\phi^2 n}{16A_c^2} \]

- Hysteresis loss decreases with increase in number of pole. To obtain same hysteresis loss for all value of number of pole, area of armature core with large number of pole is required to decrease.

(c) Weight of copper

- Portion of conductor accommodated in slot contributes in production of emf and torque. Therefore it is called active copper.

![Figure 3.5 Length of overhang for two pole and four pole machine](image-url)
3. Design of DC Machines

- While portion of conductor in overhang only provides connections, it does not contribute in production of emf and torque. Therefore it is called inactive copper. As the ratio of inactive copper to active copper is less, machine becomes cheaper.
- For constant diameter, pole pitch decreases with the increase in number of pole which further reduces the length of conductor in overhang portion. So, it is clear that copper used in two pole machine is more than copper used in four pole machine. Also overhang projected outside the core is more in case of two pole machine which gives large overall length of machine.
- As field mmf is inversely proportional to number of pole, two pole machine will have more field mmf required compared to four pole machine. This means weight of copper, number of turns, mean turn length and field copper loss is more in case of two pole machine.

(d) Frequency

\[ f = \frac{pn}{2} \]

- Frequency of flux reversal increases with the increase in number of pole. In DC machine large frequency of reversal leads to excessive iron loss in armature core and teeth.
- To maintain above said losses, frequency of reversal is kept 25-50 Hz for common ratings. While some high speed DC turbo generator is designed with two pole machine at the cost of high iron loss in armature.

(e) Length of commutator

- Current collected by each brush arm depends on number of pole.
- More number of pole means current per brush arm is less and hence less area and thickness of brush. Reduction in brush thickness results in reduction in length of commutator and overall length of machine.
3. Design of DC Machines

(f) Flash over

- Number of brush arm is equal to number of pole. For same diameter distance between adjacent brush arm decreases with increased number of pole, hence it leads to the possibility of flash over.

(g) Labor charge

\[ E = \frac{\phi Z p N}{60a} \]

\[ \therefore Z\alpha \frac{E_a}{N} \text{ since } \phi p \text{ is constant} \]

- For lap winding \( a=p \), so number of conductor and armature coil increases with the increase in number of pole.
- For wave winding \( a=2 \), so number of conductor and armature coil is independent of number of pole.
- Number of commutator segments are same as number of armature coil, means more number of commutator segments for more number of pole.
- Also, number of field coil is same as number of pole, means more number of field coils needs to be assembled for more number of pole.
- Hence, labor charge for armature coil and field coil winding increases with increase in number of pole.

Guideline for selection of number of pole

- Frequency of flux reversal in armature core should be between 25 to 50 Hz.
- Current per parallel path should not be more than 200 A.
- Current per brush arm should not be more than 400 A.
- Armature mmf should not be too large.

3.7. Factors affecting selection of length of air gap for DC machines

- In rotating machine air gap is undesirable yet unavoidable due to required movement between the stationary and the rotating element. The value of air gap depends on several factors.

(a) Armature reaction

- To overcome effect of distorted field form due to armature reaction, field mmf is made larger than armature mmf.
- Large air gap draws large field mmf, hence machine with large air gap has less distorting effect of armature reaction. But large field mmf results in increased cost and size of machine.

(b) Circulating current

- Small irregularity in air gap results in large circulation current in lap wound machine. So, large air gap is preferred for lap wound machine.
3. Design of DC Machines

(c) Pole face loss

- With large air gap, variation in gap flux density due to slotting is very less. Hence, pulsation loss in pole faces reduces if large value of air gap is selected.

(d) Noise

- Operation of machine with large air gap is silent.

(e) Cooling

- Cooling of machine with large air gap is better.

(f) Mechanical aspects

- Small air gap value leads to unbalanced magnetic pull and causes rotor to foul with stator.

Estimation of air gap length

\[ AT_g = \text{Armature mmf per pole (AT)} \]
\[ AT_a = \text{Air gap mmf (AT)} \]
\[ B_g = \text{Maximum flux density in the air gap (Wb/m}^2\text{)} \]
\[ l_g = \text{Air gap length (mm)} \]
\[ k_g = \text{Gap contraction factor} \]

\[ AT_g = (0.5 \text{ to } 0.7) AT_a = (0.5 \text{ to } 0.7) \left( \frac{ac \times \tau}{2} \right) \]

Also,

\[ AT_g = 8,00,000 B_g l_g k_g \]

So,

\[ 8,00,000 B_g l_g k_g = (0.5 \text{ to } 0.7) \left( \frac{ac \times \tau}{2} \right) \]

\[ l_g = \left( \frac{0.5 \text{ to } 0.7} {8,00,000 B_g k_g} \right) \left( \frac{ac \times \tau}{2} \right) \]

- Field form i.e. air gap flux distribution curve should have a shape that will improve commutation.
- In order to have improved commutation, flux density in the air gap must reduce gradually from maximum to zero under pole center to inter pole axis.
- Abrupt drop of field form from maximum to zero leads to magnetic noise and commutation issues.
- In order to attain good field form, air gap length is not kept uniform under the entire pole phase. Shape of pole phase is maintained such a way that it gives gradually increasing distance towards both pole tips.
- Air gap length at the pole tips is 1.5 to 2 times the air gap length at center of pole.
3.8. **Effect of armature reaction on air gap flux in DC machines**

- When the armature of DC machine carries the current, it sets up its own magnetic field. The effect of this armature flux on the distribution of main field flux is called armature reaction.

(a) **Reduced emf**
- As load increases, armature reaction increases and hence flux per pole decreases. Reduction in flux per pole will decrease generated emf.

(b) **Increased iron loss**
- Armature reaction distorts the field flux, which increases flux density in teeth and pole shoe. Therefore iron loss, particularly in teeth increases. Sometimes it is 1.5 times iron loss at no load.

(c) **Sparking**
- Armature reaction increases the maximum value of gap flux density and so maximum value of voltage between adjacent commutator segments. If this voltage goes beyond 30 V, spark over may take place at commutator periphery.

(d) **Delayed commutation**
- Commutation means reversal of current from one direction to the direction opposite to the original. Armature reaction has tendency to maintain current in its original direction and thus it delays commutation.

3.9. **Methods to improve armature reaction in DC machines**

- In all DC machine, armature mmf is approximated as a symmetrical triangular wave with an amplitude, AT= \((I_a/a) (Z/2p)\).

- Axis of the armature mmf is the inter pole axis so that the armature mmf and field mmf are displaced by 90° in space.

- Hence, the armature reaction has cross magnetization effect.

(a) **Increased air gap length at pole tip**
- Distorting effect of armature reaction can be reduced if reactance of the path of cross magnetizing field is increased. Air gap at pole tips offers reluctance to path of cross magnetization flux, so air gap at pole tip is made 1.5 to 2 times air gap at pole center.

(b) **Increased reactance of pole tips**
- Reluctance of pole tips can be increased by adopting special construction of pole shoe. This is done by alternately omitting laminations at leading and trailing pole tips portion.

(c) **Compensating winding**
- A compensating winding with ampere conductor equals to the ampere conductors under the pole shoe is used to compensate armature reaction effect. It is connected in series with armature winding and housed in axial slots in pole faces.

Compensating winding mmf per pole,

\[
AT_c = \frac{\text{Pole arc}}{\text{Pole pitch}} \times \text{Armature mmf per pole}
\]
3. Design of DC Machines

(d) Interpoles

- In order to attain commutation at zero flux, armature reaction at the brush axis must be neutralized. To achieve this another mmf opposite to that of armature mmf is applied at brush axis.
- Inter poles placed at geometric neutral axis produces opposing mmf. Inter pole winding is connected in series with armature winding to produce opposing mmf for all load.

(e) Brush shift

- As armature reaction not only distorts main field but also shifts the magnetic neutral axis against the direction of rotation. This phenomenon leads to delay in commutation, heavy sparking and short circuit at brushes.
- So, brushes are shifted backward opposite to the direction of rotation to bring them in magnetic neutral zone.
- Effect of this brush shift resolve armature in two winding component demagnetizing component \((AT_{ad})\) and cross magnetizing component \((AT_{aq})\).

![Figure 3.8 Distribution of armature current with brush shift](image)

Demagnetizing mmf per pole

\[ AT_{ad} = AT_a \left( \frac{2\alpha}{180} \right) \]

Cross magnetizing mmf per pole

\[ AT_{aq} = AT_a - AT_{ad} = AT_a - AT_a \left( \frac{2\alpha}{180} \right) = AT_a \left( 1 - \frac{2\alpha}{180} \right) \]

3.10. Design aspects of armature winding for DC machines

- Modern DC machine employs two type of winding, lap winding and wave winding. These windings differ each other by number of circuits between positive and negative brush terminal and the manner in which coil ends are connected to commutator segment.
3. Design of DC Machines

![Figure 3.9 Lap winding](image)

![Figure 3.10 Wave winding](image)

In lap winding, \( a = p \)

\[
I_z = \frac{I_a}{a} = \frac{I_a}{p}
\]

\[
Z = \left( \frac{60E}{\phi pN} \right) a = \left( \frac{60E}{\phi pN} \right) (p)
\]

In wave winding, \( a = 2 \)

\[
I_z = \frac{I_a}{a} = \frac{I_a}{2}
\]

\[
Z = \left( \frac{60E}{\phi pN} \right) a = \left( \frac{60E}{\phi pN} \right) (2)
\]

- Cross section area of conductor in lap winding is \( 2/p \) times the cross section area in wave winding. Number of conductor in lap winding is \( p/2 \) times the conductor in wave winding. Therefore volume of copper in both winding is same.
- Wave winding has small number of conductor, hence number of coils are less and low labor cost. It is used for machine with rated current less than 400 A.
- Lap winding has reduced overhang at both end, hence preferred for traction application where reduction in size and weight matters. It is used for machine with rated current greater than 400 A.

**Number of armature coil**

\( Z_c \) = Number of conductors between adjacent commutator segments

\( T = \) Turns between adjacent commutator segments

\( C = \) Number of armature coil

\[
T_c = \text{Turns per coil} = \frac{Z}{2C}
\]

\( N_c = \) Number of coil between adjacent commutator segments

\( e_c = \) Average voltage per conductor at no load (V)

\( B_{gm} = \) Maximum flux density at load (Wb/m²)

\( B_g = \) Maximum flux density at no load (Wb/m²)

\( K_f = \) Field form factor = 0.67

\( E_c = \) Average voltage between adjacent commutator segments at no load

\[
E_c = \frac{Z}{2T} e_c = \left( \frac{2T}{Z_c} \right) e_z = \left( \frac{2T N_c}{Z_c} \right) \left( B_{gm} L V_z \right)
\]
3. Design of DC Machines

\[ E_{cm} = \text{Maximum voltage between adjacent commutator segments at load} \]
\[ = \left( 2T_c N_c \right) \left( 1.3 B_a LV_a \right) \]
\[ = \left( 2T_c N_c \right) \left( 1.3 \frac{B_{av}}{K_f} LV_a \right) \]
\[ = 3.88 \left( T_c N_c \right) \left( B_{av} LV_a \right) \]
\[ E_{cm} = 4 \left( T_c N_c \right) \left( B_{av} LV_a \right) \]

- Maximum value of voltage between commutator segments at load must not exceed 30 V.

\[ E_{cm} = 4 \left( T_c N_c \right) \left( B_{av} LV_a \right) \]
\[ \approx 4 \left( \frac{Z}{2C} \right) N_c \left( \frac{p \phi}{\pi DL} \right) \left( L \right) \left( \pi D \frac{N}{60} \right) \]
\[ \approx 4 \left( \frac{Z}{2C} \right) N_c \left( p \phi \right) \left( \frac{N}{60} \right) \]
\[ \approx 4 \left( \frac{N_c}{2C} \right) \left( \phi Z N p \right) \left( \frac{60}{60} \right) \]

\[ 30 = 4 \left( \frac{N_c}{2C} \right) \left( Ea \right) \]
\[ C = \frac{EaN_c}{15} \]

For lap winding, \( a=p \) and \( N_c = 1 \)
\[ C = \frac{Ep}{15} \]

For wave winding, \( a=2 \) and \( N_c = \frac{p}{2} \)
\[ C = \frac{Ep}{15} \]

### 3.11. Factors affecting selection of number of armature slots for DC machines

- Influences such as accommodation of conductor, flux density in tooth, reactance voltage drop etc. can be governed by proper selection of number of armature slots.

(a) **Slot pitch**
- The slot pitch is a peripheral distance between centers of two adjacent slot in DC machine. A large number of slots results in smaller slot pitch and so the width of tooth is also small. This may lead to difficulty in construction.

(b) **Cooling of armature conductors**
- Large number of slots leads to less number of conductors per slot, so less conductors are bunched together in a slot. Hence the cooling of armature conductors is better.

(c) **Flux pulsations**
- Change in air gap flux due to slotting is called flux pulsation. This phenomenon gives rise to eddy current loss in pole phase and magnetic noise.
3. Design of DC Machines

- When flux passes from pole to armature through 5 teeth i.e. (a), if armature moves half slot pitch to the right i.e. (b) flux passes from pole to armature through 6 teeth. Reluctance through air gap at position (a) is greater than at position (b). Hence flux pulsates in air gap as armature rotates.

- When flux passes from pole to armature through 5+1/2 teeth i.e. (a), if armature moves half slot pitch to the right i.e. (b) flux passes from pole to armature through 5+1/2 teeth. Reluctance through air gap at position (a) and at position (b) is almost same. Hence no flux pulsations in air gap as armature rotates.

- So, number of slots under pole shoe should be integer+1/2.

(d) Commutation
- For spark less commutation the flux pulsations and oscillations under the inter pole needs to be avoided. This can be achieved with large number of slots per pole. In fact, the number of slots in the region between the tips of two adjacent poles should be at least 3.

(e) Cost
- Labor cost and slot insulation cost reduces when less number of slots are selected.

Guideline for selection of number of slots
- Slot pitch should lie between 25 to 35 mm.
- Slot loading should not exceed 1500 A.
- Number of slot per pole pair should be odd integer.
- Slot per pole should lie between 9 to 16.
- For wave winding number of slot should not be multiple of pole pair and for lap winding number of slot should be multiple of pole pair.
3.12. Factors affecting dimensions of armature slots for DC machines

- Slot dimensions such as depth and width depend upon the conductor per slot arrangement, conductor cross section area, type of conductor insulation, type of slot insulations etc.

(a) Excessive flux density
- Conductor accommodation in slot fixes the slot dimension. For parallel sided slots, teeth are tapered. The depth of slot should be selected such that teeth flux density at 1/3 height from root does not exceed 2.1 Wb/m².

(b) Flux pulsation
- Slot with narrow opening will have less pulsation of flux in the air gap that will again reduce iron loss and commutation issues.

(c) Eddy current loss in conductor
- Conductor with large depth will have more eddy current loss that will also restrict the selection in conductor depth. The depth of slot is also limited to accommodate two layer of conductor.

(d) Reactance voltage
- Deep slot will have more specific permeance and subsequently reactance voltage. This will lead to retardation in commutation. Generally ratio of slot depth to slot width is kept not more than four.

(e) Mechanical difficulties
- For deep slot, thickness of teeth at the root becomes small, so it may not be possible to support them without impairing the ventilation.
3. Design of DC Machines

### 3.13. Design aspects of pole for DC machines

- Pole design involves the determination of pole height, pole cross section area and field winding.

\[
\phi_p = \text{Flux per pole (Wb)}
\]

\[
A_p = \text{Area of pole (m}^2\text{)}
\]

\[
B_p = \text{Flux density in pole (Wb/m}^2\text{)}
\]

\[
L_p = \text{Length of pole (m)}
\]

\[
h_p = \text{Height of pole (m)}
\]

\[
AT_n = \text{Field mmf at full load (AT)}
\]

\[
h_i = \text{Height of field winding (m)}
\]

\[
d_f = \text{Depth of field winding (m)}
\]

\[
L_{mi} = \text{Length of mean turn of field winding (m)}
\]

\[
R_f = \text{Resistance of field winding (}\Omega\text{)}
\]

\[
T_f = \text{Number of turns in each field coils}
\]

\[
a_f = \text{Cross section area of conductor of field winding (mm}^2\text{)}
\]

\[
I_f = \text{Field current (A)}
\]

\[
Q_f = \text{Copper loss in each field coil (Watt)}
\]

\[
q_f = \text{Permissible loss per unit surface for normal temperature rise (Watt/m}^2\text{)}
\]

\[
S_f = \text{Space factor}
\]

\[
A_p = \frac{\phi_p}{B_p}
\]

\[
L_p = L
\]

\[
L_{pi} = 0.9L
\]

\[
b_p = \frac{A_p}{L_{pi}}
\]

\[
\frac{AT_n}{AT} = 1.1 \text{ to } 1.25
\]

Cooling surface of field winding excluding top and bottom, \(S = 2L_{mi}h_i\)

Permissible copper loss in each field coil, \(S_q_f = (2L_{mi}h_i)q_f\)

Cross section area of field coil = \(h_id_f\)

Copper area of each field coil, \(S_fh_id_f = T_f a_f\)
3. Design of DC Machines

Copper loss in each field coil,

\[ I_f^2 R_f = (a_f \delta_f)^2 \left( \frac{T_f \rho L_{mt}}{a_f} \right) = \delta_f^2 a_f T_f \rho L_{mt} = \delta_f^2 S_f h_f d_f \rho L_{mt} \]

\[ \therefore (2 L_{mt} h_f) q_f = \delta_f^2 S_f h_f d_f \rho L_{mt} \]

\[ \delta_f = \sqrt{\frac{2 L_{mt} h_f q_f}{S_f h_f d_f \rho L_{mt}}} = \sqrt{\frac{2 q_f}{S_f d_f \rho}} \]

Also,

Mmf per meter height of field winding,

\[ \frac{AT_{\phi}}{h_f} = \frac{I_f T_f}{h_f} \]

\[ = \left( a_f \delta_f \right) \left( \frac{S_f h_f d_f}{a_f} \right) \]

\[ = \delta_f S_f d_f \]

\[ = \left( \sqrt{\frac{2 q_f}{S_f d_f \rho}} \right) \left( S_f d_f \right) \]

\[ \frac{AT_{\phi}}{h_f} = \sqrt{\frac{2 q_f S_f d_f}{\rho}} \]

\[ \therefore h_f = \frac{AT_{\phi}}{\sqrt{\frac{2 q_f S_f d_f}{\rho}}} \]

Height of pole,

\[ h_p = \text{Height of field winding} + \text{Heigth of pole shoe} + \text{Space wasted in curvature} \]

\[ h_p = h_f + h_s + h_c \]

\[ h_p = h_f + (0.1 \text{ to } 0.2) h_p + (0.1 \text{ to } 0.15) r \]

3.14. Design aspects of shunt field winding for DC machines

- For small DC machine, shunt field winding with circular cross section and for large machine rectangular cross section is preferred. Varnish is the most common insulation preferred for both type of cross section.

- In shunt machine, entire winding space along the pole height is occupied, while in compound machine shunt winding occupies 80% space and series winding occupies 20% space along pole height.

- For shunt machine field current is some percentage of load current, so it is designed for more number of turns with smaller conductor cross section area.

- Large number of turns and less cross section area offers high resistance to a winding.
Mean turn length of field winding, \( L_{mt} = 2 \left( L_p + b_p \right) + 4 d_f \)

Voltage across shunt field winding, \( V_r = \left( 0.8 \text{ to } 0.85 \right) \text{ Supply voltage} \)

Voltage across each shunt field coil, \( E_f = \frac{(0.8 \text{ to } 0.85) V}{p} \)

Resistance of each field coil, \( R_f = \frac{T_f \rho L_{mt}}{a_f} \)

Also, \( R_f = \frac{E_f}{I_f} \)

\[ \therefore \frac{E_f}{I_f} = \frac{T_f \rho L_{mt}}{a_f} \]

\[ \therefore a_f = \frac{(I_f T_f) \rho L_{mt}}{E_f} = \frac{AT_f \rho L_{mt}}{E_f} \]

Height of field winding, \( h_f = h_p - \left( 0.1 \text{ to } 0.2 \right) h_p - \left( 0.1 \text{ to } 0.15 \right) r \)

Number of turns of field winding, \( T_f = \frac{S_f \left( h_f d_f \right)}{a_f} \)

Current density in field winding, \( \delta_f = \frac{I_f}{a_f} \)

Copper loss in each field coil, \( Q_f = I_f^2 R_f \)

Cooling surface of each field coil, \( S = 2L_{mt} \left( h_f + d_f \right) \)

Cooling co-efficient, \( C_f = \frac{0.14 \text{ to } 0.16}{1 + 0.1 V_a} \)

Temperature rise of field coil, \( \theta_f = \frac{Q_f C_f}{S} \)

### 3.15. Design aspects of series field winding for DC machines

- Series field winding uses rectangular conductor cross section. Paper or cotton is the most common insulation preferred for conductor.
- Mmf of series field winding at full load usually lies between 15 to 25 percentage more of armature mmf in order to compensate armature reaction.
- Series field winding carries the current same as armature current, so it is designed for less number of turns with large conductor cross section area.
- Less number of turns and large conductor cross section area offers low resistance to a winding.
3. Design of DC Machines

Mmf of series field per pole, \( AT_f = (1.15 \text{ to } 1.25)AT_a \)

Number of turns of field winding, \( T_f = \frac{AT_f}{I_f} \)

Cross section area of field winding conductor, \( a_f = \frac{I_f}{\delta_f} \)

Mean turn length of field winding, \( L_{mt} = 2(L_p + b_p) + 4d_f \)

Resistance of each field coil, \( R_f = \frac{T_f \rho L_{mt}}{a_f} \)

Height of field winding, \( h_f = \frac{T_f a_f}{s_f d_f} \)

Copper loss in each field coil, \( Q_f = I_f^2 R_f \)

Cooling surface of each field coil, \( S = 2L_{mt}(h_f + d_f) \)

Cooling co-efficient, \( C_f = \frac{0.14 \text{ to } 0.16}{1 + 0.1V_a} \)

Temperature rise of field coil, \( \theta_f = \frac{Q_f C_f}{S} \)

3.16. Design aspects of inter pole to improve commutation in DC machines

What is commutation?

- The term commutation can be defined as the change that takes place in a winding element during the period of a short circuit by a brush.
- For the explanation consider, width of the commutator bar equals to the width of the brush and commutator moves from left to right i.e. brush moves from right to left.
- Current flowing through the conductor is \( I_z \).
- At position (a), brush comes in contact with commutator bar b, hence armature current flows through bar b.
- As the armature starts to move right, at position (b) brush comes in contact with commutator bar b and bar a, hence armature current flows through bar b and bar a.
- As the brush contact area with bar a increases and with bar b decreases, contribution of armature current flowing through bar a increases and through bar b decreases. When contact area is equal for both the commutator bar, at position (c) same armature current flows through both the bars.
- As the brush contact area with bar a increases and with bar b decreases further, at position (d) current flowing through the coil 2 changes its direction and starts to flow counter clockwise.
3. Design of DC Machines

- At position (e), brush totally comes under the bar a and disconnected with the bar b, current $I_z$ flows through the coil 2 in counter clockwise and short circuit is removed.
- Hence, reversal of current or the process of commutation is done.
- Total current collected by brush $2I_z$ remains same for all brush position with commutator.
- During the period of the short circuit of an armature coil by a brush, the current in the coil must be reversed and also has to gain its full value in reverse direction.
- In practice, the current in the short circuited coil after commutation can’t reach its full value. This is because of the self-inductance of coil. The rate of chance of current is that much high, its self-inductance sets up back emf, which opposes the reversal.
- To compensate the back emf in a coil, a reversing emf can be produced by two ways by brush shifting and by using inter poles.
3. Design of DC Machines

**Inter pole design aspect**

- Interpoles are narrow poles placed between the main poles. Its winding is connected in series with the armature, because inter poles must produce fluxes that are directly proportional to the armature current.

- The armature and the inter poles mmf is affected simultaneously by the same armature current. Thus the armature flux in the commutating zone, which tends to shift the magnetic neutral axis, is neutralized by an appropriate component of inter pole flux.

\[
\text{Average reactance voltage, } E_{rav} = \frac{4T_c \alpha L I_z Z_z}{T_c}
\]

\[
\text{Maximum reactance voltage, } E_{rm} = 1.3 E_{rav}
\]

\[
\text{Maximum flux density under inter pole, } B_{gim} = \frac{E_{rm}}{LV_a}
\]

\[
\text{Mmf required for gap under inter pole, } A T_{gi} = 8,00,000 B_{gim} I_a K_{gi}
\]

\[
\text{Mmf required for inter pole, } A T_i = A T_a + A T_{gi}
\]

\[
\text{Number of turns on each inter pole, } T_i = \frac{A T_i}{I_a}
\]

\[
\text{Area of inter pole winding conductor, } a_i = \frac{I_a}{\delta_i}
\]

3.17. Design aspects of commutator and brushes in DC machines

- Commutator in DC machine rectifies alternating current of armature conductor. It is cylindrical shaped structure made of copper bars or segments.

- Commutator segments are separated by insulating material from one another and connected with armature conductor with the help of riser.

- Number of commutator segment is equal to number of coils.

- In order to avoid ionization of the skin at commutator surface, peripheral voltage gradient around commutator is limited to 3 V/mm. Hence, commutator diameter is selected with regards of peripheral speed and thickness of segment.

- Diameter of commutator is taken as 60 to 80 percentage of armature diameter. Commutator diameter varies from 62% for 350/700 V, 68% for 200/250 V and 75% for 100/125 V machine.

- Thickness of commutator should not be less than 3 mm to achieve mechanical stability under all condition of speed and temperature.

- Length of commutator depend on space required by the brushes and surface required to dissipate heat generated by commutator.

- Thickness of brush and commutator pitch are the governing factors that determines width of commutating zone and number of coils undergoing commutation at a time.
3. Design of DC Machines

- Thickness of brush should cover 2 to 3 commutator segment for good design.
- Number of brushes should be selected such that current in each brush does not carry current more than 70 A.

Commutator diameter, \( D_c = (0.6 \text{ to } 0.8)D \)

Commutator peripheral, \( V_c = \pi D_c n \)

Commutator pitch, \( \beta_c = \frac{\pi D_c}{C} \)

Current per brush arm, \( I_b = \frac{2I}{n} \)

Area of each brush, \( a_b = \frac{I_b}{\delta_b} \)

Thickness of each brush, \( t_b = (2 \text{ to } 3) \beta_c \)

Width of each brush, \( w_b = \frac{a_b}{t_b} \)

Total contact area of brush in one spindle, \( A_b = n_b w_b t_b \)

Length of commutator, \( L_c = n_b (w_b + C_b) + C_1 + C_2 + C_3 \)

Brush contact loss, \( P_{bc} = 2V_b I_a \)

Brush friction loss, \( P_{bf} = \mu P_b p A_b V_c \)

Total loss at commutator, \( P_c = P_{bc} + P_{bf} \)

Barrel surface of commutator, \( S_c = \pi D_c L_c \)

Where,
- \( C_b \) = Clearance between brushes (mm)
- \( C_1 \) = Clearance for staggering of brushes (mm)
- \( C_2 \) = Clearance for end play (mm)
- \( C_3 \) = Clearance for risers (mm)
- \( V_b \) = Per brush contact voltage drop (V)
- \( \mu \) = Friction co-efficient
- \( P_b \) = Brush contact pressure (N/m²)